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Problems set # 2 **Physics 307** September 14, 2016

Complex Analysis II

1. Calculate the following contour integrals: *(i)* $\int_C \Im\mathrm{m}\,(z)\,dz$, where C is the upper half of the unit circle, traced counterclockwise; *(ii)* $\oint_C z^*\,dz$, where C is the unit circle traced counterclockwise; *(iii)* $\int_C \{[\Re\mathrm{e}(z)]^2 - [\Im\mathrm{m}(z)]^2\} dz$, where C is the straight line from 0 to i; *(iv)* $\int_C (z+3)\,dz$, where C is the upper half of the circle of radius 2 centered at the origin.

2. Evaluate the following integrals using the Cauchy integral theorem: *(i)* $\oint_C \frac{z^2}{z-3}\,dz$, where the contour C is the circle $|z| = 2$; *(ii)* $\oint_C ze^{-z}\,dz$, where the contour C is the circle $|z| = 10$; *(iii)* $\oint_C \frac{1}{z^2+2z+2}\,dz$, where the contour C is the circle $|z| = 1$; *(iv)* $\oint_C (z^3+3)\,dz$, where the contour C is the circle of radius 2 centered at the origin; *(v)* $\oint_C e^{1/z}\,dz$, where C is the circle of radius 2 centered at $3-2i$.

3. Let f be a function which is analytic everywhere in a region G except at one point $z_0 \in G$. Suppose there is a number M and an open ball D around z_0, such that for all $z \in D$, $|f(z)| \leq M$. Use corollary 1.5 and the ML-inequality to prove that $\oint_C f(z)\,dz = 0$ for any simple closed curve C in G containing the point z_0.

4. Find the following integrals: *(i)* $\oint_C \frac{e^z}{z^2}\,dz$, where C is a the cricle centered at the origin with radius 1; *(ii)* $\oint_C \frac{1}{z^2-2i}\,dz$, where C is the circle centered at $(1,0)$ with radius 2; *(iii)* $\oint_C \frac{1}{z^2(z^2+16)}dz$, where C is the cricle centered at the origin with radius 2.

5. Find the Taylor series for the following functions around the given points and determine their domains of convergence: *(i)* e^z around $z_0 = 1$; *(ii)* $\frac{\sin z}{z}$ around $z_0 = 1$; *(iii)* $\frac{1}{(1-z)}$ around $z_0 = i$.