| Problems set # 12 | **Physics 307** | December 7, 2016 |

Partial Differential Equations III

1. Solve Laplace's equation inside a rectangle $0 \leq x \leq L$, $0 \leq y \leq H$, with the following boundary conditions: $u_x(0, y) = 0$, $u_x(L, y) = 0$, $u(x, 0) = 0$, $u(x, H) = f(x)$.

2. Solve the Laplace's equation inside a semicircle of radius a ($0 < r < a$, $0 < \theta < \pi$) subject to the boundary conditions: $u = 0$ on the diameter and $u(a, \theta) = g(\theta)$.

3. Solve the Laplace's equation inside a 90° sector of a circular annulus ($a < r < b$, $0 < \theta < \pi/2$) subject to the boundary conditions: $u(r, 0) = 0$, $u(r, \pi/2) = 0$, $u(a, \theta) = 0$, $u(b, \theta) = f(\theta)$.

4. A sphere of radius a is held at zero potential except for a strip around its mid-section which is held at the constant potential V. The strip exists between the polar angles of $\pi/2 - \alpha$ and $\pi/2 + \alpha$ where α is some constant angle. Find the potential $\Phi(r, \theta, \phi)$ at any point inside the sphere. [*Hint:* Because the problem contains no charge, it simplifies down to solving the Laplace's equation $\nabla^2 \Phi = 0$, applying the boundary condition.]

5. Two concentric spheres of radii a and b are centered at the origin. The inner sphere (radius a) is held at zero potential and the outer sphere is held at the potential $V \cos\theta$ where θ is the polar angle in spherical coordinates. Find the potential everywhere in the region between the spheres in terms of Legendre Polynomials.

6. The surface of a hollow conducting sphere of inner radius a is divided into an even number of equal segments by a set of planes; their common line of intersection is the z-axis and they are distributed uniformly in the angle ϕ. (The segments are like the skin on wedges of an apple, or the earth's surface between successive meridians of longitude.) The segments are kept at fixed potentials $\pm V$, alternately. Set up a series representation for the potential inside the sphere for the general case of $2n$ segments, and carry the calculation of the coefficients in the series far enough to determine exactly which coefficients are different from zero. For the nonvanishing terms, exhibit the coefficients as an integral over $\cos\theta$.

7. Find a series solution for the potential anywhere inside a cylinder satisfying the following boundary conditions: *(i)* $\Phi(r, \theta, 0) = 0$, $\Phi(r, \theta, b) = V(r, \theta)$, $\Phi(a, \theta, z) = 0$; $0 \leq r \leq a$, $0 \leq \theta < 2\pi$, $0 \leq z \leq b$. Comment on the particular case $\Phi(r, \theta, b) = V(r)$. *(ii)* $\Phi(r, \theta, b) = V(r)\sin\theta$, $\Phi(r, \theta, -b) = -V(r)\sin\theta$, $\Phi(a, \theta, z) = 0$; $0 \leq r \leq a$, $0 \leq \theta < 2\pi$, $-b \leq z \leq b$.