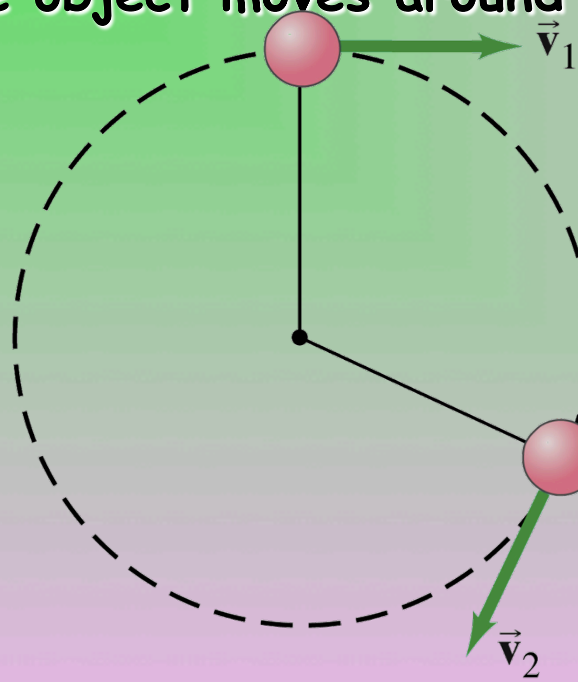

Circular motion
Luis Anchordoqui

# Kinematics of Uniform Circular Motion

An object that moves in a circle at constant speed
is said to experience uniform circular motion

The magnitude of the velocity remains constant in this case
but the direction of the velocity continuously changes
as the object moves around the circle




An object revolving in a circle is continuously accelerating
even when the speed remains constant

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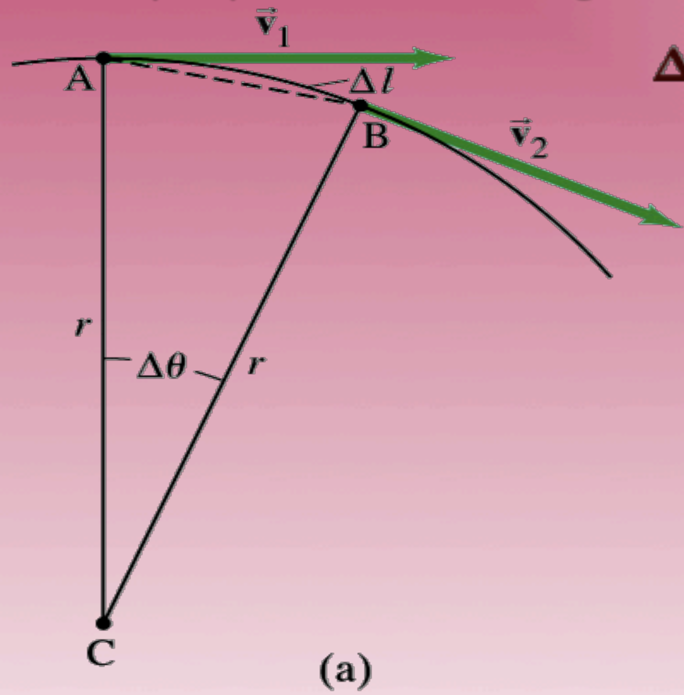
# Kinematics of Uniform Circular Motion (cont'd)

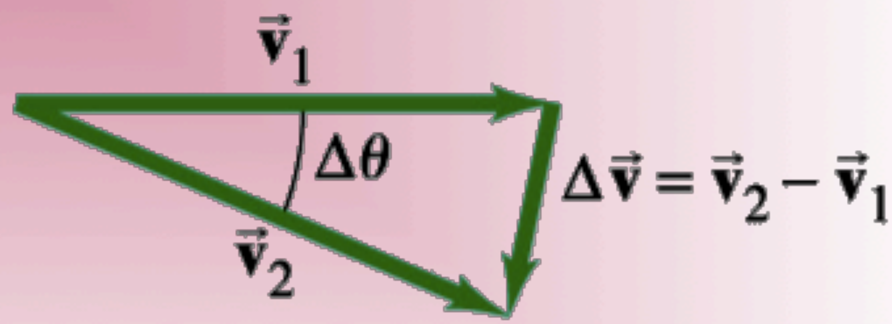
The acceleration is defined as $\vec{a} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ for a short interval of time

We will eventually consider the situation when $\Delta t$ approaches to zero to obtain the instantaneous acceleration

For the purposes of making a clear drawing consider a non-zero time interval




$\Delta\vec{v}$ will be essentially perpendicular to $\vec{v}_1$ & $\vec{v}_2$ pointing to the center of the circle




If we let $\Delta t$ be very small ➔ $\Delta l$ and $\Delta\Theta$ are also very small and $\vec{v}_1$ will be almost parallel to $\vec{v}_2$

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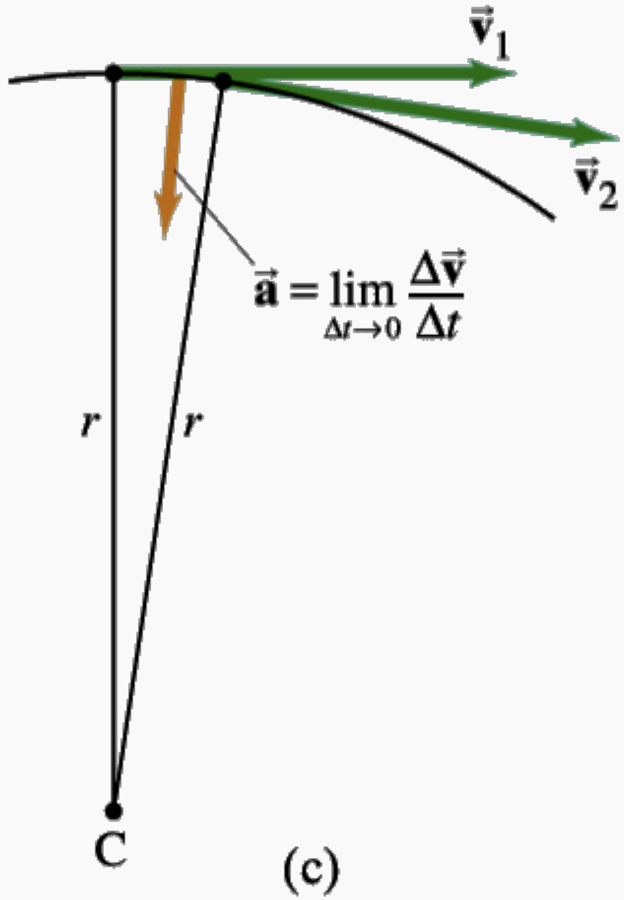
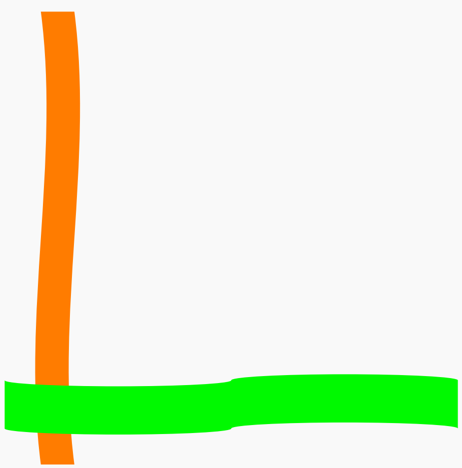
# Kinematics of Uniform Circular Motion (cont'd)

$\vec{a}$ must too point to the center of the circle



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# Kinematics of Uniform Circular Motion (cont'd)

The magnitude of the velocity is not changing we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta l}{r}$$

This is an exact equality when $\Delta t$ approaches zero

Let $\Delta t$ approach zero and solve for $\Delta v$

$$\Delta v = \frac{v}{r} \Delta l$$

To get the centripetal acceleration we divide by $\Delta t$

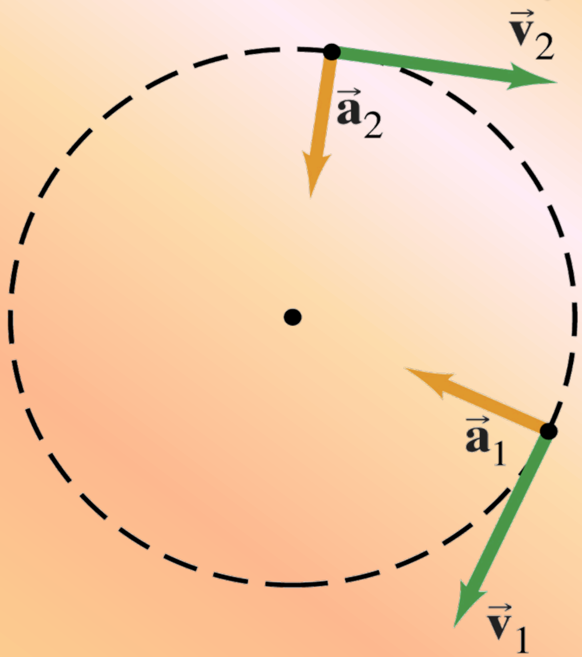
$$a_R = \frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta l}{\Delta t}$$

$\frac{\Delta l}{\Delta t}$ is just the linear speed

$$a_R = \frac{v^2}{r}$$

# Kinematics of Uniform Circular Motion (cont'd)

The acceleration vector points towards the center of the circle
The velocity vector always points in the direction of motion




Circular motion is often described in terms of the frequency (f)

the number of revolutions per second

The period of an object (T) revolving in a circle is the time required for one complete revolution

$$T = \frac{1}{f}$$

For an object revolving in a circle at constant speed we have

$$v = \frac{2\pi r}{T}$$

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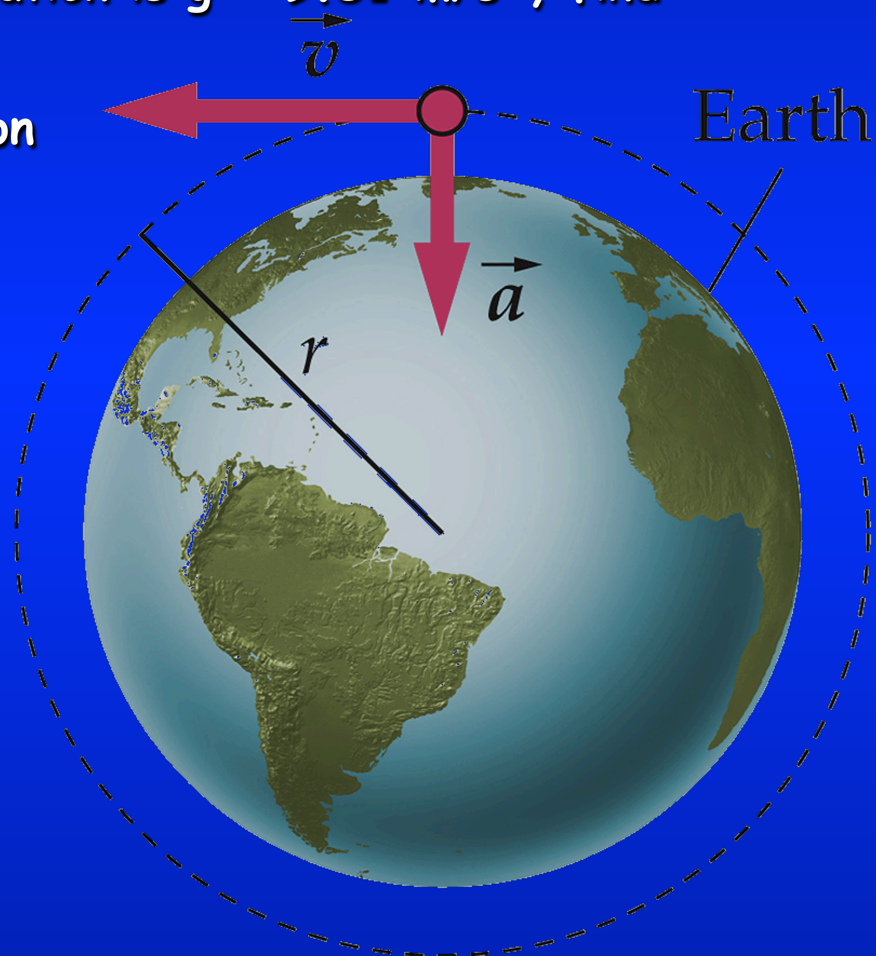
# A Satellite's Motion

A satellite moves at constant speed in a circular orbit about the center of Earth near the surface of Earth.

If the magnitude of its acceleration is $g = 9.81\ m/s^2$, find

(a) its speed and

(b) the time for one complete revolution




$v = 7.91$ km/s

$T = 5060$ s $= 84.3$ min

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# Moon's Centripetal Acceleration

The Moon's nearly circular orbit about the Earth has a radius of about 384,000km and a period T of 27.3 days.

Determine the acceleration of the Moon towards the Earth.

$$a = 2.78 \times 10^{-4} g$$

Earth as seen from Apollo 11 orbiting the Moon on July 16, 1969 (NASA)

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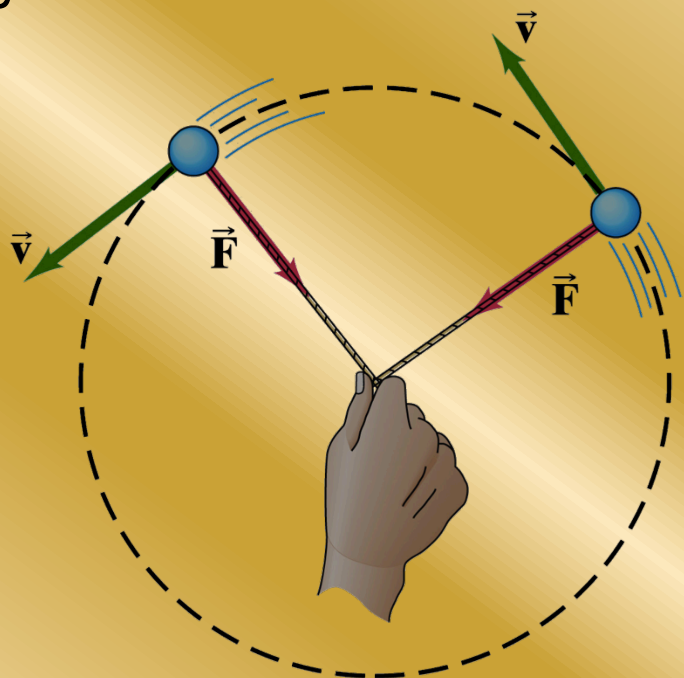
# Dynamics of Uniform Circular Motion

According to Newton's second law

an object that is accelerating must have a net force acting on it

An object moving in a circle such as a ball on the end of a string




must therefore have a force applied to it to keep it moving on that circle

The magnitude of the force can be calculated using Newton's second law for the radial component

$$\sum F_R = m a_R = m \frac{v^2}{r}$$

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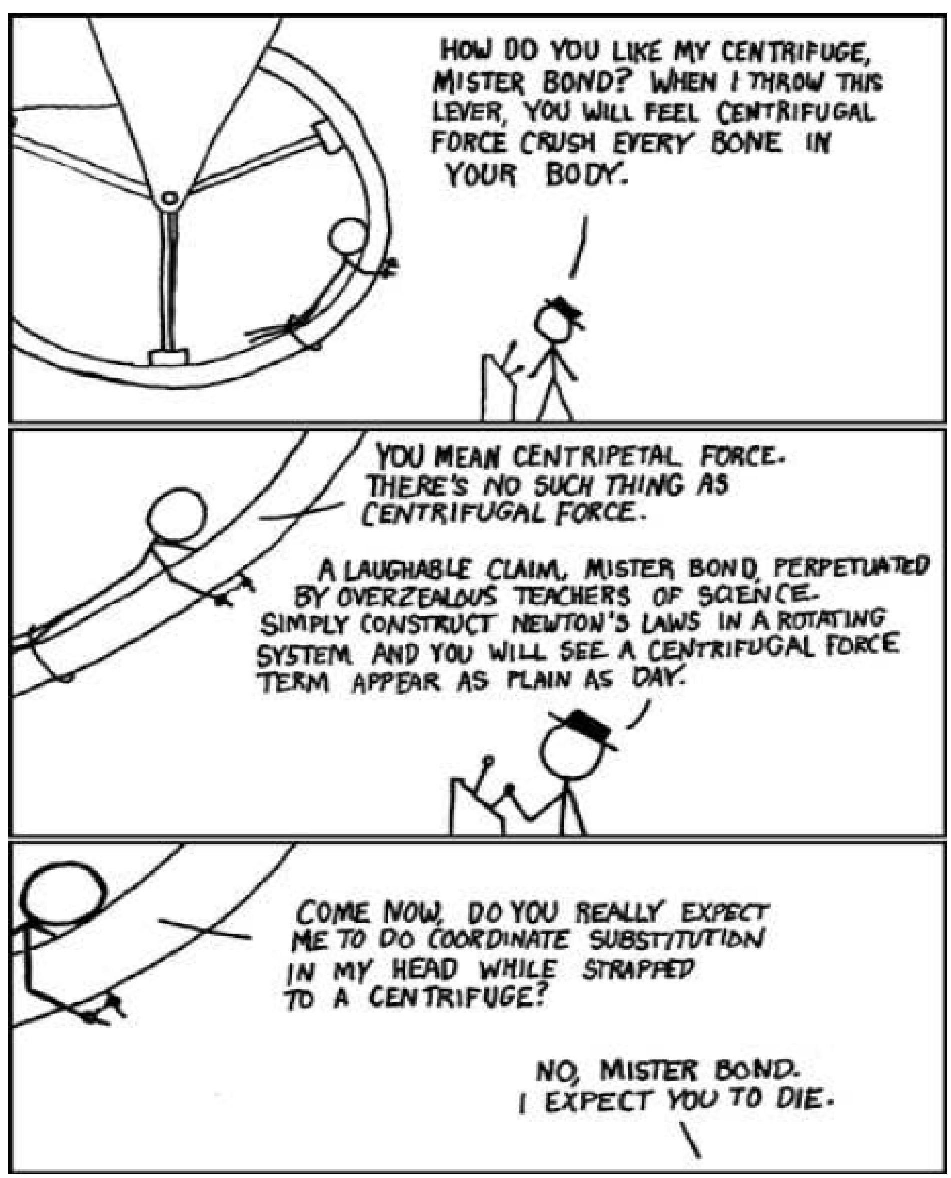
# Dynamics of Uniform Circular Motion (cont'd)

**There is a common misconception that an object moving in a circle has an outward force acting on it: centrifugal ("center feeling") force**




**There is no outward force on the revolving object**

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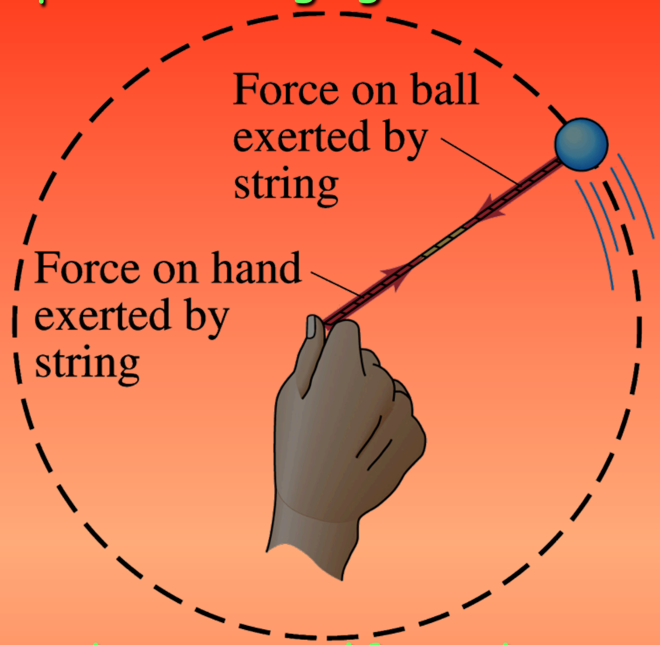
# Dynamics of Uniform Circular Motion (cont'd)

Consider for example a person swinging a ball on the end of a string around her head




If you ever done this yourself you know that you feel a force pulling outward on your hand

The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmited along the string to your hand

To keep the ball moving on a circle you pull inwardly on the string and the string exerts this force on the ball

The ball exerts an equal and opposite force on the string (Newton's third law) and this is the outward force your hand feels

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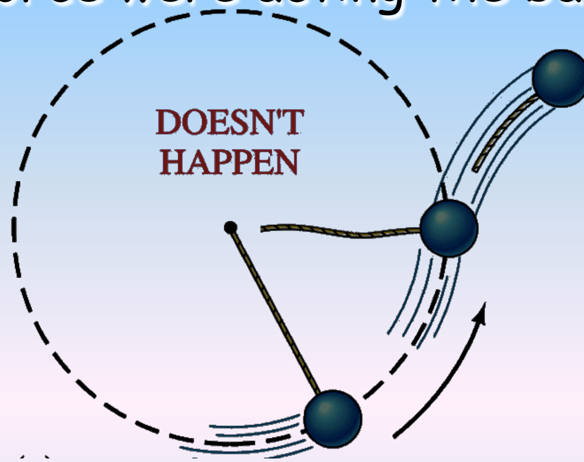
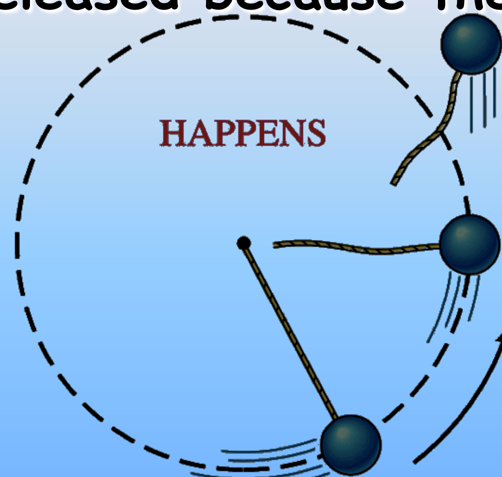
# Dynamics of Uniform Circular Motion (cont'd)

To see even more convincing evidence that a "centrifugal force" does not act on the ball ➔ consider what happens when you let go of the string

If a centrifugal force were acting the ball would fly outward




**The ball flies off tangentially in the direction of the velocity it had at the moment it was released because the inward force no longer acts**




**Try it and see!**

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# Dynamics of Uniform Circular Motion (cont'd)

Sparks fly in straight lines tangentially from the edge of a rotating grinding wheel

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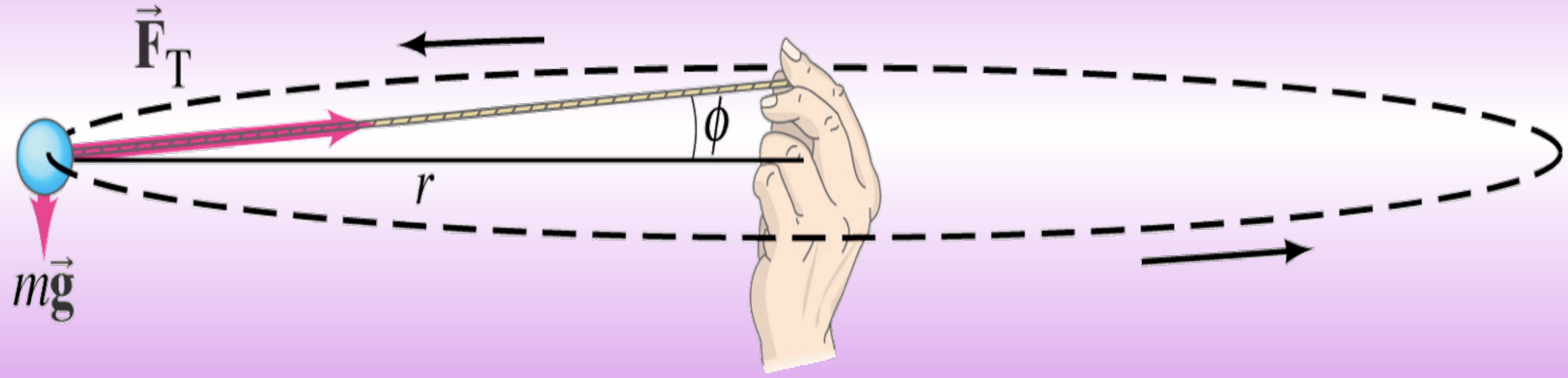
# Revolving ball (horizontal circle)

Estimate the force a person must exert on a string attached to a 0.15 kg ball to make the ball revolve in a horizontal circle of radius 0.6 m.

The ball makes 2 revolutions per second

The ball's weight complicates matters and makes it impossible to revolve the ball with a cord perfectly horizontal.




**We assume the weight is small and set Φ approximately zero**

$$F_T = 14 \text{ N}$$

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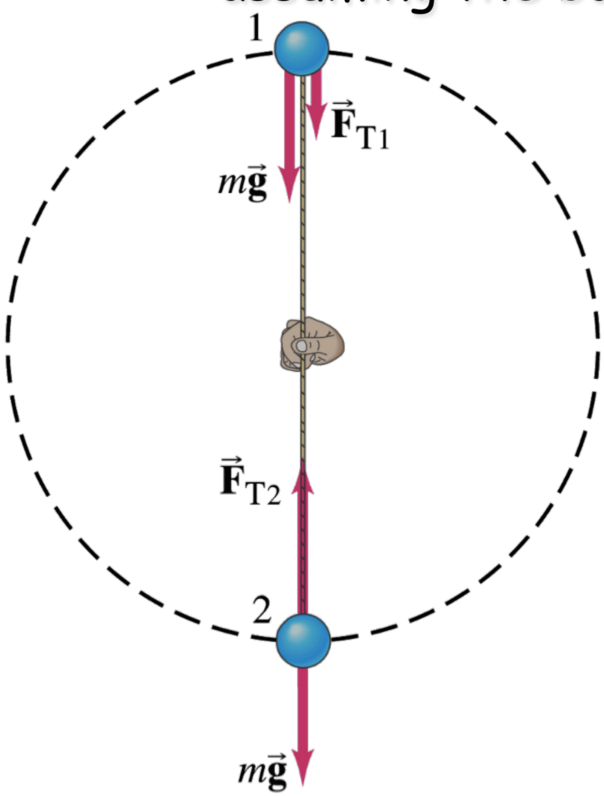
# Revolving ball (vertical circle)

A 0.15 kg ball on the end of a 1 m long cord (of negligible mass) is swung in a vertical circle.

(a) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle

(b) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice speed of part




$v_{top} = 3.13$ m/s

$F_{T2} = 7.34$ N

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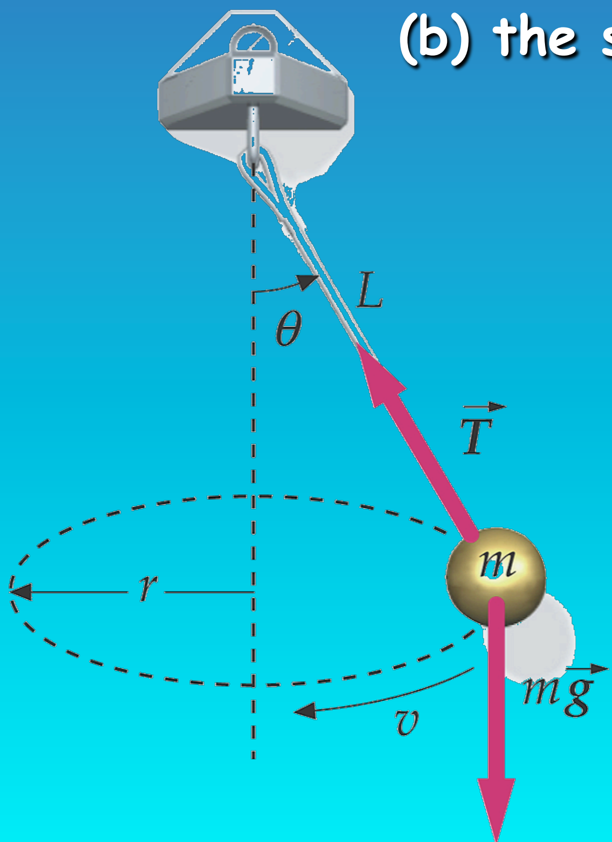
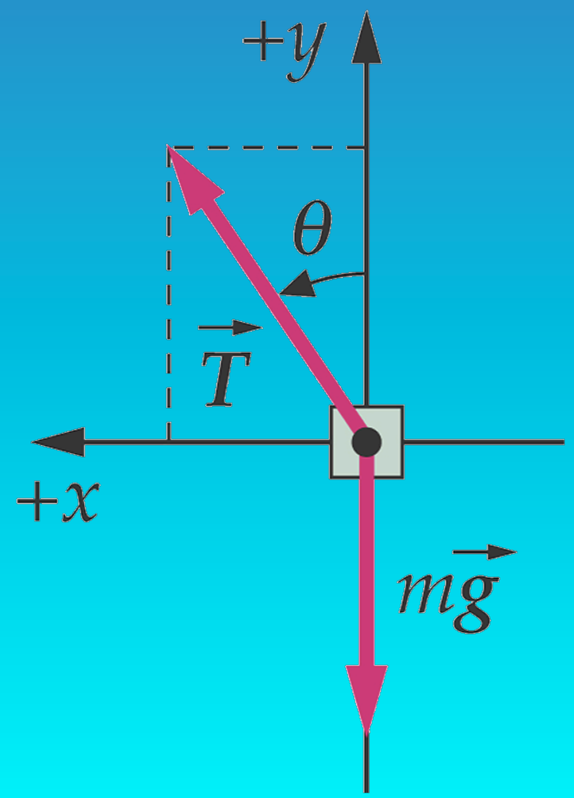
# Tetherball

A tetherball of mass m is suspended from a length rope and travels at constant speed v in a horizontal circle of radius r

Find (a) the tension of the rope $T = mg/\cos\theta$

(b) the speed of the ball $v=(r\ g\ \tan\theta)^{1/2}$

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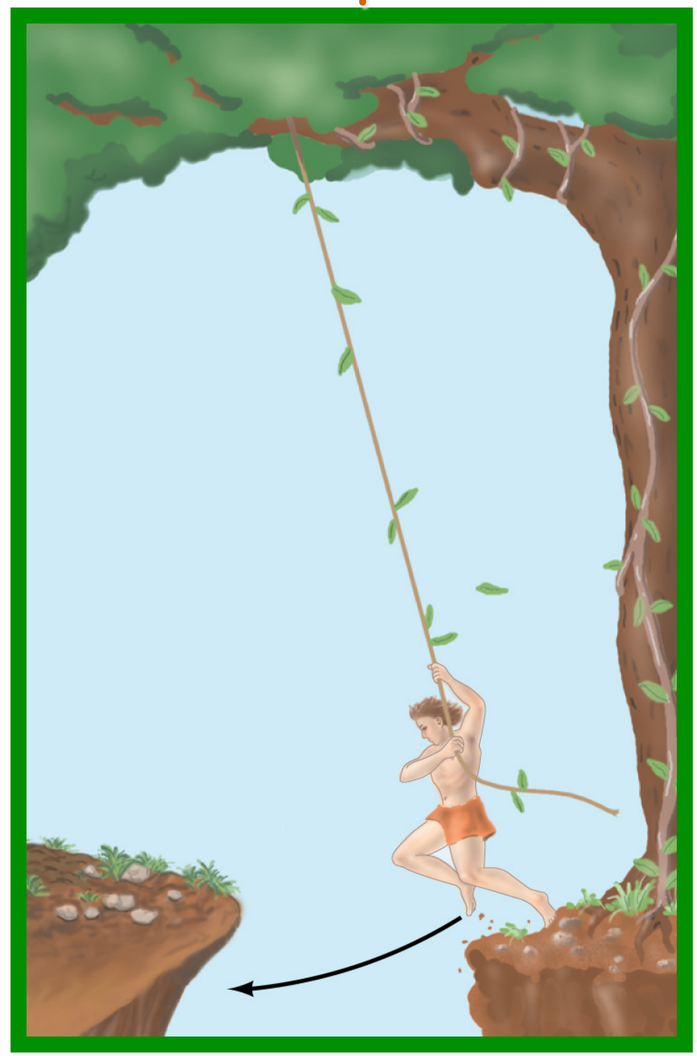
Tarzan plans to cross a gorge by swinging in an arc from a hanging vine.

If his arms are capable of exerting a force of 1400 N on the vine, what is the maximum speed he can tolerate at the lowest point of his swing?



His mass is 80 kg and the vine is 5.5m long

$v_{max}$ = 6.5 m/s

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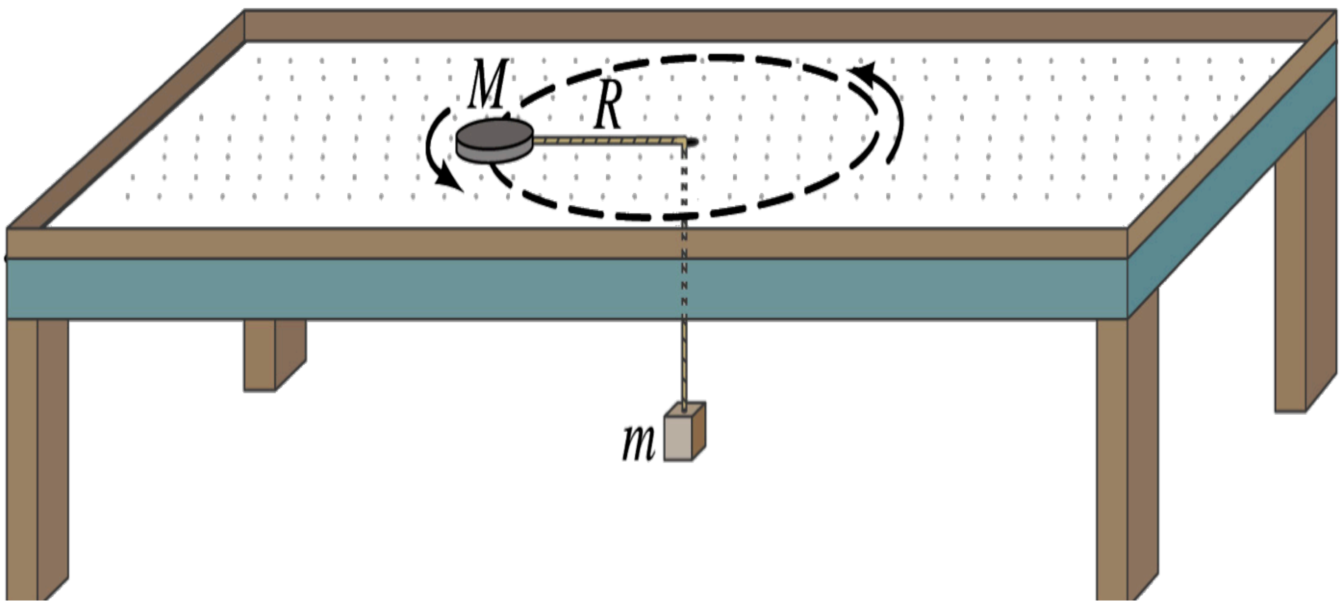
A flat puck (mass M) is rotated in a circle on a frictionaless air-hockey tabletop, and is held in this orbit by a light cord connected to a dangling block (mass m) through a central hole.




Show that the speed of the puck is given by $v = (mgR/M)^{\frac{1}{2}}$

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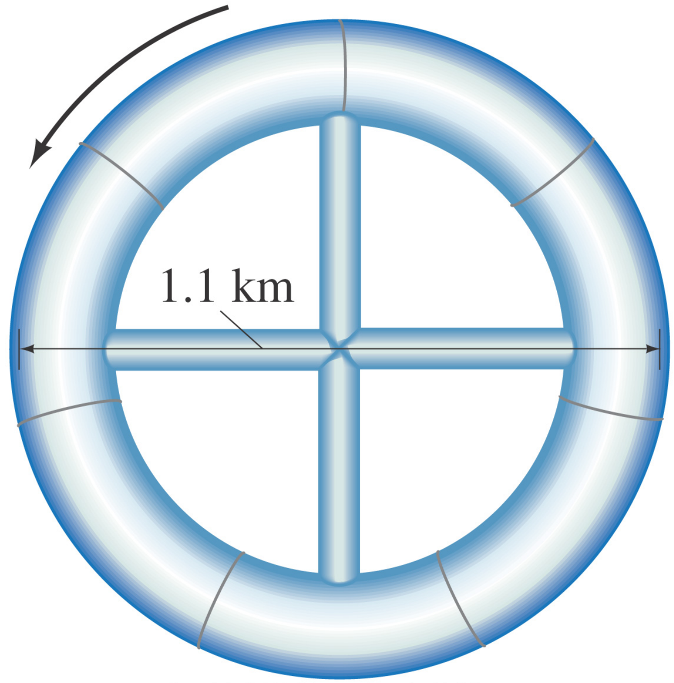
A projected space station consists of a circular tube that will rotate about its center (like a tubular bicycle tire)




The circle formed by the tube has a diameter of about 1.1 km.

What must be the rotation speed (revolutions per day) if an effect equal to gravity at the surface of the earth is to be felt?

**$1.8 \times 10^3$ rev/d**

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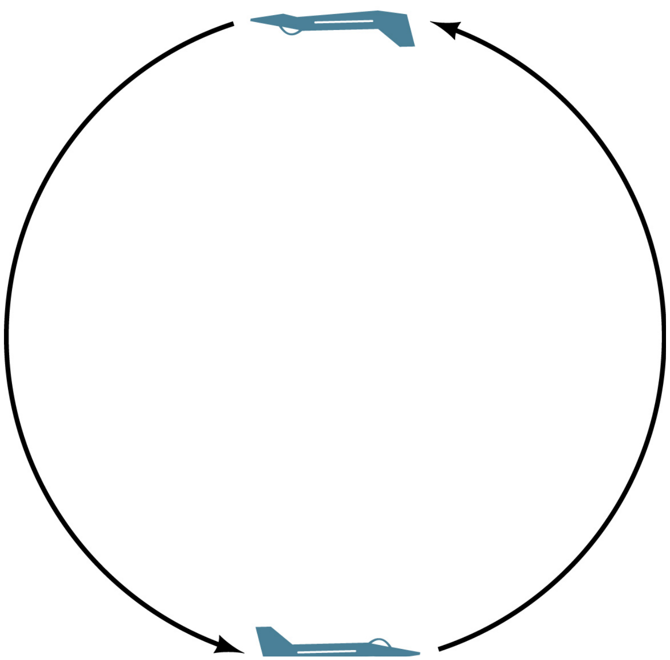
**A jet pilot takes his aircraft in a vertical loop**

(a) If the jet is moving at a speed of 1300 km/h at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at lowest point does not exceed 6.0 g's

(b) Calculate the 78 kg pilot's effective weight (the force with which the seat pushes up on him at the bottom of the circle)

(c) at the top of the circle (assume the same speed)



$$r = 2.2 \times 10^3 \text{ m}$$

$$F_N = 5.4 \times 10^3 \text{ N}$$

$$F_N = 3.8 \times 10^3 \text{ N}$$

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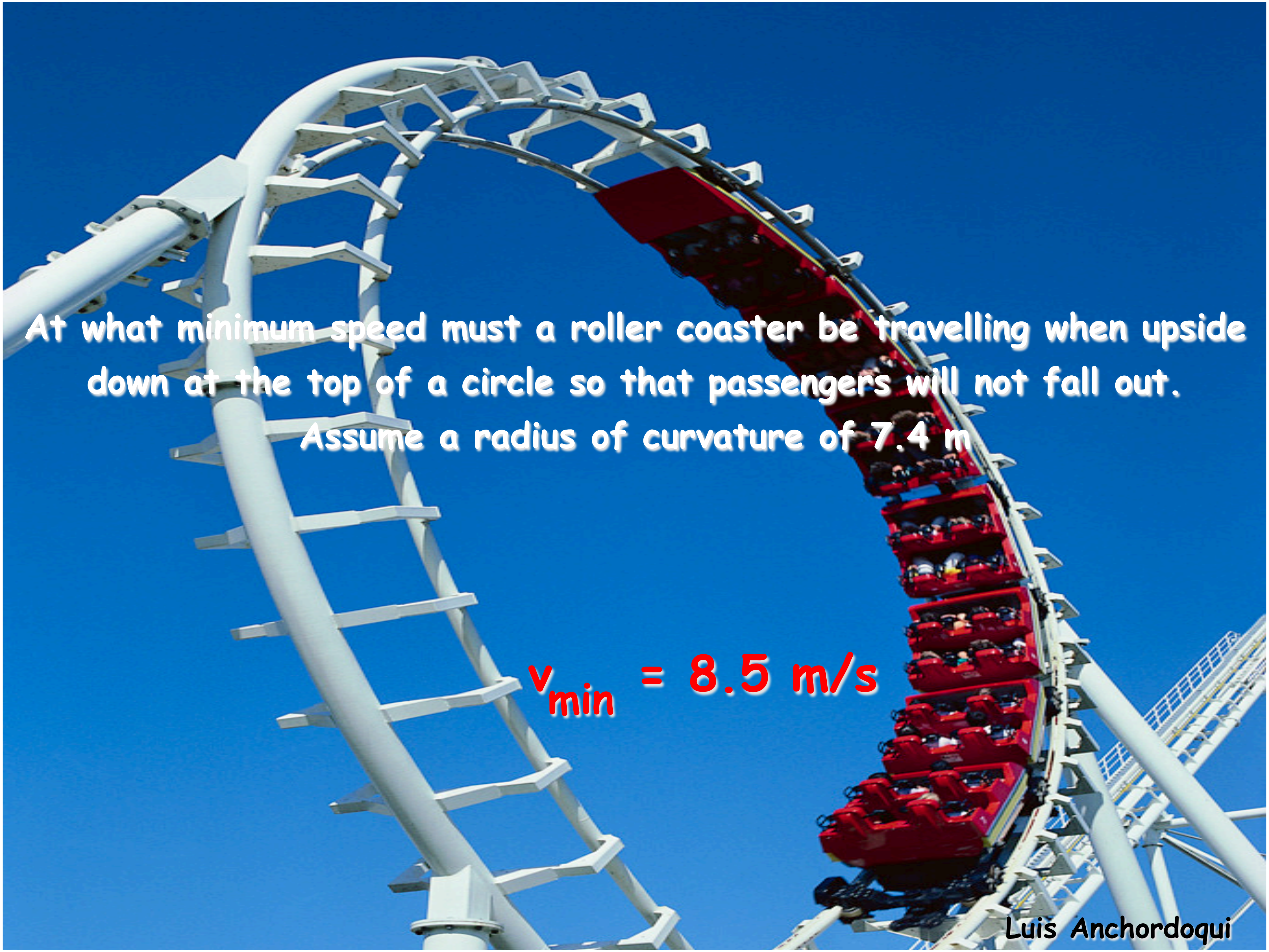
At what minimum speed must a roller coaster be travelling when upside down at the top of a circle so that passengers will not fall out. Assume a radius of curvature of 7.4 m
$v_{min}$ = 8.5 m/s
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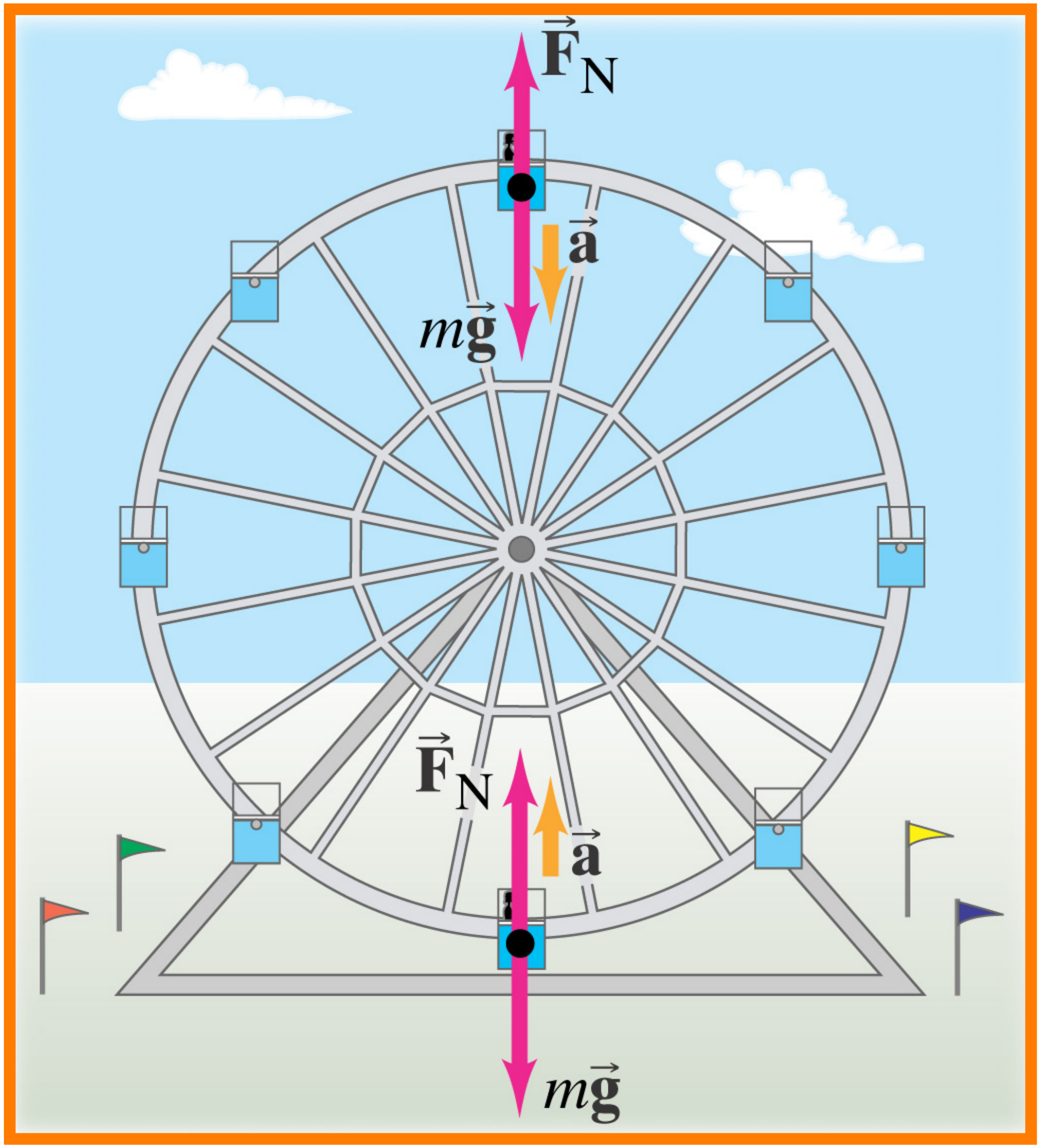
How many revolutions per minute would a 15-m-diameter Ferris whell need to make for the passengers to feel ‘ ‘ weightless ´´ at the topmost point




**11 rpm**

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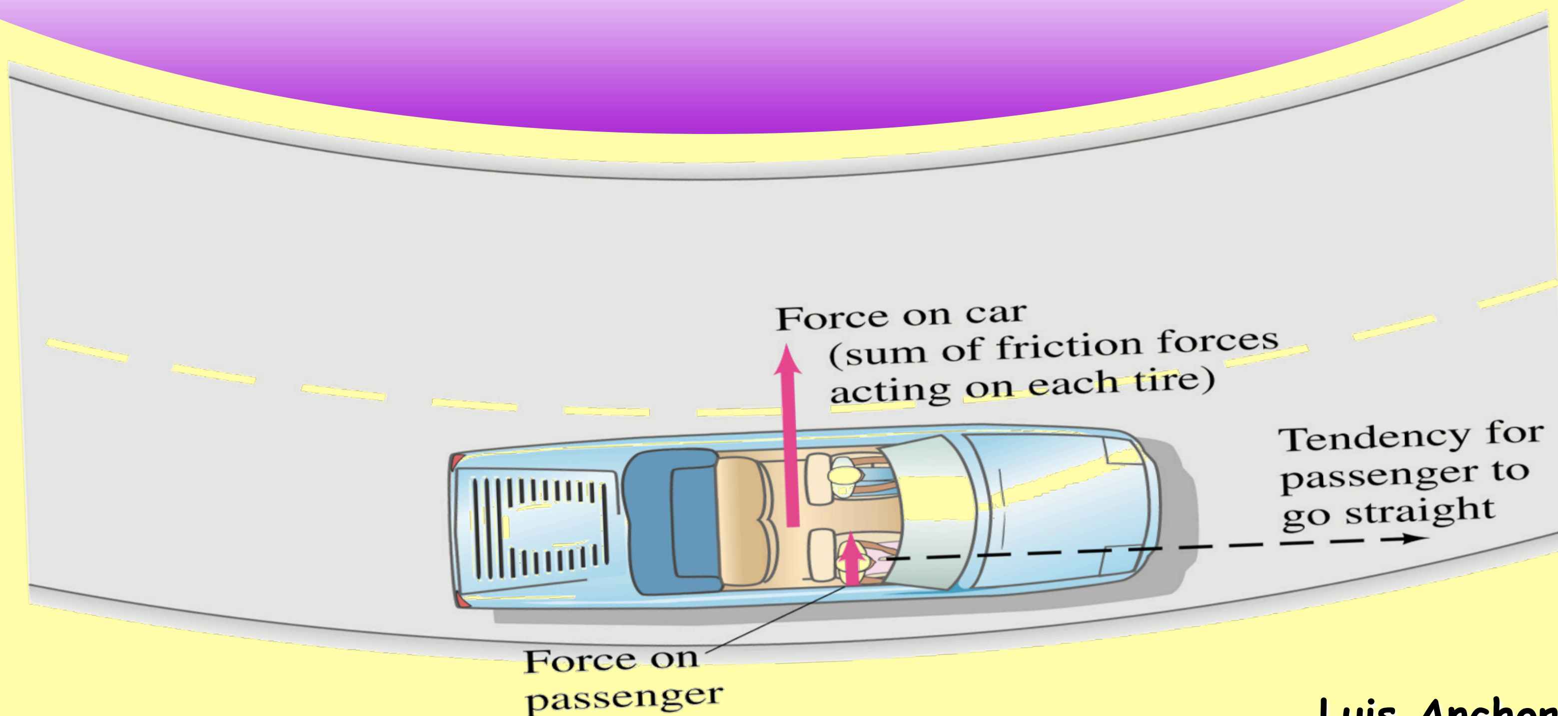
# Highway Curves, Banked and Unbanked

When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc.

If the road is flat, that force is supplied by friction.

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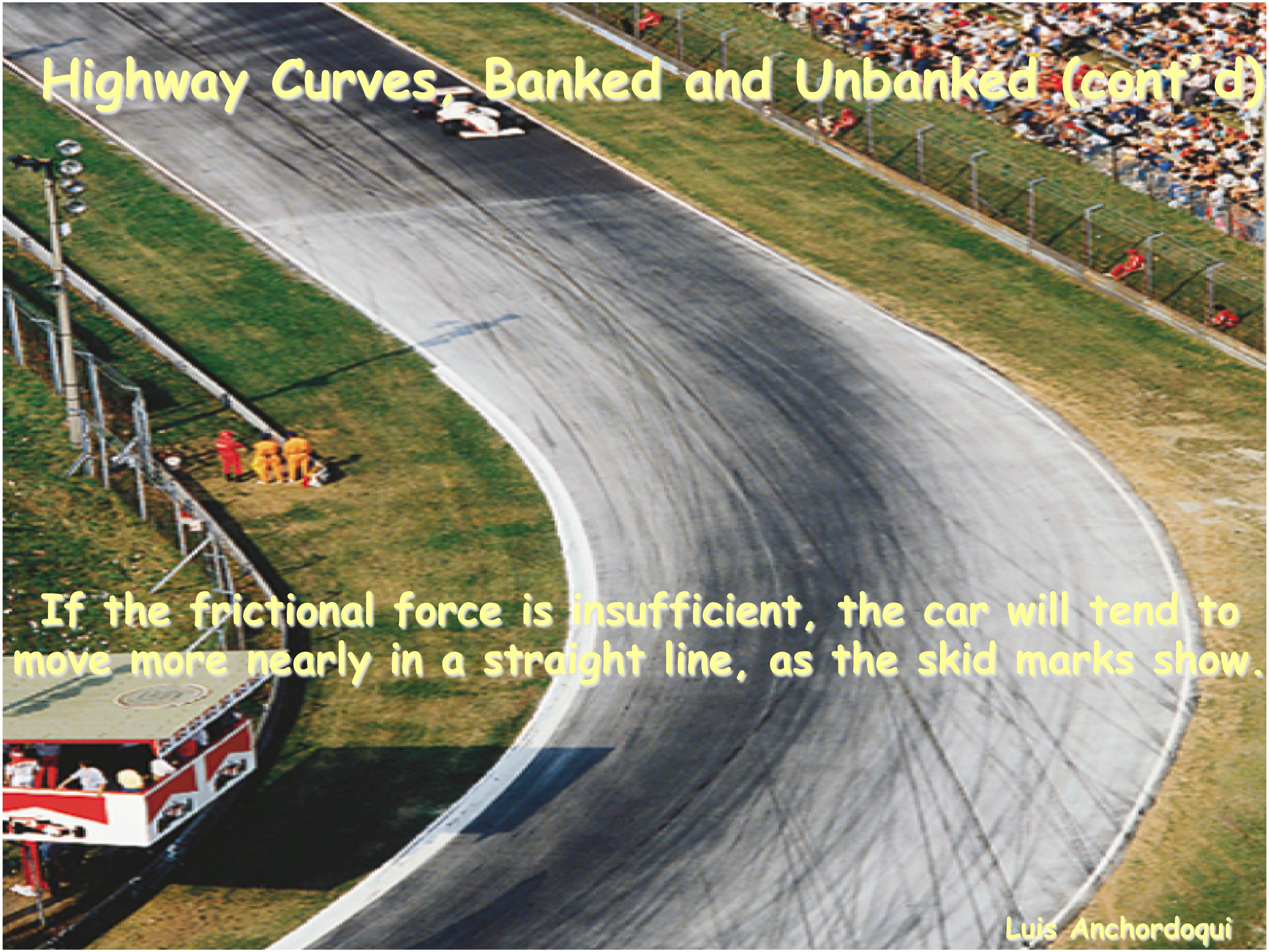
# Highway Curves, Banked and Unbanked (cont'd)



If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.

# Highway Curves, Banked and Unbanked (cont'd)

As long as the tires do not slip, the friction is static. If the tires do start to slip, the friction is kinetic, which is bad in two ways:

1. The kinetic frictional force is smaller than the static.

2. The static frictional force can point towards the center of the circle, but the kinetic frictional force opposes the direction of motion, making it very difficult to regain control of the car and continue around the curve.

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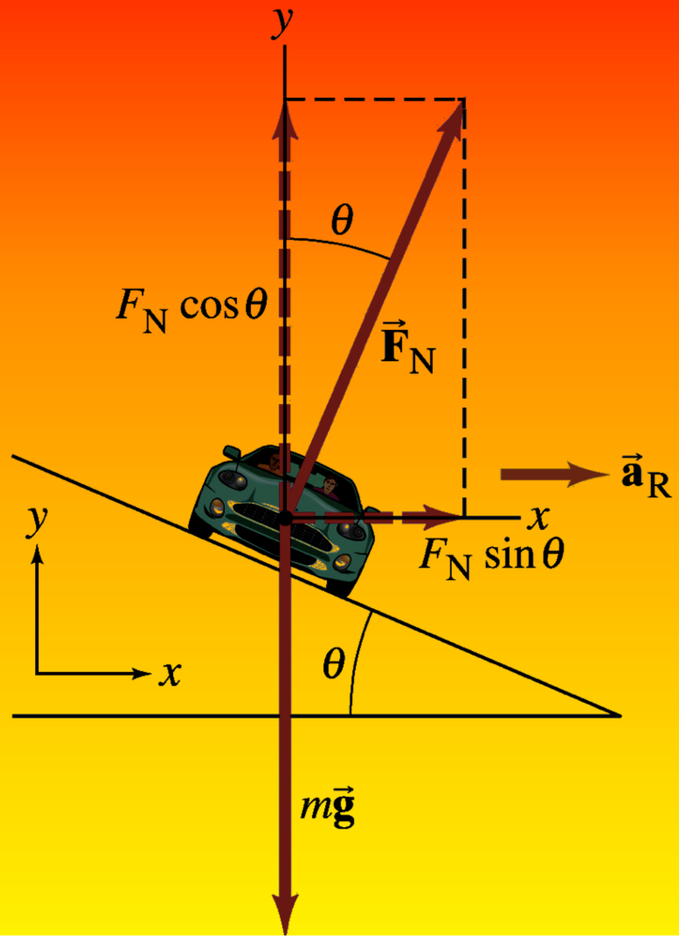
# Highway Curves, Banked and Unbanked (cont'd)

Banking the curve can help keep cars from skidding.

In fact, for every banked curve, there is one speed where the entire centripetal force is supplied by the horizontal component of the normal force, and no friction is required.

This occurs when:




$$F_N \sin\theta = m\frac{v^2}{r}$$

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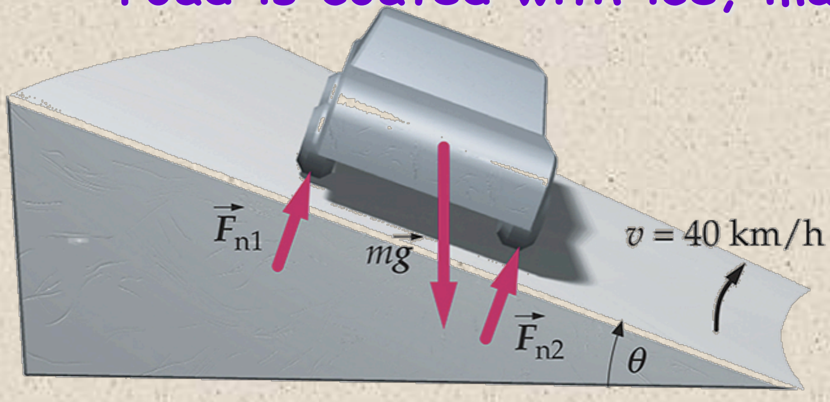
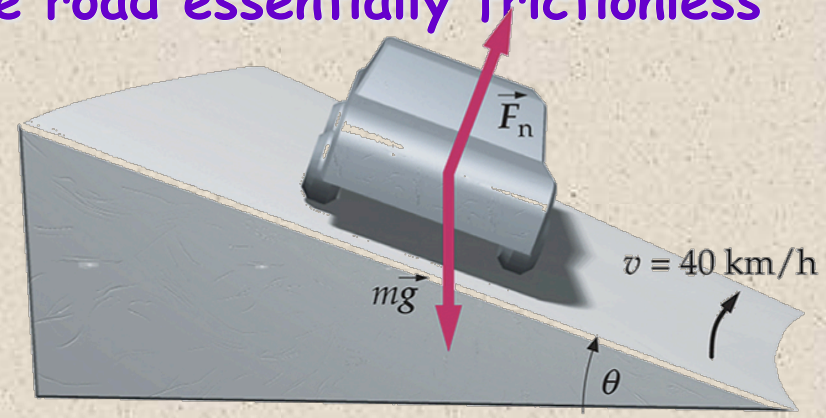
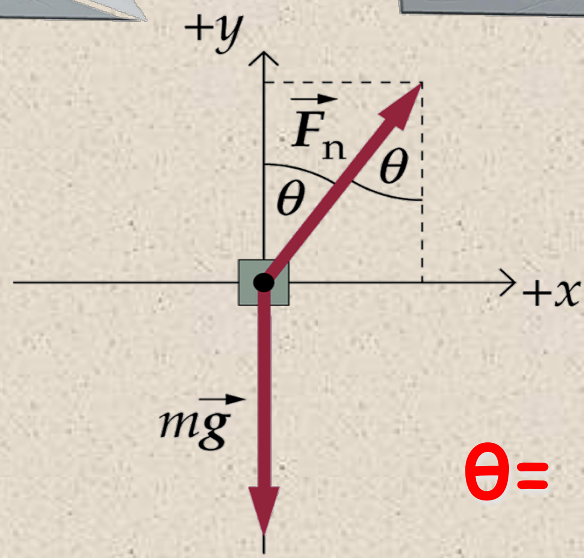
# Rounding a Banked Curve

A curve of radius 30.0 m is banked at an angle θ.

Find θ such that a car can round the curve at 40 km/h even if the road is coated with ice, making the road essentially frictionless




**θ= 22.8 degrees**

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# A Road Test

You have a summer job with NASCAR as part of an automobile tire testing. You are testing a new model of racing tires to see whether or not the coefficient of static friction between the tires and dry concrete pavement is 0.90 as claimed by the manufacturer.

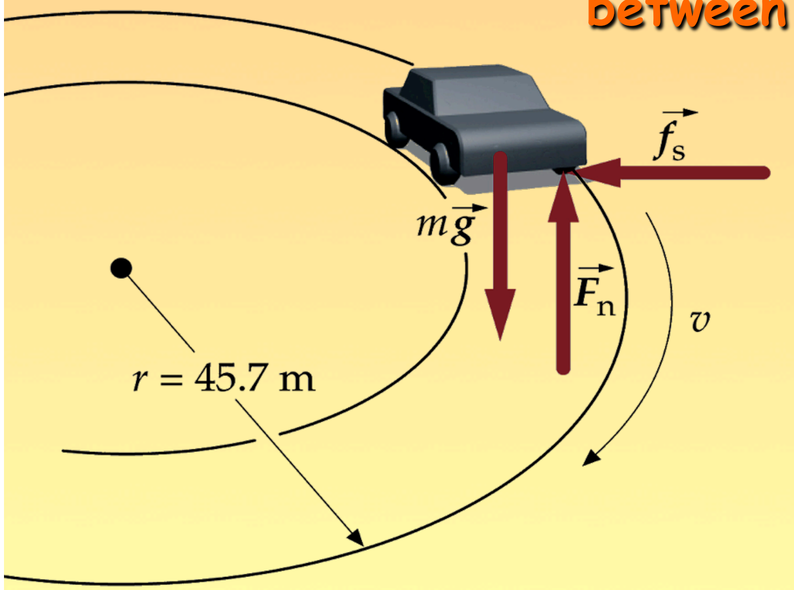
In a skidpad test, a racecar is able to travel at constant speed in a circle of radius 45.7 m in 15.2 s without skidding.

Assume air drag and rolling friction are negligible and assume that the road surface is horizontal.

In a skidpad test a car travels in a circle on a flat, horizontal surface at maximum possible speed without skidding.

(a) What was its speed? (b) What was its acceleration

(c) What was the minimum value for the coefficient of static friction between the tires and the road?

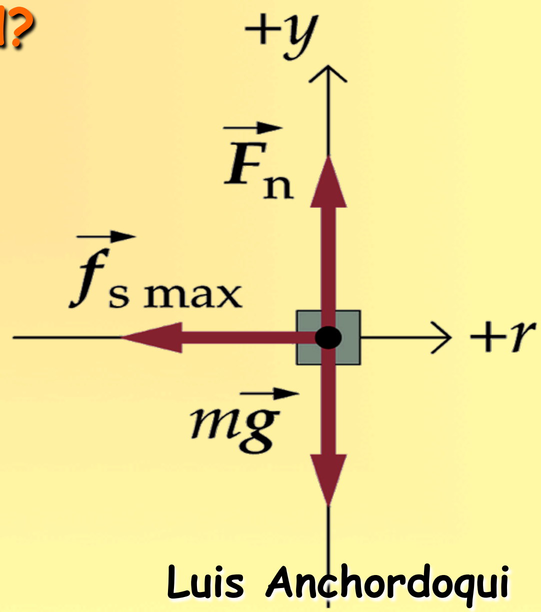



$v = 18.9$ m/s

$a_c = 7.81$ m/s²

$a_T = 0$

$\mu_s = 0.796$

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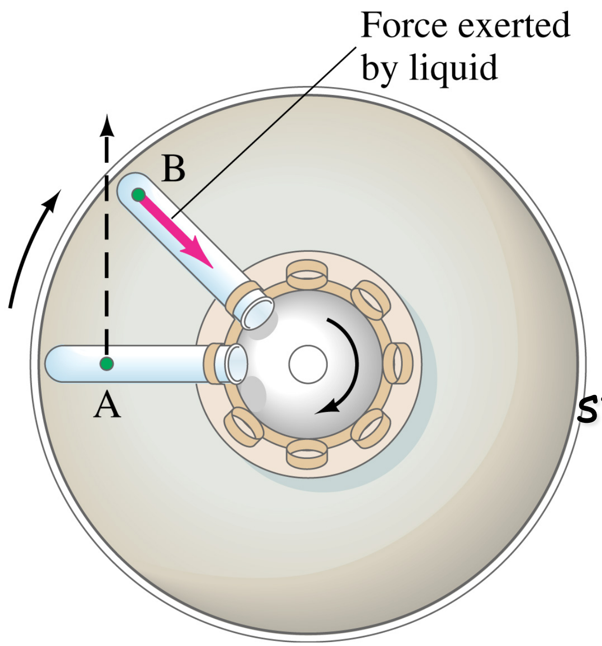
# Centrifugation

These devices are used to sediment materials quickly or to separate materials

Test tubes are held in the centrifugal rotor which is accelerated to very high rotational speeds




The small green dot represents a small particle (macromolecule) in a fluid filled test tube.

When the tube is at position A and the rotor is turning the particle has a tendency to move in a straight line in the direction of the dashed arrow.

But the fluid that resists the motion of these particles exerts a centripetal force that keeps the particles moving nearly in a circle.

Usually the resistance of the tube does not quite equal $mv^2/r$ and the particles eventually reach the bottom of the tube.

The purpose of a centrifuge is to provide and ‘ ‘ effective gravity´´ much larger than normal gravity because of the high rotational speeds, thus causing more rapid sedimentation

In 1993 a descendent probe containing instruments went deep into the Jovian atmosphere of Jupiter.
The fully assemble probed was tested at accelerations up to 200 *g*'s in this large centrifuge at Sandia National Laboratories

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# Ultracentrifuge

The rotor of an ultracentrifuge rotates at 50,000 rpm.
The top of the 4cm long test tube is 6cm from the rotation axis and is perpendicular to it.
The bottom of the tube is 10 cm from the axis of rotation.

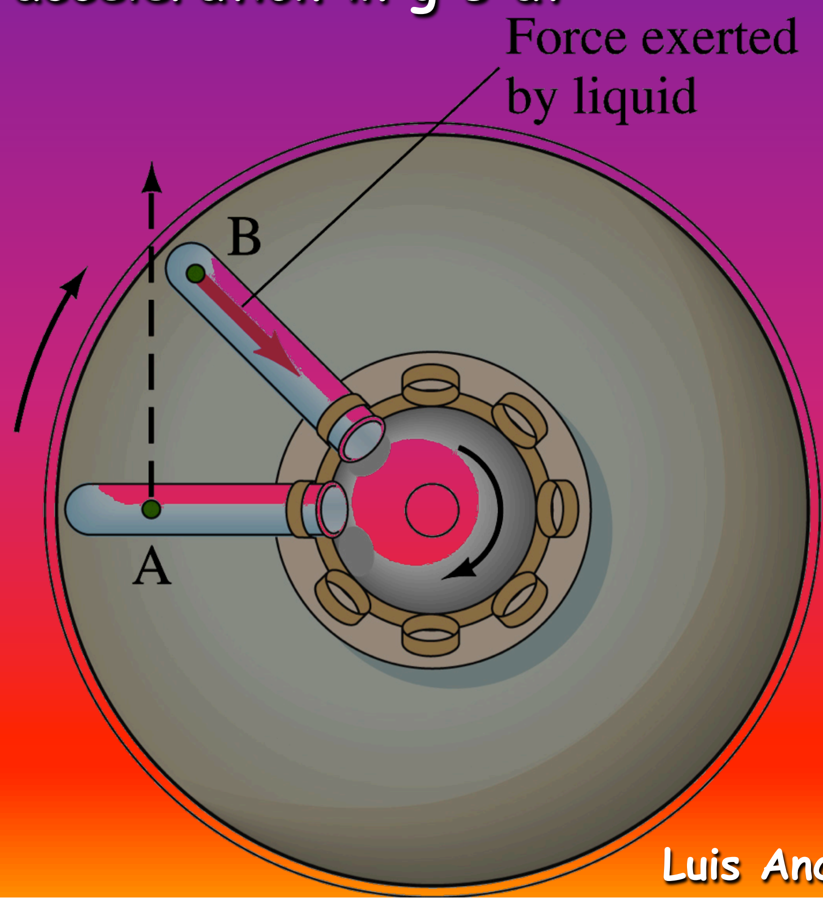
Calculate the centripetal acceleration in g's at

(a) the top

(b) the bottom of the tube

$$a_R = 1.67 \times 10^5 \, g$$

$$a_R = 2.8 \times 10^5 \, g$$

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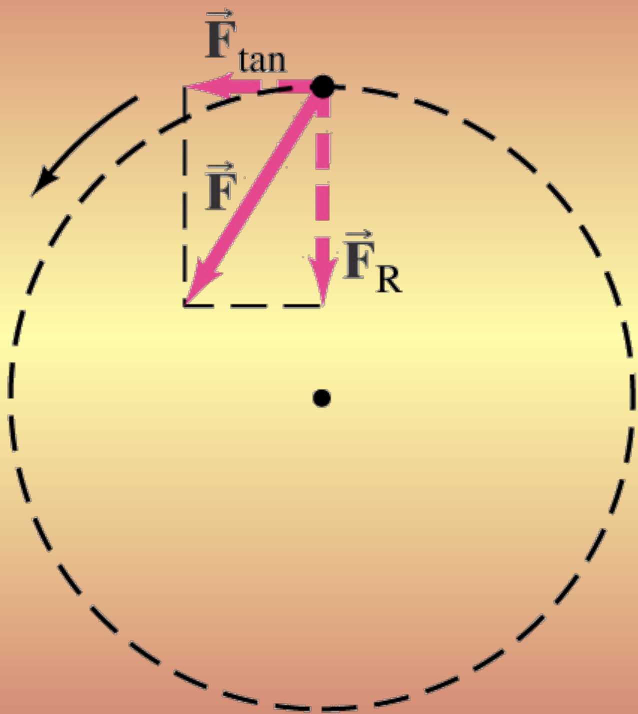
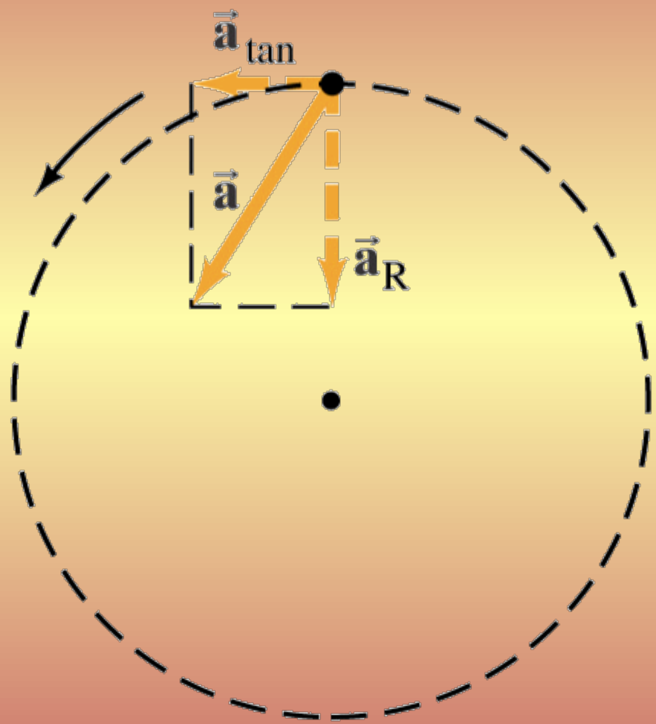
# Nonuniform Circular Motion

If an object is moving in a circular path but at varying speeds, it must have a tangential component to its acceleration as well as the radial one




$$a_{tan} = \frac{dv}{dt}$$

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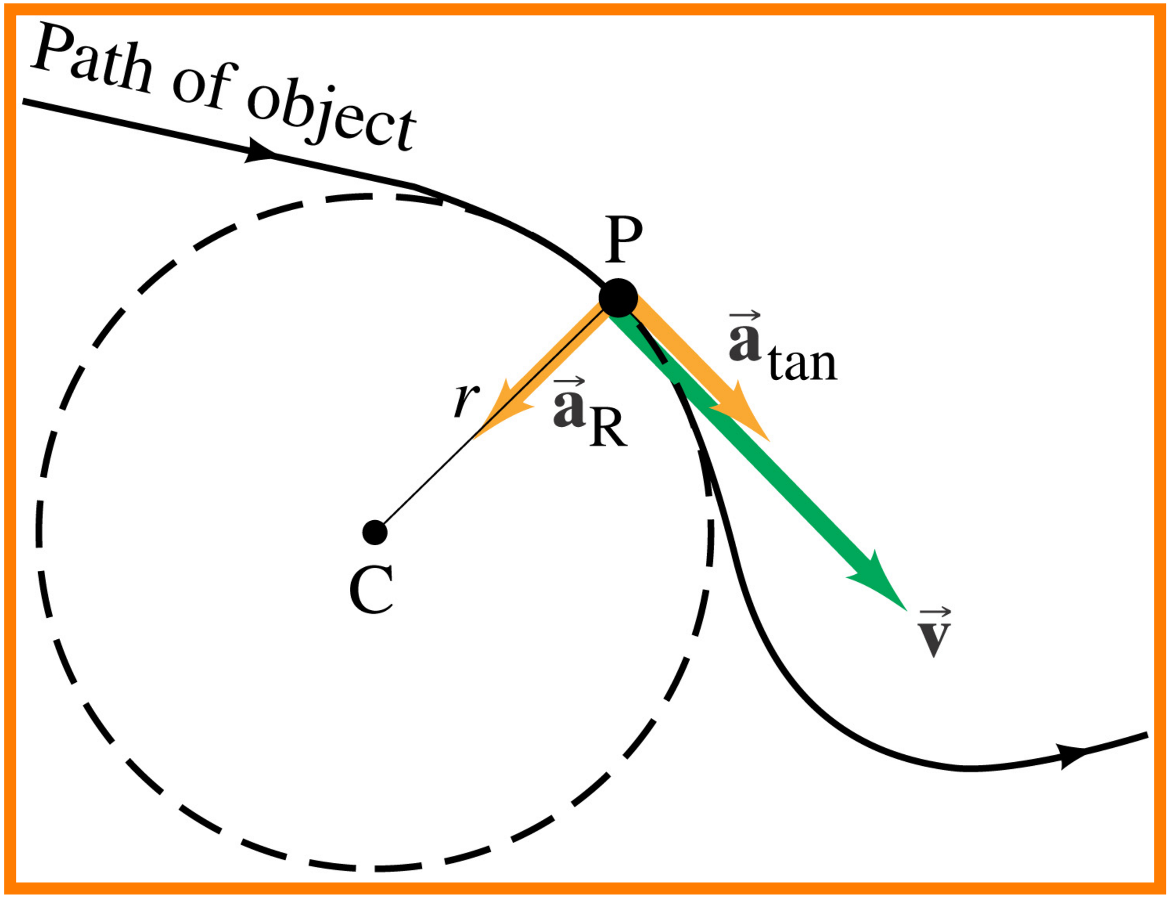
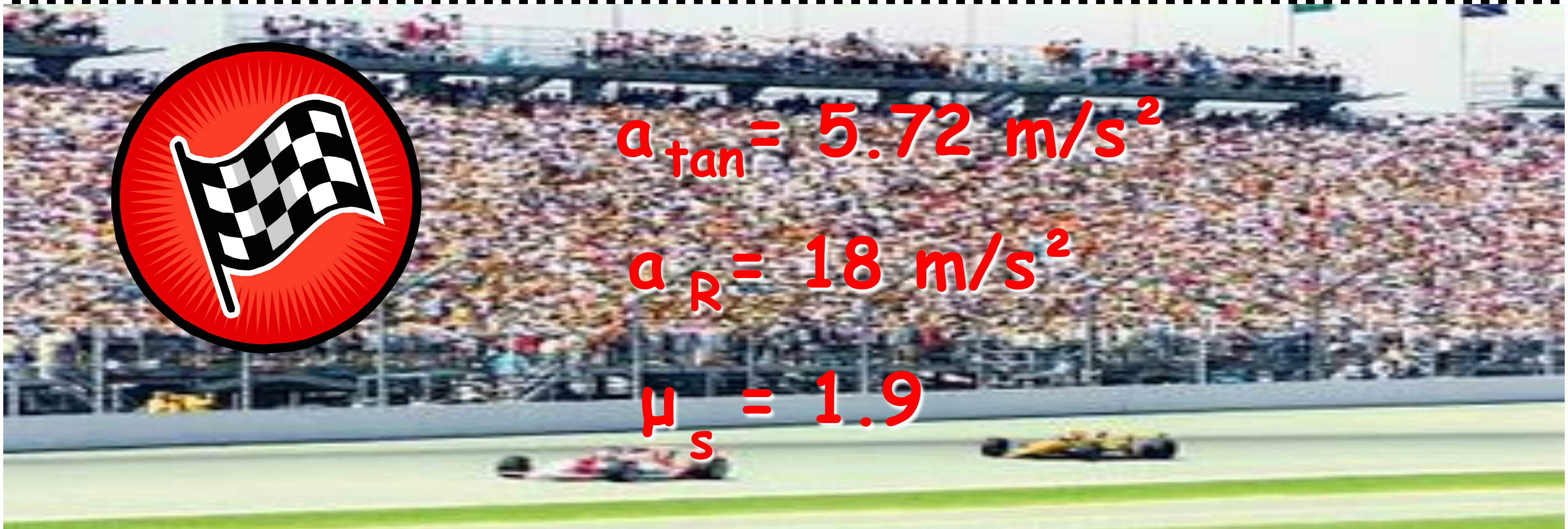
# Nonuniform Circular Motion (cont'd)

**This concept can be used for an object moving along any curved path, as a small segment of the path will be approximately circular.**






A car at Indianapolis accelerates uniformly from the pit area, going from rest to 320 km/h in a semicircular arc with a radius of 220 m.

Determine the tangential and radial acceleration of the car when it is halfway through the turn, assuming constant tangential acceleration.

If the curve where flat, what would the coefficient of static friction have to be between the tires and the road to provide this acceleration with no slipping or skidding?

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