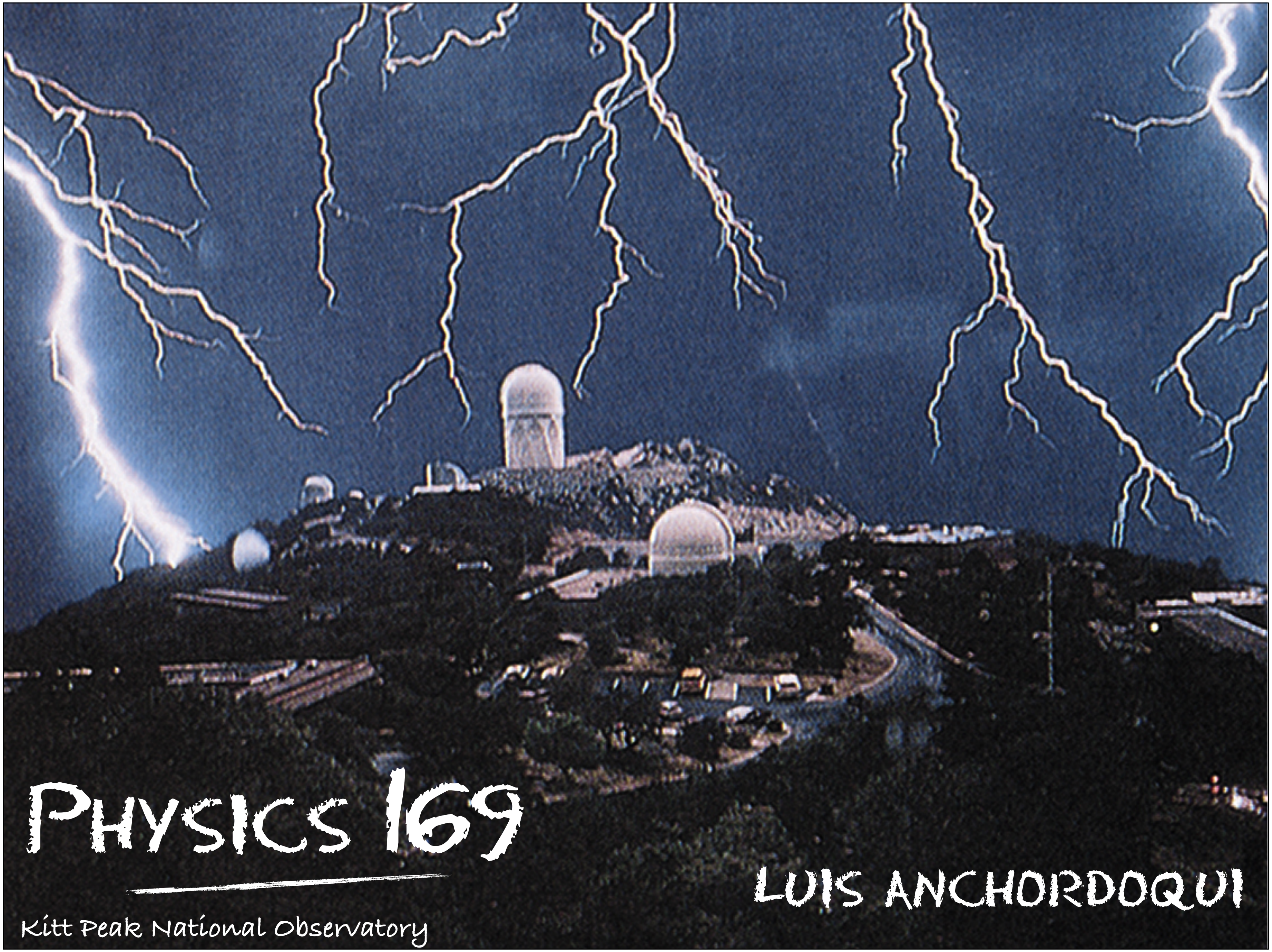

PHYSICS 169
LUIS ANCHORDOQUI
Kitt Peak National Observatory

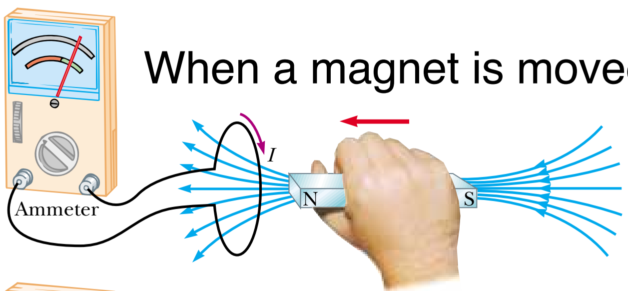
**Question**

Steady electric current can give steady magnetic field

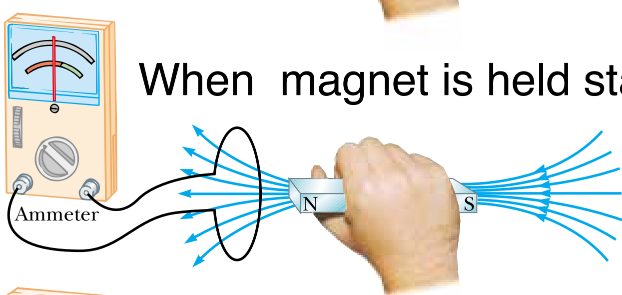
Because of symmetry between electricity and magnetism ☛ we can ask:

Steady magnetic field can give steady electric current

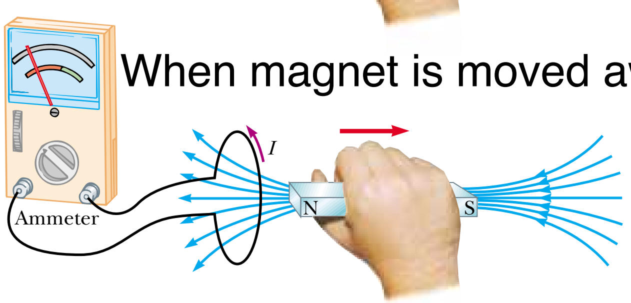
OR Changing magnetic field can give steady electric current




When a magnet is moved toward a loop of wire ☛ sensitive ammeter deflects indicating that current is induced in the loop




When magnet is held stationary ☛ there is no induced current in the loop even when the magnet is inside the loop




When magnet is moved away from loop ☛ ammeter deflects in opposite direction indicating that induced current is in opposite direction

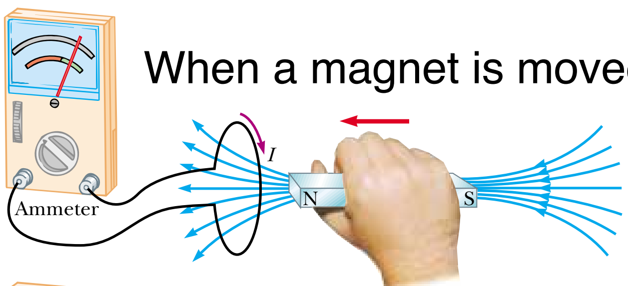
## Answer

Steady electric current can give steady magnetic field

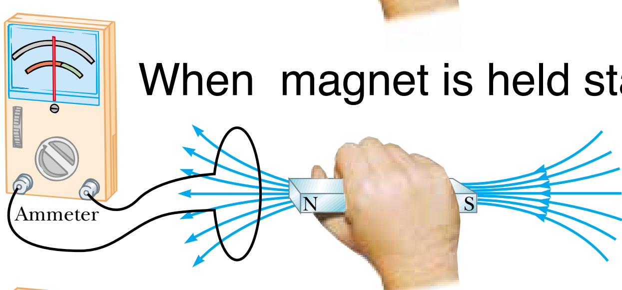
Because of symmetry between electricity and magnetism ☛ we can ask:

Steady magnetic field can give steady electric current ✗

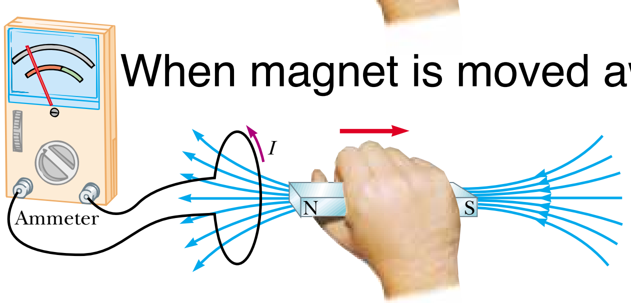
OR Changing magnetic field can give steady electric current ✓




When a magnet is moved toward a loop of wire ☛ sensitive ammeter deflects indicating that current is induced in the loop



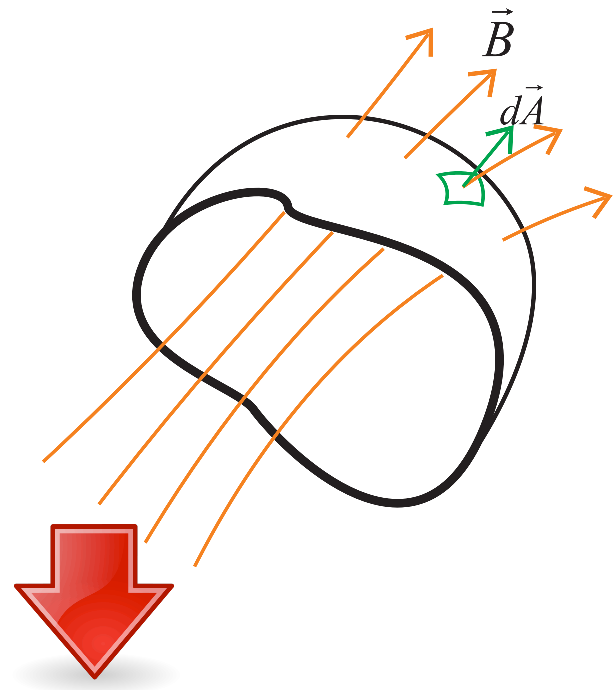

When magnet is held stationary ☛ there is no induced current in the loop even when the magnet is inside the loop




When magnet is moved away from loop ☛ ammeter deflects in opposite direction indicating that induced current is in opposite direction

## 9.1 Magnetic Flux

① Magnetic flux through surface $S$




$$\Phi_m = \int_S \vec{B} \cdot d\vec{A}$$

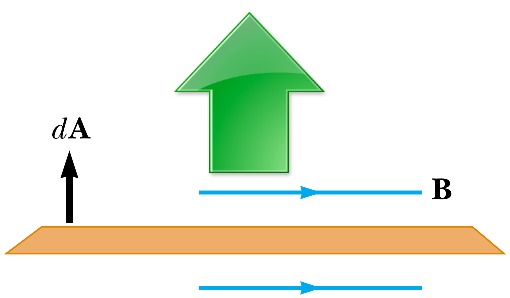
Unit of $\Phi_m$ ☛ $\mathrm{Weber\,(Wb)}$

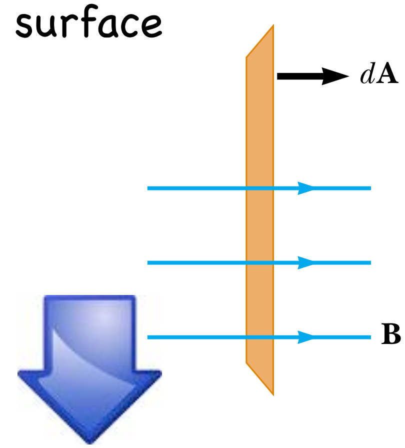
$$1\,\mathrm{Wb} = 1\,\mathrm{Tm}^2$$

② Graphical

$\Phi_m$ ☛ number of magnetic field lines passing through surface

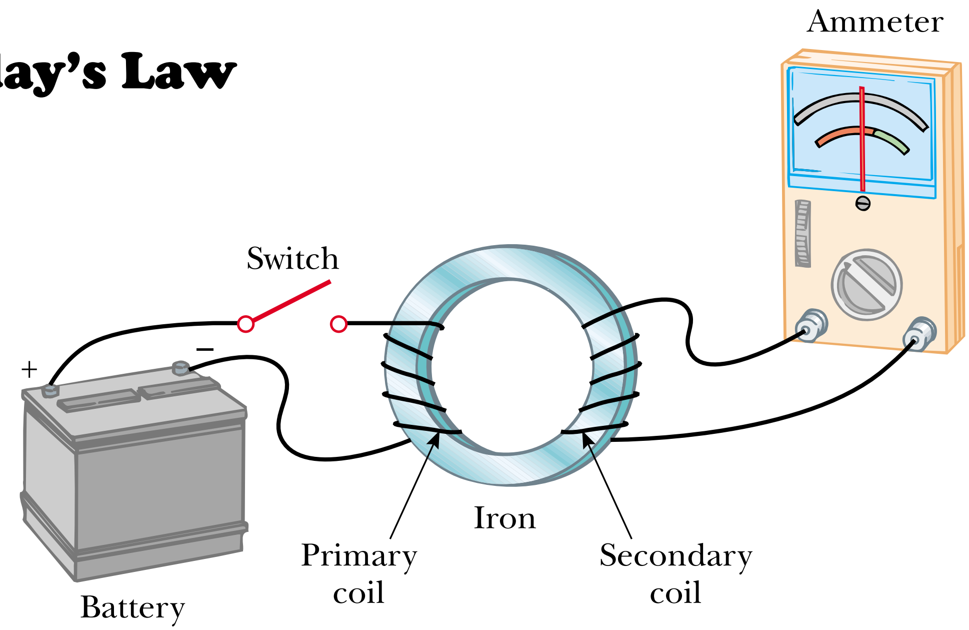
Flux through plane is zero when magnetic field is parallel to plane surface




Flux through plane is maximum when magnetic field is perpendicular to plane

## 9.2 Faraday's Law

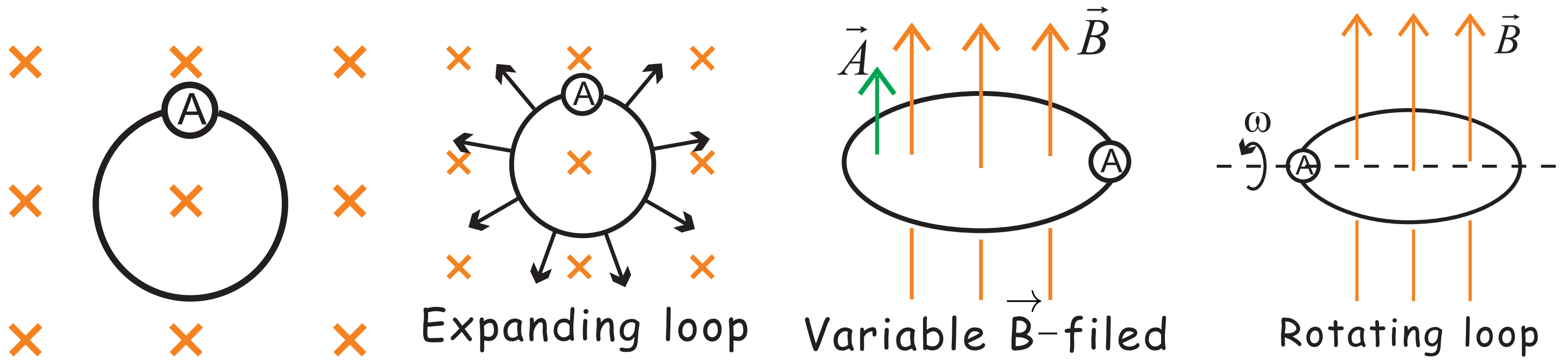



**Faraday's experiment** ➡ When switch in primary circuit is closed ammeter in secondary circuit deflects momentarily

emf induced in secondary circuit is caused by changing magnetic field through secondary coil

**Faraday's law of induction** ➡ Induced emf ☛ $$|\mathcal{E}| = N \left| \frac{d\Phi_m}{dt} \right|$$

$N$ ⬇ number of coils in circuit




$\vec{B} = \text{Constant}$
$\vec{A} = \text{Constant}$

$\mathcal{E} = 0$

$\vec{B} = \text{Constant}$
$\hat{A} = \text{Constant}$
$dA/dt \neq 0$
$\therefore\ |\mathcal{E}| > 0$

$\hat{B} = \text{Constant}$
$dB/dt \neq 0$
$\vec{A} = \text{Constant}$
$\therefore\ |\mathcal{E}| > 0$

$\vec{B} = \text{Constant}$
$A = \text{Constant}$
$d\hat{A}/dt \neq 0$
$\therefore\ |\mathcal{E}| > 0$

**Note**

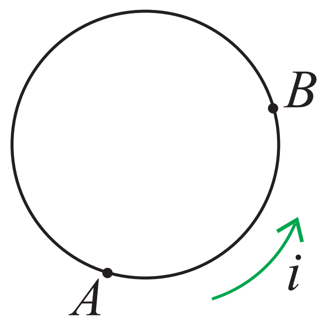
Induced emf drives a current throughout circuit similar to function of a battery

Difference here is that induced emf is distributed throughout circuit

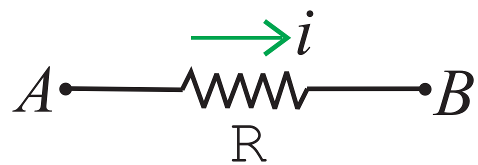
consequence

we cannot define a potential difference between any two points in circuit

Suppose there is an induced current in loop ☛ can we define $\Delta V_{AB}$ **?**




Recall




$$\Delta V_{AB} = V_A - V_B = iR > 0$$

$$\Rightarrow V_A > V_B$$

**Going anti-clockwise (same as $i$ )**

If we start from $A$ going to $B$ then we get $V_A > V_B$

If we start from $B$ going to $A$ then we get $V_B > V_A$

$\therefore$ **We cannot define $\Delta V_{AB}$ !!**

This situation is like when we study **interior of a battery**

A battery } provides energy needed to drive { chemical reactions

A loop } charge carriers around circuit by { changing magnetic flux

**sources of emf** **non-electric means**

## 9.3 Lenz's Law

① Flux of magnetic field due to induced current **opposes** change in flux that causes induced current

② Induced current is in such a direction as to **oppose** changes that produces it

③ Incorporating Lenz's law into Faraday's Law ☛

$$\mathcal{E} = -N\frac{d\Phi_m}{dt}$$

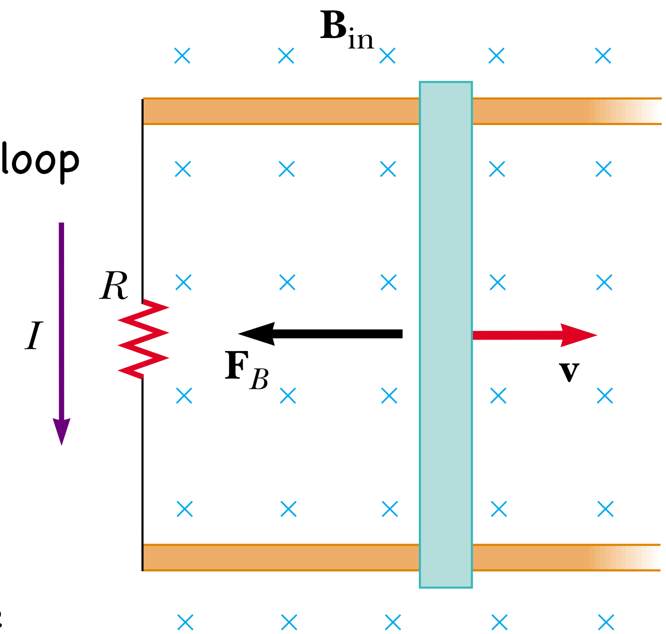
If $\frac{d\Phi_m}{dt} > 0$, $\Phi_m \uparrow \Rightarrow \mathcal{E}$ appears $\Rightarrow$ Induced current appears

$\Rightarrow$ $\vec{B}$-field due to induced current $\Rightarrow$ change in $\Phi_m$ $\overset{\text{so that}}{\Rightarrow}$ $\Phi_m \downarrow$

## ④ Lenz's Law is consequence from **principle of conservation of energy**

Suppose bar is given slight push to right

This motion sets up a counterclockwise current in the loop





What happens if we assume that current is clockwise

such that direction of magnetic force exerted on bar is to the right?

This force would accelerate the rod and increase its velocity

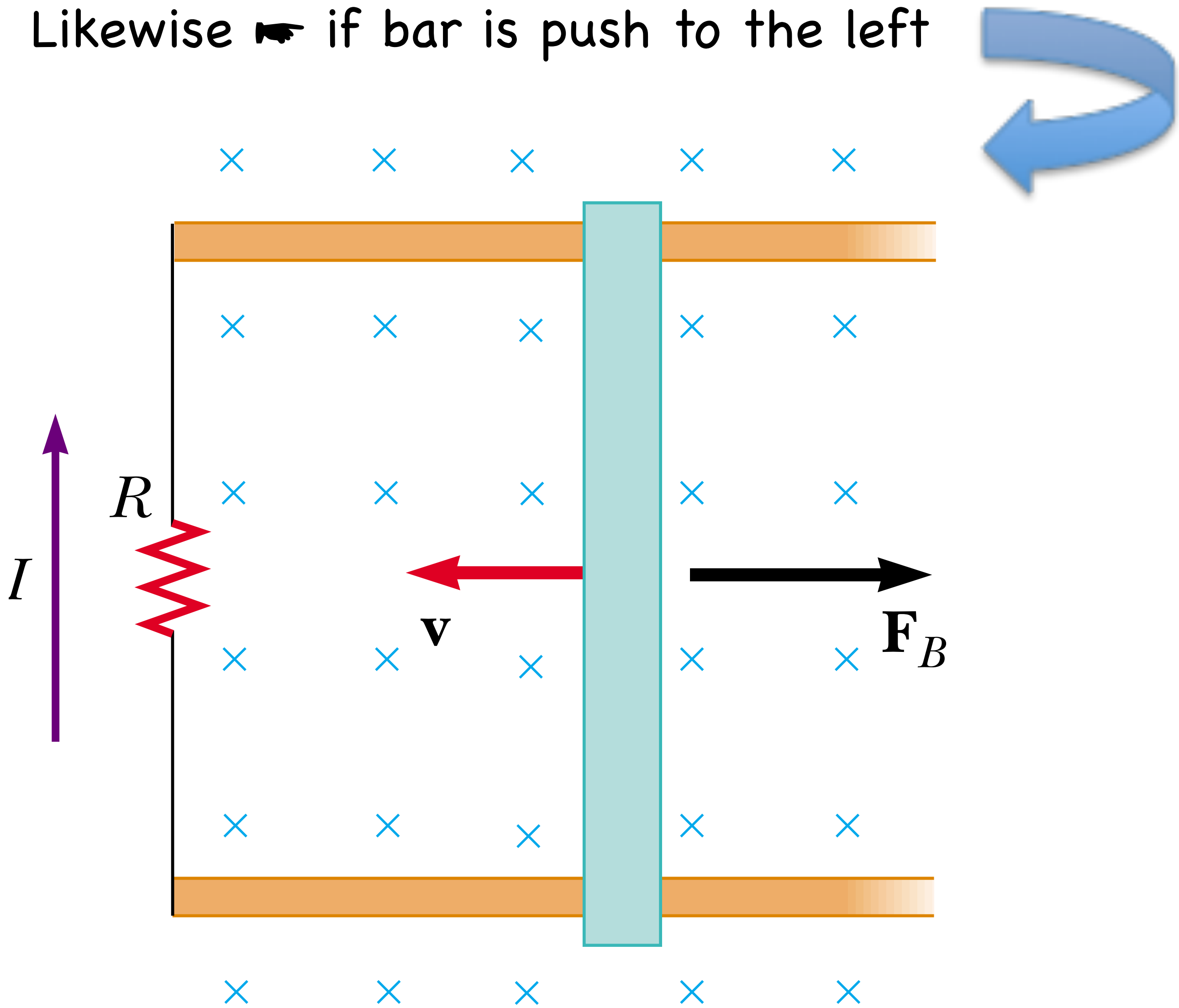
This (in turn) would cause area enclosed by loop to increase more rapidly
this would result in increase in induced current
which would cause increase in force
which would produce increase in current … and so on…

System would acquire energy with no input of energy

This is clearly inconsistent with all experience and violates law of energy conservation

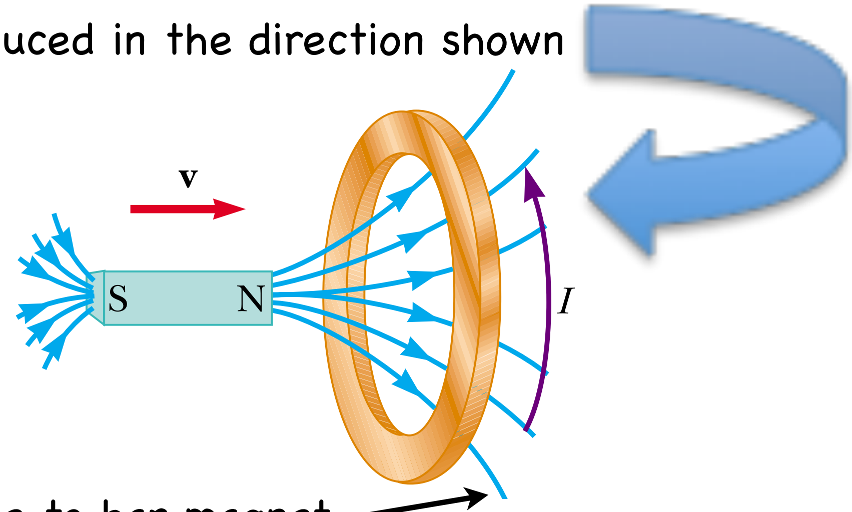
We are forced to conclude that current must be counterclockwise

Likewise ☛ if bar is push to the left



When magnet is moved toward stationary conducting loop

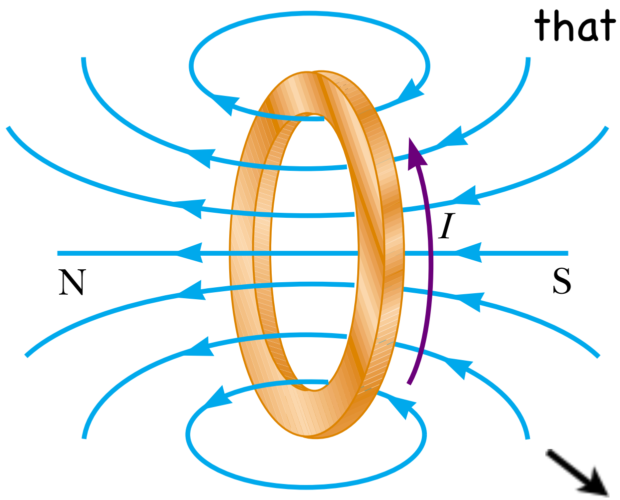
current is induced in the direction shown




Magnetic field lines shown are those due to bar magnet

This induced current produces its own magnetic field directed to the left

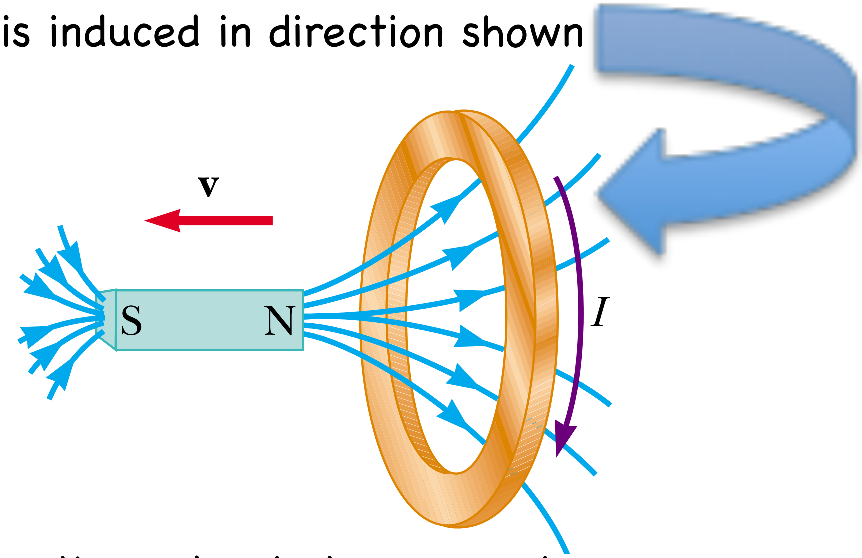
that counteracts the increasing external flux




Magnetic field lines shown are those due to induced current in ring

When magnet is moved away from stationary conducting loop

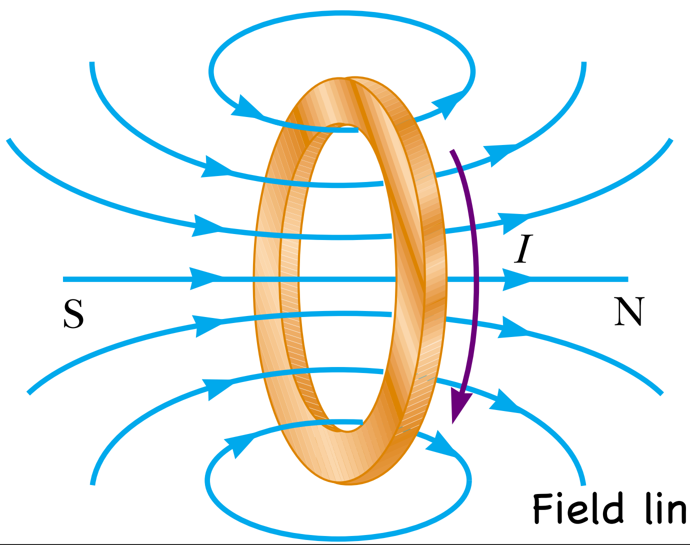
current is induced in direction shown




Magnetic field lines shown are those due to bar magnet

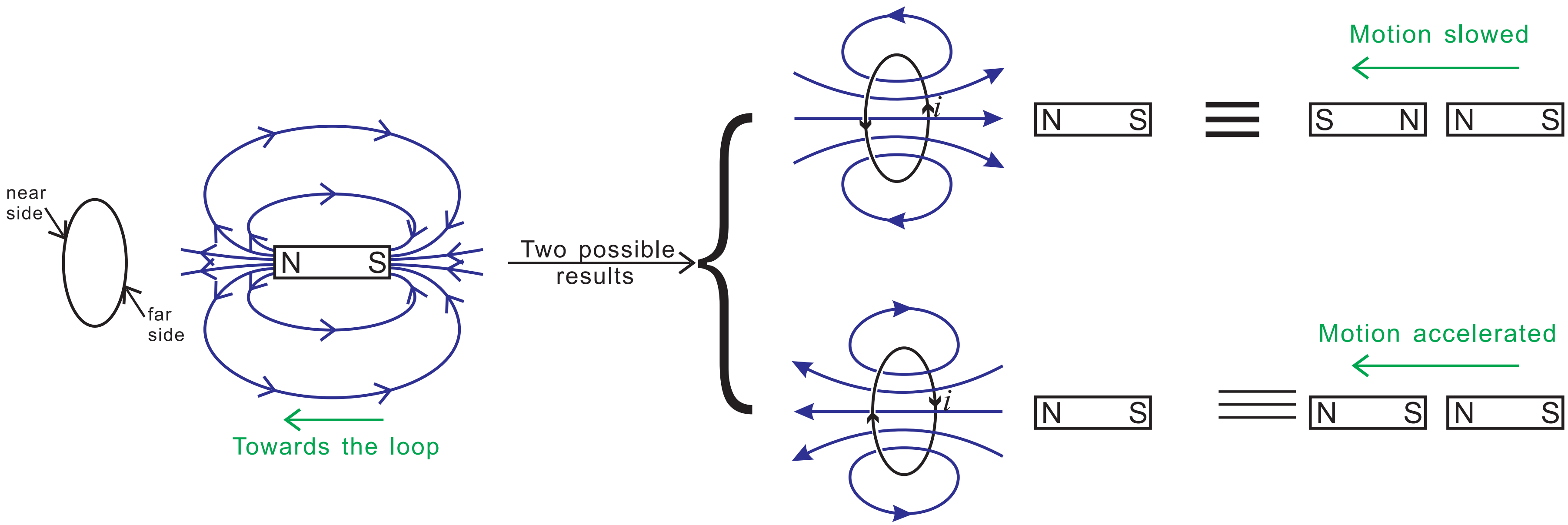
This induced current produces magnetic field directed to the right

and so counteracts decreasing external flux



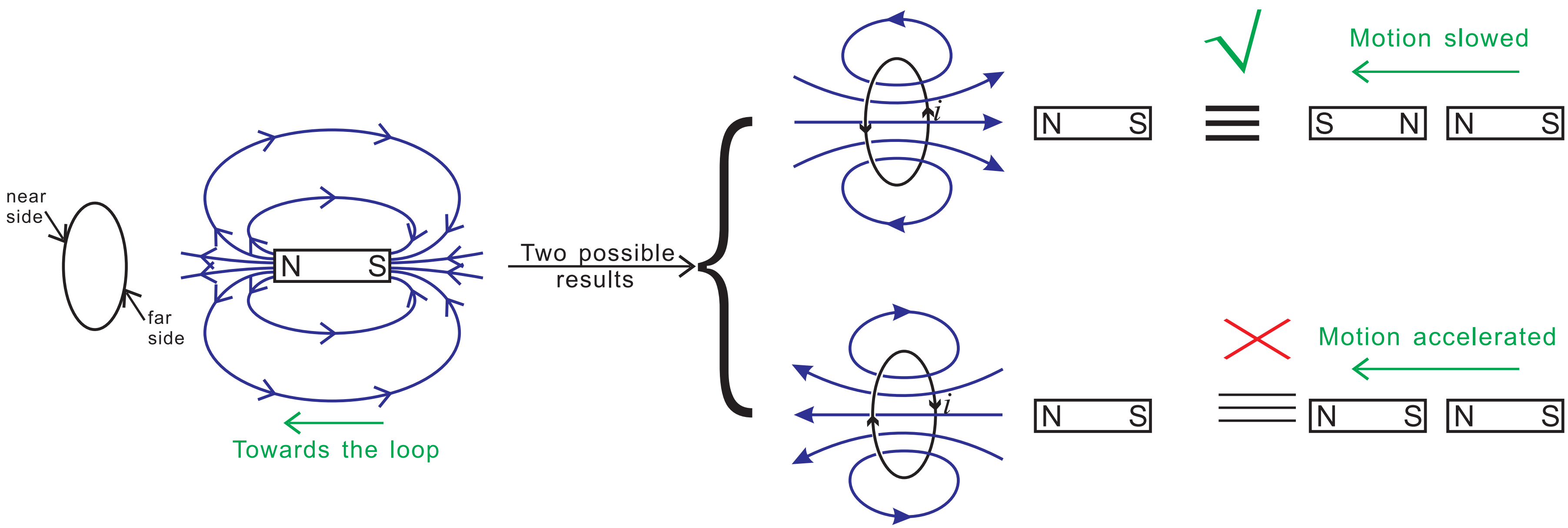

Field lines shown are those due to induced current in ring

# Question

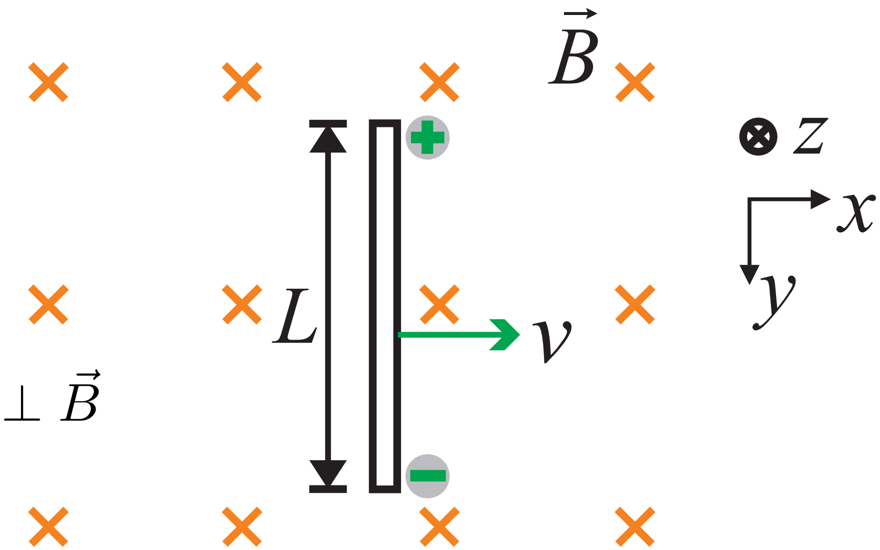


near side
far side
N
S
Towards the loop
Two possible results
i
N
S
S
N
N
S
Motion slowed
i
N
S
N
S
N
S
Motion accelerated

# Answer


near side
far side
N S
Towards the loop
Two possible results
i
N S
S N
N S
Motion slowed
i
N S
N S
N S
Motion accelerated

## 9.4 Motional EMF




Straight conductor of length $L$
is moving through uniform $\vec{B}$-field
directed into the page

Assume conductor is moving with constant $\vec{v} \perp \vec{B}$
under influence of some external agent

Electrons in conductor experience force ➡ $\vec{F}_B = q\vec{v} \times \vec{B}$
directed along the length $L$ perpendicular to both $\vec{v}$ and $\vec{B}$

Under influence of this force
electrons move to lower end of conductor and accumulate there
leaving net positive charge at upper end

Because of this charge separation electric field $\vec{E}$ is produced inside conductor

Charges accumulate at both ends
until downward magnetic force $qvB$ on charges remaining in conductor
is balanced by the upward electric force $qE$

At this point ☛ electrons move only with random thermal motion

Equilibrium requires that ➡ $\vec{F}_E + \vec{F}_B = 0$

$$\Rightarrow q\vec{E} + q\vec{v} \times \vec{B} = 0$$

$$\Rightarrow \vec{E} = -\vec{v} \times \vec{B}$$

Voltage across ends of conductor ☛ $\Delta V = -\int_0^L \vec{E} \cdot d\vec{s}$

$$\Delta V = -EL$$

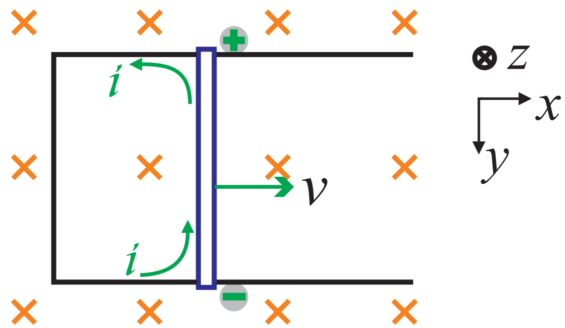
$\therefore$ **Voltage** ☛ $\Delta V = vBL$

**Potential difference is maintained between ends of conductor as long as the conductor continues to move through the uniform magnetic field**

Suppose moving wire **slides without friction**

on stationary $U$-shape conductor

Motional emf can drive electric current $i$ in $U$-shape conductor




$\Rightarrow$ Power is dissipated in circuit

$\Rightarrow$ $P_{out} = Vi$ ☛ Joule's heating

What is source of this power?

Look at the forces acting on conducting rod:

- Magnetic force $\vec{F}_m = i\vec{L} \times \vec{B}$

  $F_m = iLB$ ☛ (pointing left)

- For wire to continue to move at constant velocity

  we need to **apply an external force**

  $\vec{F}_{ext} = -\vec{F}_m = iLB$ ☛ (pointing right)

$\therefore$ Power required to keep rod moving

$$\begin{aligned} P_{in} &= \vec{F}_{ext} \cdot -\vec{v} \\ &= iBLv \\ &= iBL\frac{dx}{dt} \\ &= iB\frac{d(xL)}{dt} \qquad (xL = A \text{ area enclosed by circuit}) \\ &= i\frac{d(BA)}{dt} \qquad (BA = \Phi_m \text{ magnetic flux}) \end{aligned}$$

Since energy is not being stored in system

$$\therefore \quad P_{in} + P_{out} = 0$$

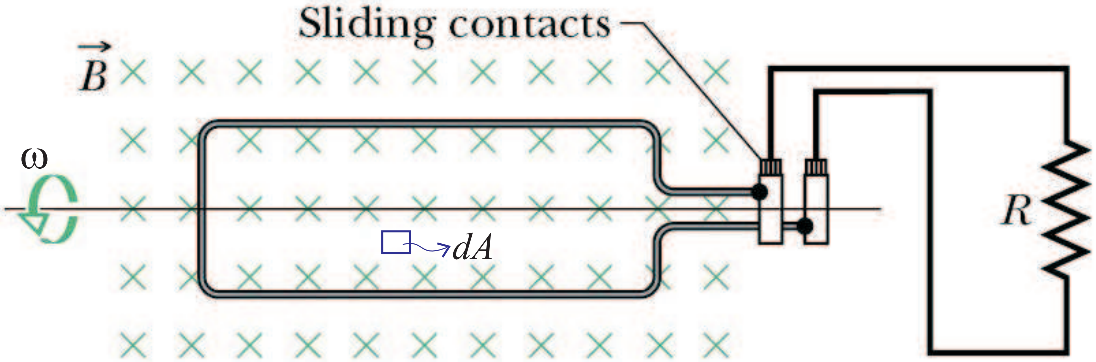
$$iV + i\frac{d\Phi_m}{dt} = 0$$

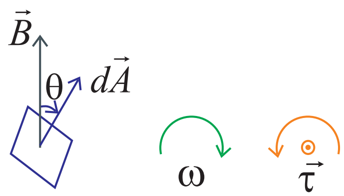
We recover **Faraday's Law** $\Rightarrow \quad V = -\frac{d\Phi_m}{dt}$

# Generators and Motors

Assume circuit loop is **rotating at constant angular velocity** $\omega$

(Source of rotation ☛ steam produced by burner or water falling from dam)




changes with time! $\theta = \omega t$

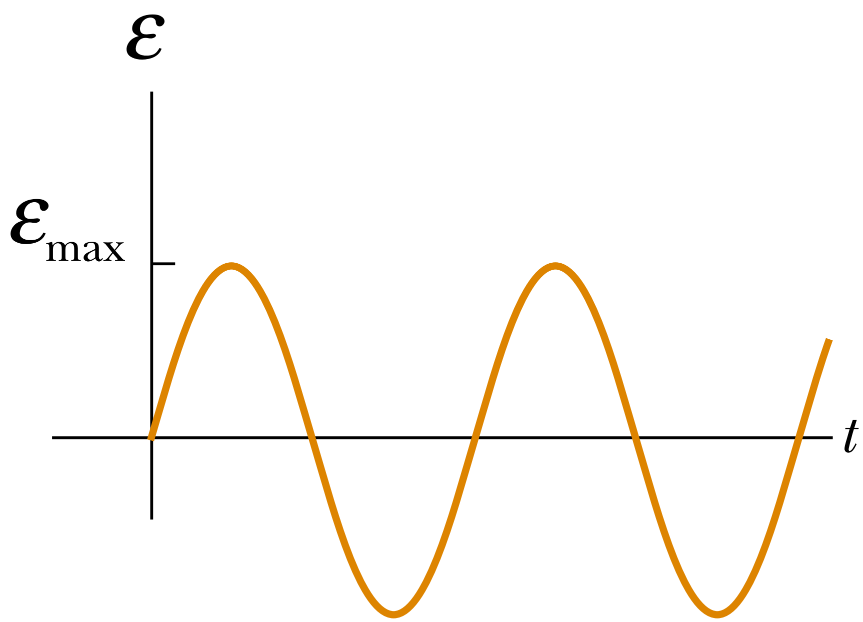
number of coils

Magnetic flux through loop ☛ $\Phi_B = N \int_{\text{loop}} \vec{B} \cdot d\vec{A} = NBA\cos\theta$

Induced emf ☛ $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -NBA\dfrac{d}{dt}(\cos\omega t) = N\,B\,A\,\omega\,\sin\omega t$

Induced current ☛ $i = \dfrac{\mathcal{E}}{R} = \dfrac{NBA\omega}{R}\sin\omega t$

## Alternating current (AC) voltage generator




Power has to be provided by source of rotation

to overcome torque acting on a current loop in a magnetic field

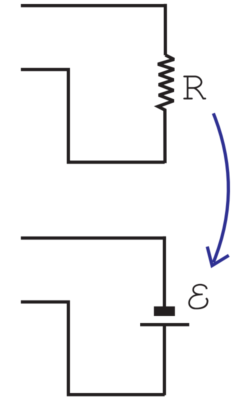
$$\vec{\tau} = \overbrace{Ni\vec{A}}^{\vec{\mu}} \times \vec{B}$$

$$\therefore \quad \tau = NiAB\sin\theta$$

Net effect of torque is to oppose rotation of coil

**Electric motor** is a **generator operating in reverse**

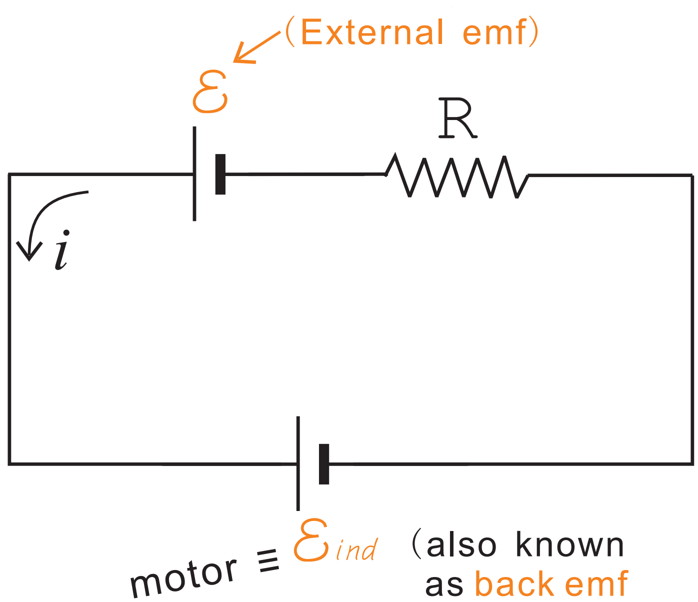
Replace load resistance $R$ with a battery of emf $\mathcal{E}$ $\Rightarrow$




With battery ☛ there is a current in coil and it experiences torque in B-field

$\Rightarrow$ Rotation of coil leads to an induced emf ☛ $\mathcal{E}_{ind}$ in direction opposite that of battery

**Lenz's law**




(also known as back emf for motors)

$$\therefore \quad i = \frac{\mathcal{E} - \mathcal{E}_{ind}}{R}$$

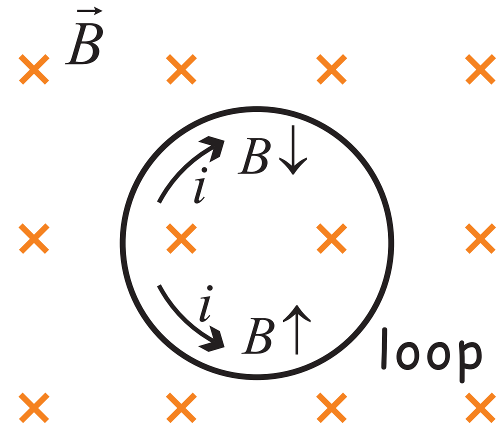
$\Rightarrow$ As motor speeds up $\mathcal{E}_{ind} \uparrow, \quad \therefore i \downarrow$

$$P_{electric} = i^2 R + P_{mechanical}$$

**Electric power input** **Mechanical power delivered**

## 9.5 Induced Electric Field

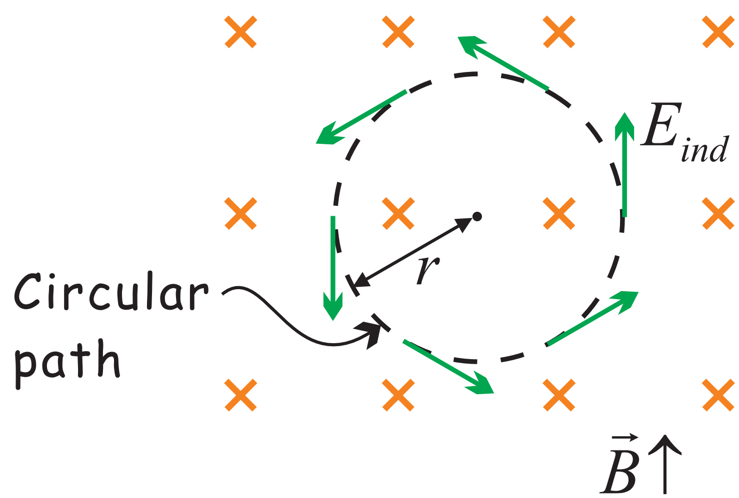
We have seen that a changing magnetic flux induces an emf and a current in a conducting loop




$\therefore \vec{B}\uparrow \Rightarrow$ anti-clockwise current
$\vec{B}\downarrow \Rightarrow$ clockwise current

In the same way

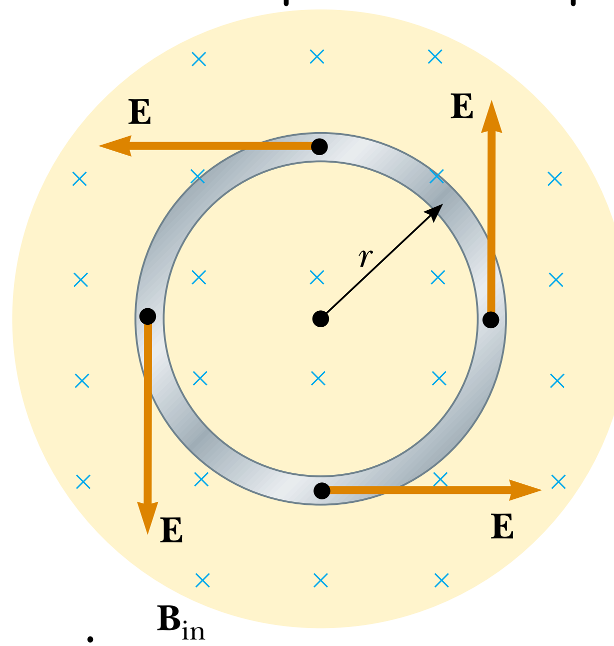
we can relate induced current in conducting loop to an electric field by claiming that electric field is created in conductor as a result of the changing magnetic flux



Induced electric field is nonconservative

unlike electrostatic field produced by stationary charges

We can illustrate this point by considering conducting loop of radius $r$ situated in uniform magnetic field that is perpendicular to plane of loop




If magnetic field changes with time ☛ according to Faraday's law

emf $\mathcal{E} = -d\Phi_B/dt$ induced in loop

Induction of current in loop

implies presence of induced electric field $\vec{E}$

which must be tangent to the loop

because this is direction in which charges in the wire move in response to electric force

Work done by $\vec{E}$-field in moving test charge $q$ once around loop $= q\mathcal{E}$

Because electric force acting on charge $= q\vec{E}$

work done by electric field in moving charge once around loop $= qE2\pi r$

These two expressions for work done must be equal

$\therefore$ we see that ☛ $q\mathcal{E} = qE2\pi r$

$$E = \frac{\mathcal{E}}{2\pi r}$$

$$= -\frac{1}{2\pi r}\frac{d\Phi_B}{dt}$$

$$= -\frac{r}{2}\frac{dB}{dt}$$

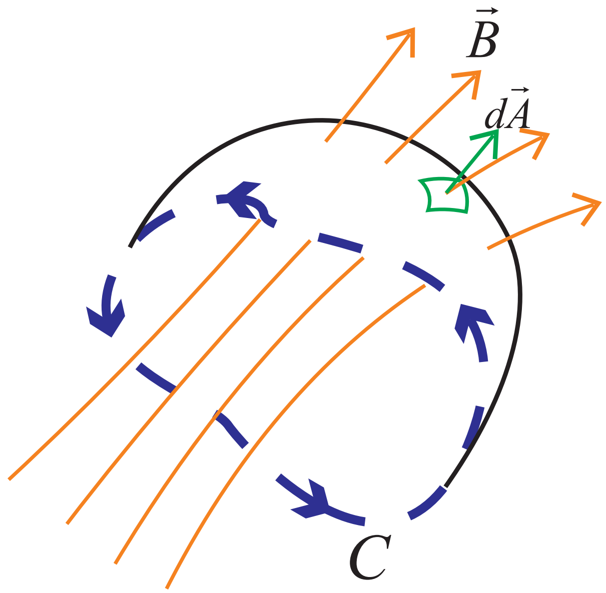
In general ☛ emf for any closed path can be expressed as line integral

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}$$

Induced electric field is a nonconservative field

that is generated by a changing magnetic field




$\therefore$ Faraday's Law becomes ☛ $$\oint_C \vec{E}_{ind} \cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$

L.H.S = Integral around a closed loop $C$

R.H.S = Integral over a surface bounded by $C$

Direction of $d\vec{A}$ determined by direction of line integration $C$

(Right-Hand Rule)

## SUMMARY

**Regular $\vec{E}$-field**

created by charges

$\vec{E}$ -field lines start from

$+q$ and end on $-q$ charge

can define electric potential
so that we can discuss potential
difference between two points

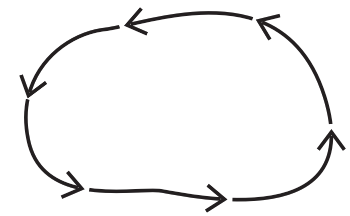
$\Downarrow$

Conservative force field

Induced $\vec{E}$-field

created by changing B-field

$\vec{E}$-field lines form closed loops



Electric potential cannot be defined
(or, potential has no meaning)

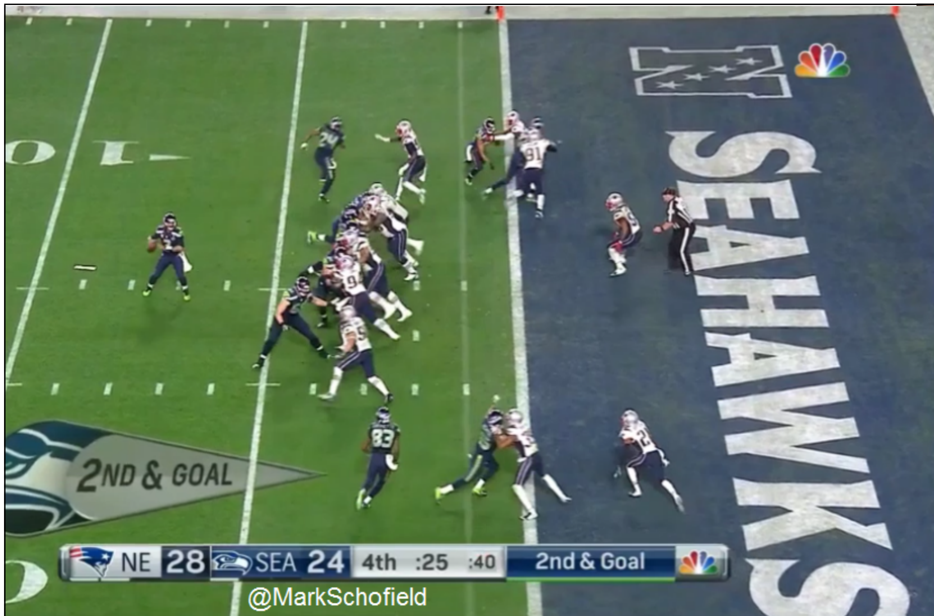
$\Downarrow$

Non-conservative force field

Classification of electric and magnetic effects
**depend on frame of reference of observer** !!!

**e.g.** For motional emf ☛ observer in reference frame of moving loop
will **NOT** see an induced $\vec{E}$-field but just a **regular** $\vec{E}$-field

**To be continued next semester in Special Relativity**
**same bat-time, same bat-channel**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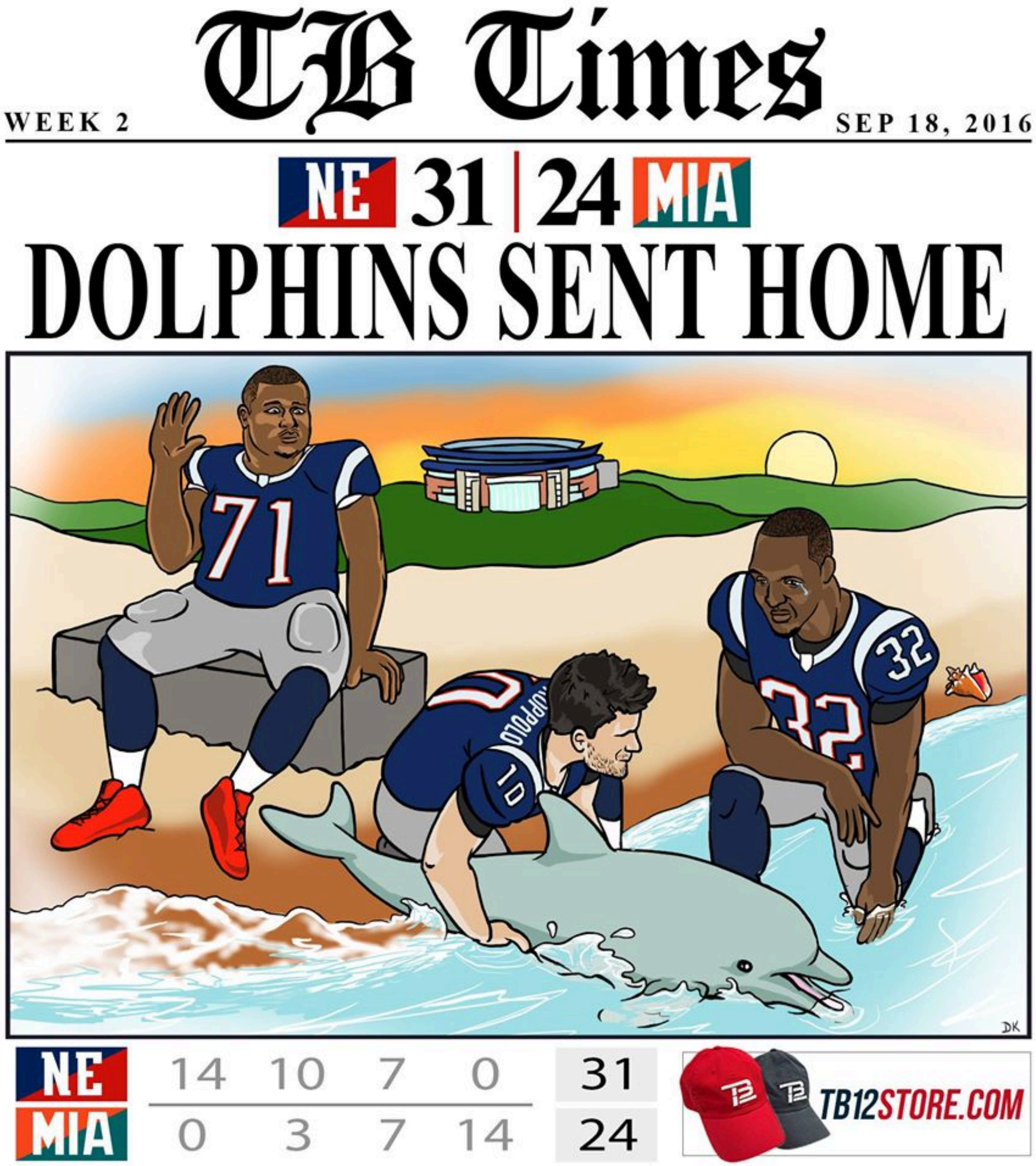
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