Modern Physics

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Lesson II
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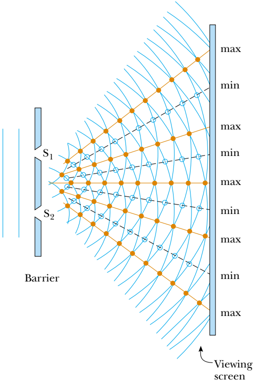
Young's interferometer

- Monochromatic light from a single concentrated source illuminates a barrier containing two small openings
- Light emerging from two slits is projected onto distant screen
- Distinctly ☞ we observe light deviates from straight-line path and enters region that would otherwise be shadowed



Harmonic Oscillator

- Light of a given color is intrinsically an oscillating system
- We have seen that different colors of light can be associated with different frequencies f
- Each color of light is identified with: certain time period $T = f^{-1}$ or wavelength $\lambda = cT$
- Light is described by an amplitude

$$A(t) = A_0 \cos(\omega t) = A_0 \cos(2\pi f t) \tag{1}$$

- Light propagates between two points in space by having its amplitude travel over all available paths and while travelling oscillates with frequency f
- What you see and can measure is the square of that amplitude
- For double slit experiment ☞ there is constant level of brightness
- BUT light has amplitude varying harmonically with time

- Rate of energy flow per unit area

$$S(t) = A^2(t) = A_0^2 \cos^2(\omega t) \tag{2}$$

- At optical frequencies S is extremely rapidly varying function of t its instantaneous value would be impractical quantity to measure
- This suggests that we employ average procedure
- For visible light:
  - wavelength $\approx 6 \times 10^{-7}$ m
  - frequency $\approx 5 \times 10^{14}$ s^{-1}
  - period $\approx 2 \times 10^{-15}$ s
- If time resolution of eye is milliseconds
  what we see is average of tens of millions of cycles
- Intensity we see is the long time average of many periods

$$I = \langle S(t) \rangle_t = \langle A^2(t) \rangle_t = \frac{A_0^2}{2} \tag{3}$$

If only slit 1 is open ...

1. Arrange apparatus so that amplitude at slit is

$$A_1(t) = A_0 \cos(\omega t) \tag{4}$$

2. Amplitude on screen at given time t is original amplitude at slit 1 delayed by time it takes light to go from slit to screen

$$\mathcal{A}_1(t) = A_0 \cos\left(\omega t - \frac{2\pi r_1/c}{T}\right) = A_0 \cos\left(\omega t - \frac{2\pi r_1}{\lambda}\right) \tag{5}$$

   - Amplitude oscillates with f ☞ same color of light at screen and slit
   - Only difference is time independent term ☞ starting angle
   - Signal varies so rapidly that sensors can only see the time average
   - Starting angle (a.k.a. phase) is not detectable

3. Phase shift is only change as you move to different parts of screen
4. Intensity at screen is uniform

If only slit 2 is open ...

1. Similar situation
2. Since the two slits are located symmetrically relative to source amplitude at slit 2 is same as that of slit 1
3. Amplitude at screen from slit 2 alone would be

$$\mathcal{A}_2(t) = A_0 \cos[\omega(t - r_2/c)] \tag{6}$$

4. For general point on screen ☞ r_1 and r_2 will be different
5. Illumination again is uniform and same color as original light

Intensity on screen for only one slit

$$\mathcal{I}_1 = \langle \mathcal{A}_1^2(t) \rangle_t = \frac{A_0^2}{2} = I_1 \tag{7}$$

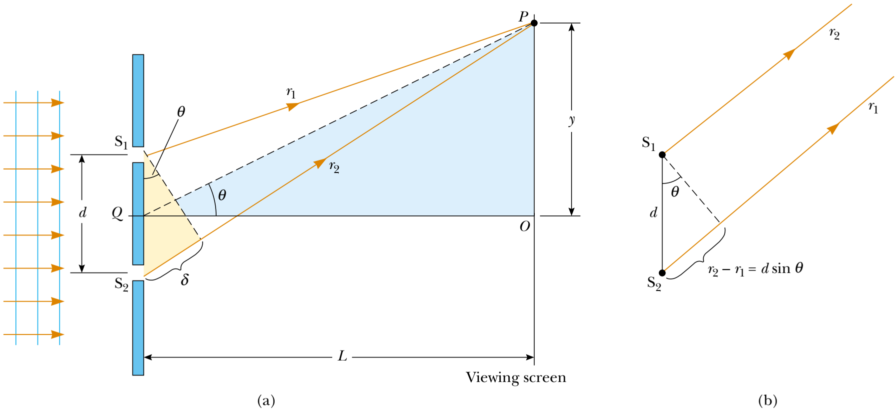
What happens when both slits are open? ☞ Superposition Principle

$$\begin{aligned} \mathcal{A}_{\rm tot} &= \mathcal{A}_1 + \mathcal{A}_2 \\ &= A_0\{\cos[\omega(t - r_1/c)] + \cos[\omega(t - r_2/c)]\} \\ &= 2A_0 \cos\left(\omega \frac{r_2 - r_1}{2c}\right) \cos\left[\omega\left(t - \frac{r_1 + r_2}{2c}\right)\right] \end{aligned} \tag{8}$$

amplitude at screen has position dependent amplitude

$$2A_0 \cos\left[\omega\left(\frac{r_2 - r_1}{2c}\right)\right]$$

Aproximations



$$d \ll L \wedge \lambda \ll d \Rightarrow \theta \approx \sin\theta \approx \tan\theta \Rightarrow \delta(y)/d \approx y/L$$

Intensity

$$\mathcal{I}_{\rm tot} = 4\mathcal{I}_1 \cos^2\left(\frac{\omega\,\delta(y)}{2c}\right) = 4\mathcal{I}_1 \cos^2\left(\frac{y\omega d}{2cL}\right) \tag{9}$$

- Bright fringes measured from O are @

$$y_{\rm bright} = \frac{\lambda L}{d} m \qquad m = 0, \pm 1, \pm 2, \cdots \tag{10}$$

  m ☞ order number
  when $\delta = m\lambda$ ☞ constructive interference

- Dark fringes measured from O are @

$$y_{\rm dark} = \frac{\lambda L}{d}(m + \tfrac{1}{2}) \qquad m = 0, \pm 1, \pm 2, \cdots \tag{11}$$

  when δ is odd multiple of $\lambda/2$ ☞ two waves arriving at point P are out of phase by π and give rise to destructive interference

1. We know how to construct amplitude for light with given frequency
2. What do you do if you do not have monochromatic light?
3. For any form of light ☞ treat it as superposition of several colors
   - Evaluate what happens for each frequency
   - add the amplitudes and then squared them
4. long time average mixed frequency terms in square drop out

$$\langle \mathcal{A}_{\omega_i} \mathcal{A}_{\omega_j} \rangle_t = 0 \; \forall \; \omega_i \neq \omega_j$$

$$\begin{aligned} \mathcal{I}_{\rm tot} &= \langle (\mathcal{A}_{\omega_1} + \mathcal{A}_{\omega_2} + \cdots + \mathcal{A}_{\omega_n})^2 \rangle_t \\ &= \langle \mathcal{A}^2_{\omega_1} \rangle_t + \langle \mathcal{A}^2_{\omega_2} \rangle_t + \cdots + \langle \mathcal{A}^2_{\omega_n} \rangle_t \\ &= \mathcal{I}_{\omega_1} + \mathcal{I}_{\omega_2} + \cdots + \mathcal{I}_{\omega_n} \, . \end{aligned} \quad (12)$$

This translates into the statement that you have heard since childhood:

light is made up of individual colors

Maxwell's Equations

1. All known laws of electricity and magnetism are summarize in

$$\vec{\nabla} \cdot \vec{E}(\vec{r}, t) = \frac{1}{\epsilon_0} \rho(\vec{r}, t) \tag{13}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}(\vec{r}, t)}{\partial t} \tag{14}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}, t) = 0 \tag{15}$$

$$\vec{\nabla} \times \vec{B}(\vec{r}, t) = \mu_0 \vec{j}(\vec{r}, t) + \mu_0 \epsilon_0 \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} \tag{16}$$

and associated force law

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{17}$$

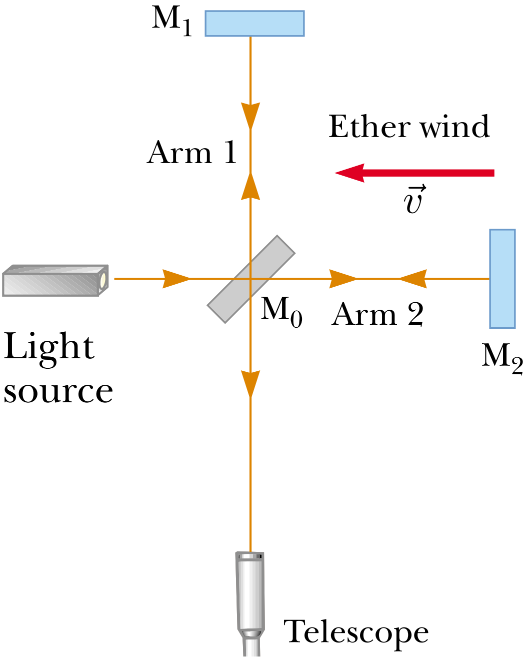
2. Young's amplitude ☞ special combination of $\vec{E}$ and $\vec{B}$
3. Fields can be measured ☞ but still too difficult at optical f

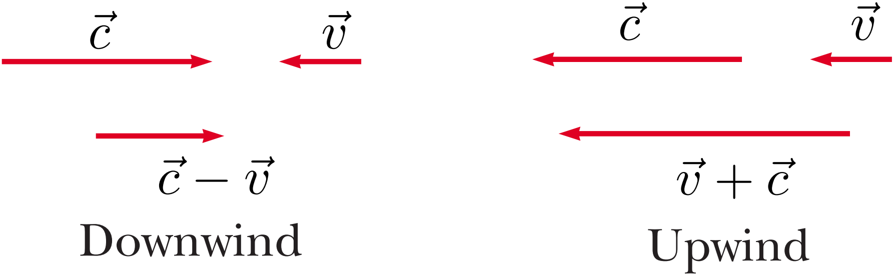
Whatcha talkin' bout Willis

- Like any system of forces
  ☞ Maxwell equations must obey Galilean invariance
  or we would be able to use electromagnetic phenomena
  to determine velocity in space
- Careful dimensional analysis ☞ $c = (\mu_0 \epsilon_0)^{-1/2}$
- If Maxwell's equations and associated force law are correct
  fundamental dimensional constants must be same in all frames
  speed of changes in EM field must be same to all observers
- Since Maxwell's equations are not Galilean invariant
  velocity could be measured and light could be used to do it
- There should be some preferred state of uniform motion
  in which Maxwell's equations are true as written
  in this frame measured speed of light would be ☞ $c = (\mu_0 \epsilon_0)^{-1/2}$

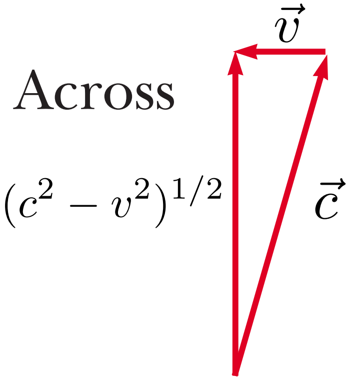
The absolute reference frame

- Scientists from 1800's believed in all notions of classical physics
- It was normal to assume that all waves traveled through mediums
- Air is clearly not the required medium for propagation of light because EM waves traveled through space to get to Earth
- To solve the problem
  ☞ it was assumed there is an æther which propagates light waves
- Æther ☞ assumed to be everywhere and unaffected by matter
- Æther could be used to determine absolute reference frame (with the help of observing how light propagates through it)
- Experiment designed to detect small changes in speed of light with motion of observer through æther was performed by Michelson and Morley


M1
Arm 1
Ether wind
v
Light source
M0
Arm 2
M2
Telescope



$$
\begin{aligned}
t_1 &= \frac{L_1}{c - v} + \frac{L_1}{c + v} = \frac{2cL_1}{c^2 - v^2} \\
&= \frac{2L_1}{c} \frac{1}{1 - v^2/c^2} = \frac{2L_1}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1}
\end{aligned}
\tag{18}
$$



$$t_2 = \frac{2L_2}{\sqrt{c^2 - v^2}} = \frac{2L_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}} \tag{19}$$

Using $1/(1-x) = \sum_{n=0}^{\infty} x^n$

$$t_1 \approx \frac{2L_1}{c}\left(1 + \frac{v^2}{c^2}\right) \tag{20}$$

Additionally

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots \tag{21}$$

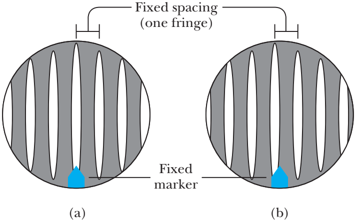
taking $m = -1/2$ and $x = -v^2/c^2$

$$t_2 \approx \frac{2L_2}{c}\left(1 + \frac{v^2}{2c^2}\right) = \frac{2L_2}{c}\left(1 + \frac{v^2}{2c^2}\right) \tag{22}$$

Earth's orbit around sun ☞ $v/c \approx 10^{-4}$
Light rays recombine at the viewer separated by

$$\Delta t = t_1 - t_2 \approx \frac{2}{c}\left(L_1 - L_2 + \frac{L_1 v^2}{c^2} - \frac{L_2 v^2}{2c^2}\right) \tag{23}$$

- Interferometer is adjusted for parallel fringes
  and telescope is focused on one of these fringes
- Time difference between the two light beams
  gives rise to a phase difference between the beams
  producing interference fringe pattern when combined @ telescope
- Different pattern should be detected
  by rotating the interferometer through $\pi/2$ in a horizontal plane



$$t_1' = \frac{2L_1}{c}\left(1 + \frac{v^2}{2c^2}\right) \text{ and } t_2' = \frac{2L_2}{c}\left(1 + \frac{v^2}{c^2}\right) \tag{24}$$

$$\Delta t' = t_1' - t_2' = \frac{2}{c}(L_1 - L_2) + \frac{v^2}{c^3}(L_1 - 2L_2) \tag{25}$$

time change produced by rotating the apparatus

$$\begin{aligned} \Delta t - \Delta t' &= \frac{2}{c}(L_1 - L_2) + \frac{2v^2}{c^3}\left(L_1 - \frac{L_2}{2}\right) \\ &- \left[\frac{2}{c}(L_1 - L_2) + \frac{v^2}{c^3}(L_1 - 2L_2)\right] \\ &= \frac{v^2}{c^3}(L_1 + L_2) \end{aligned} \tag{26}$$

1. Path difference corresponding to this time difference is

$$\delta = \frac{v^2}{c^2}(L_1 + L_2) \tag{27}$$

2. Corresponding fringe shift

$$\text{Shift} = \frac{\delta}{\lambda} = \frac{v^2}{\lambda c^2}(L_1 + L_2) \tag{28}$$

3. Michelson and Morley experiment $L = L_1 = L_2 \simeq 11$ m
4. Taking $v =$ speed of Earth about the Sun ☞ $\delta \simeq 2.2 \times 10^{-7}$ m
5. Using light of 500 nm ☞ find a fringe shift for rotation through $\pi/2$

$$\text{Shift} = \frac{\delta}{\lambda} \approx 0.40 \tag{29}$$

6. Instrument precision

   ☞ capability of detecting shift as small as 0.01 fringe
7. NO shift detected in fringe pattern
8. Conclusion:

   one cannot detect motion of Earth with respect to æther

FitzGerald contraction

- In 1892 Fitzgerald proposed that
  object moving through æther wind with velocity v
  experiences contraction in direction of æther wind of $\sqrt{1 - v^2/c^2}$
- L_1 is contracted to $L_1\sqrt{1 - v^2/c^2}$ yielding $t_1 = t_2$ when $L_1 = L_2$
  ☞ potentially explaining results of Michelson-Morley experiment
- Even under this assumption
  it turns out that Michelson-Morley apparatus with unequal arms
  will exhibit pattern shift over 6 month period
  as Earth changes direction in its orbit around the Sun
- In 1932 Kennedy and Thorndike performed such an experiment:
  they detected NO such shift

- Another suggestion to explain negative result of M&M experiment Earth "drags æther along with it" as it orbits around Sun
- Idea is rejected because of stellar aberration