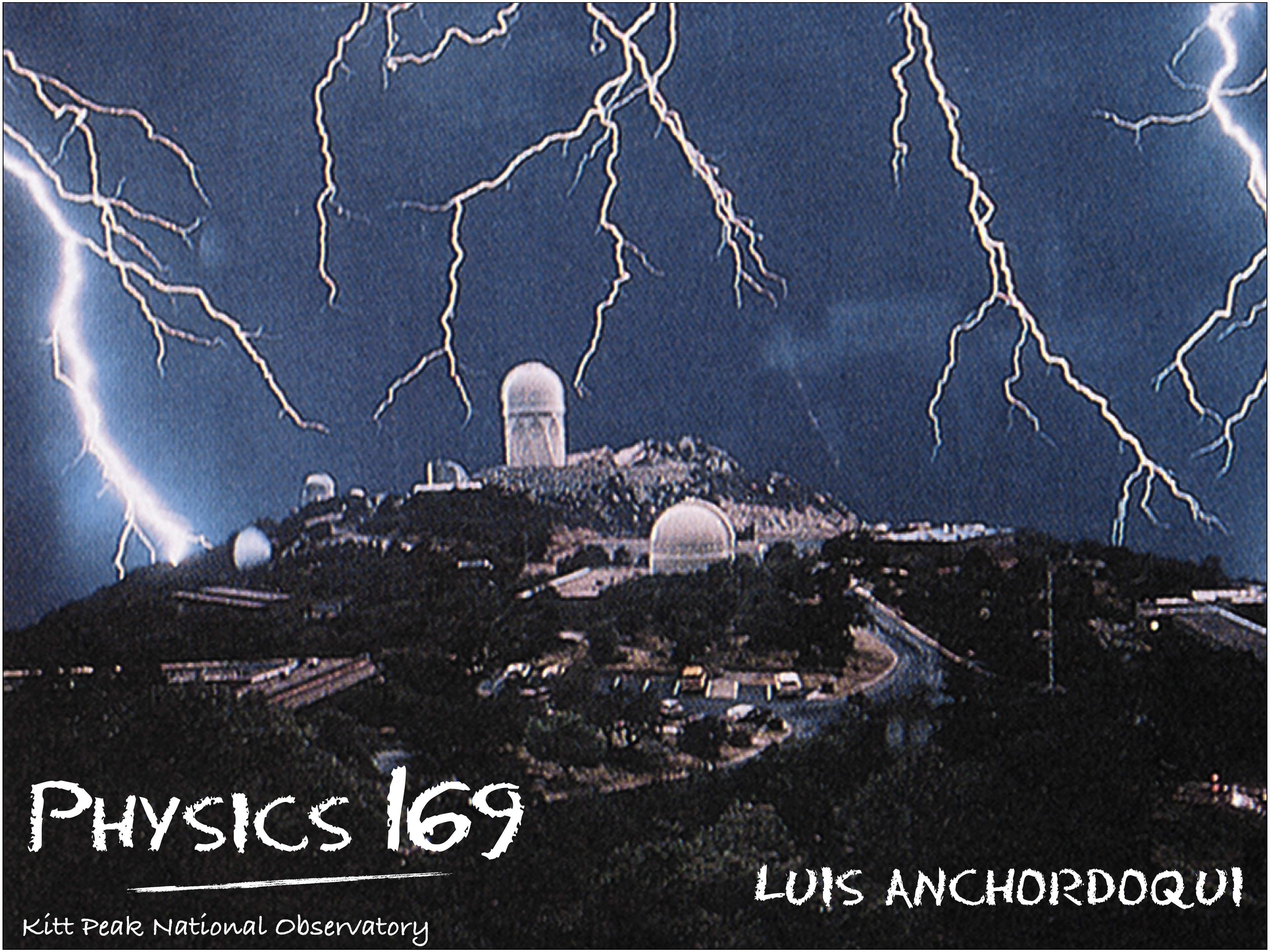

PHYSICS 169
LUIS ANCHORDOQUI
Kitt Peak National Observatory

5.1 Ohm's Law and Resistance

ELECTRIC CURRENT ☛

is defined as flow of electric charge through a cross-sectional area

$$i = \frac{dQ}{dt}$$ Unit ☛ Ampere $[\mathrm{A} = C/s]$

Convention

① Direction of current is direction of flow of positive charge

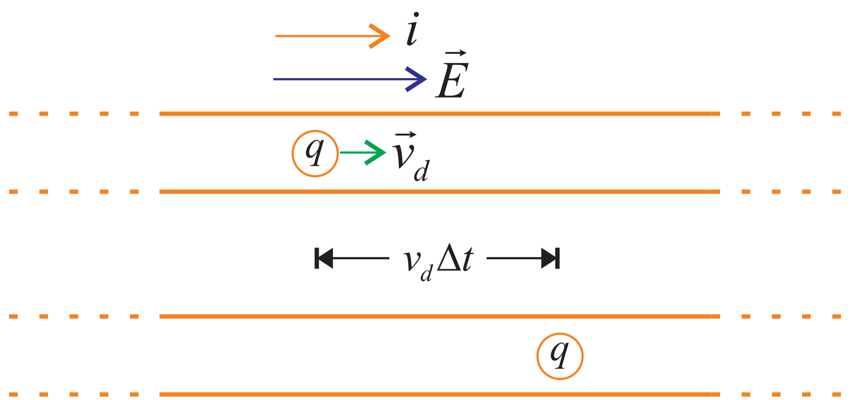
② Current is NOT a vector ☛ but the current density is a vector

$\vec{j} =$ charge flow per unit time per unit area

$$i = \int \vec{j} \cdot d\vec{A}$$

Drift Velocity

Consider a current i flowing through a cross-sectional area A



$\therefore$ In time Δt ☛ total charges passing through segment

$$\Delta Q = qA(v_d \Delta t)n$$

q ☛ charge of current carrier

n ☛ density of charge carrier per unit volume

$\therefore$ Current $\quad i = \dfrac{\Delta Q}{\Delta t} = nqAv_d$

Current Density $\quad \vec{j} = nq\vec{v}_d$

Note

For metals ☛ charge carriers are free electrons inside

$\therefore \quad \vec{j} = -ne\vec{v}_d$ for metals

$\therefore$ Inside metals $\vec{j}$ and $\vec{v}_d$ are in opposite direction

We define a general property of materials ☛ conductivity (σ)

$$\vec{j} = \sigma \vec{E}$$

Note

In general σ is NOT a constant number
but rather a **function of position and applied $\vec{E}$-field**

Resistivity (ρ) is more commonly used property defined as $\rho = \dfrac{1}{\sigma}$

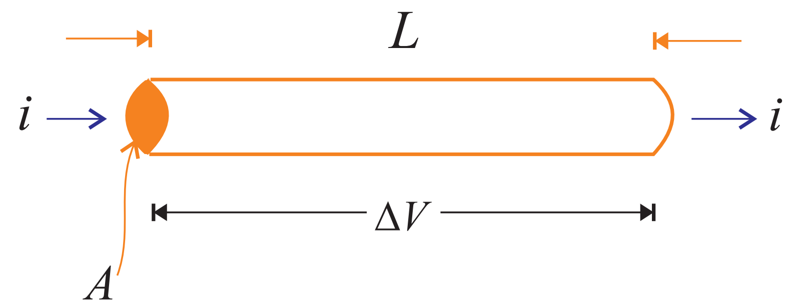
Unit of ρ : Ohm-meter (Ωm) where Ohm (Ω) = Volt/Ampere

OHM'S LAW Ohmic materials have resistivity
that are independent of applied electric field

e.g. metals (in not too high $\vec{E}$ -field)

Example

Consider a **resistor** (ohmic material) of length L and cross-sectional area A



$\therefore$ Electric field inside conductor

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s} = E \int_0^L dx = EL \Rightarrow E = \frac{\Delta V}{L}$$

Current density $j = \dfrac{i}{A}$

$$\therefore \quad \rho = \frac{E}{j}$$

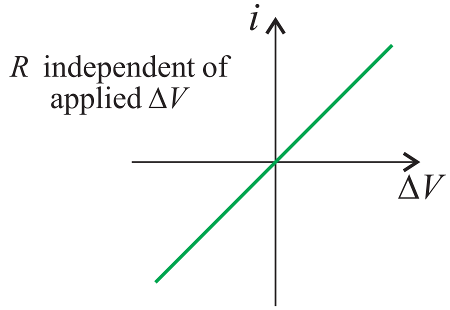
$$\rho = \frac{\Delta V}{L} \cdot \frac{1}{i/A}$$

$$\frac{\Delta V}{i} = R = \rho \frac{L}{A}$$

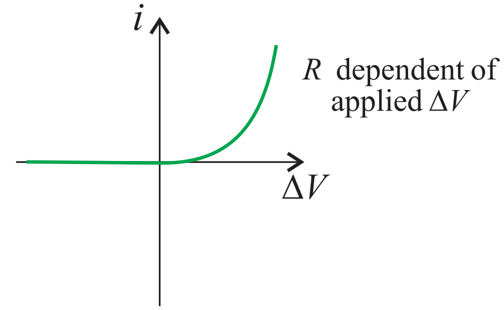
R ☛ **resistance** of conductor

Note $\Delta V = iR$ is NOT a statement of Ohm's Law

but it's just a definition for resistance



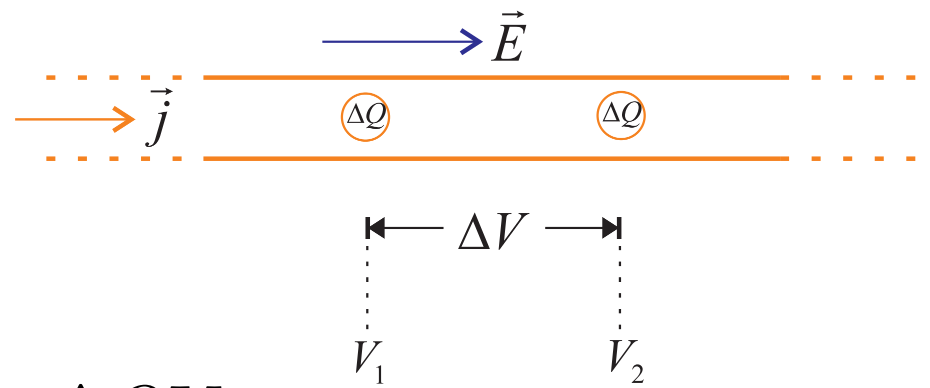
Ohmic material



Non-ohmic material

ENERGY IN CURRENT

Assuming a charge ΔQ enters with potential V_1 and leaves with potential V_2



$\therefore$ Potential energy lost in wire

$$\Delta U = \Delta Q V_2 - \Delta Q V_1$$

$$\Delta U = \Delta Q (V_2 - V_1)$$

$\therefore$ Rate of energy lost per unit time

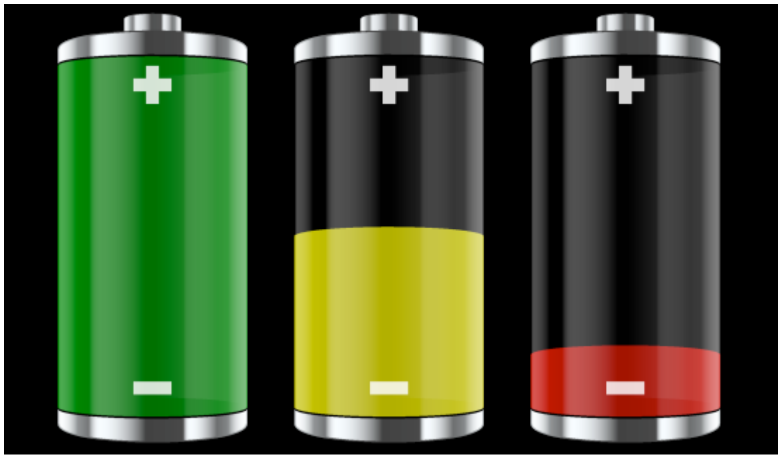
$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}(V_2 - V_1)$$

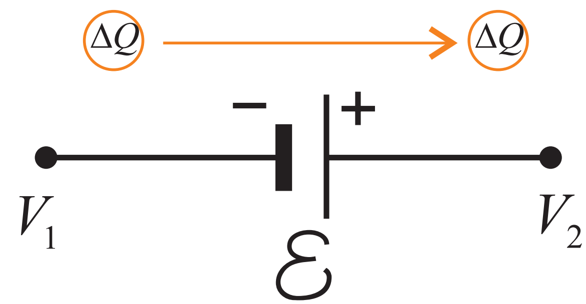
Joule's heating ☛ $P = i \cdot \Delta V$ = Power dissipated in conductor

For a resistor R ➡ $P = i^2 R = \dfrac{\Delta V^2}{R}$

5.2 DC Circuits

A **battery** is a device that **supplies electrical energy** to maintain a current in a circuit



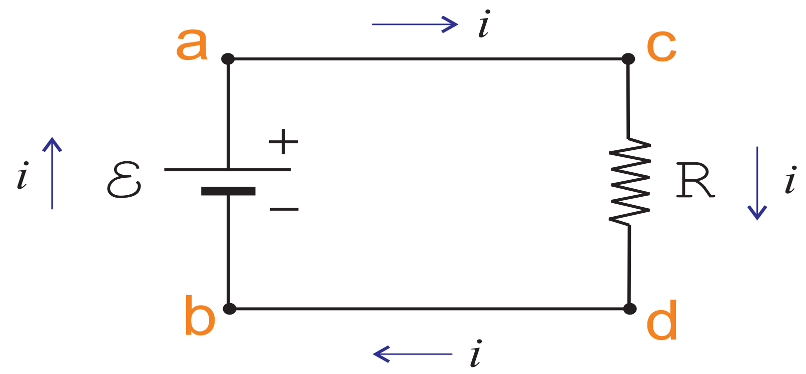


In moving from point 1 to 2 electric potential energy increase by

$$\Delta U = \Delta Q(V_2 - V_1) = \text{ Work done by } \mathcal{E}$$

Define $\mathcal{E} =$ Work done/charge $= V_2 - V_1$

Example



$$\left.\begin{aligned} V_a &= V_c \\ V_b &= V_d \end{aligned}\right\}$$ assuming perfect conducting wires

By Definition

$$V_c - V_d = iR$$

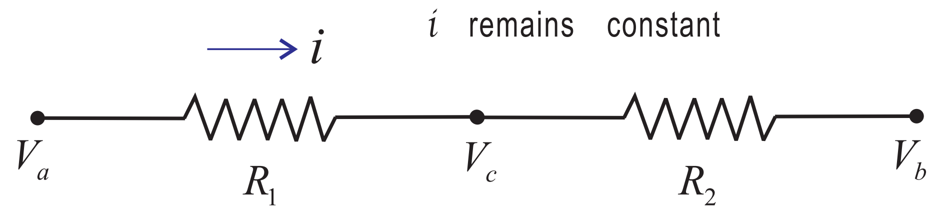
$$V_a - V_b = \mathcal{E}$$

$$\therefore \quad \mathcal{E} = iR \quad \Rightarrow \quad i = \frac{\mathcal{E}}{R}$$

Also ☛ we have assumed zero resistance inside battery

Resistance in combination

Resistors in series



Potential difference

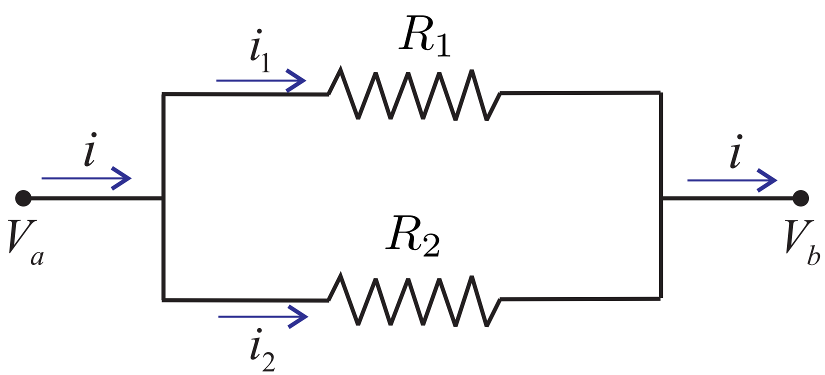
$$V_a - V_b = (V_a - V_c) + (V_c - V_b)$$
$$= iR_1 + iR_2$$

$\therefore$ Equivalent Resistance for resistors in series $R = R_1 + R_2$

Resistors in parallel

Resistors in parallel carry same voltage

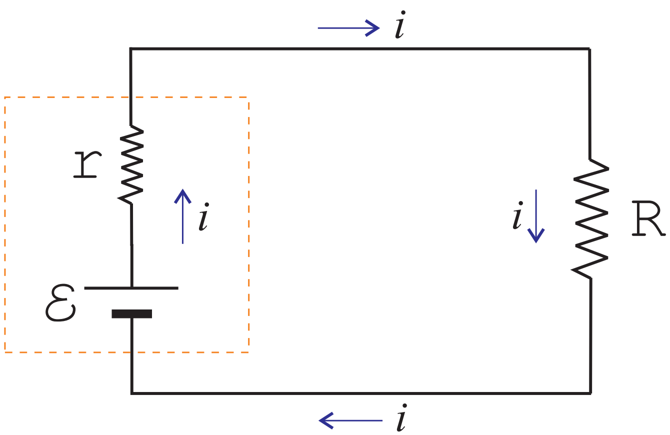
Current flowing through each resistor could be different



$$i = i_1 + i_2$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Equivalent Resistance for resistors in parallel $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Example For real battery ☛ there is an **internal resistance** that we cannot ignore



$$\therefore \quad \mathcal{E} = i(R + r)$$

$$i = \frac{\mathcal{E}}{R + r}$$

Joule's heating in resistor R ➡ $P = i \cdot$ (P.D. across resistor R)

$$= i^2 R$$

$$P = \frac{\mathcal{E}^2 R}{(R + r)^2}$$

Question ☛ What is value of R to obtain maximum Joule's heating?

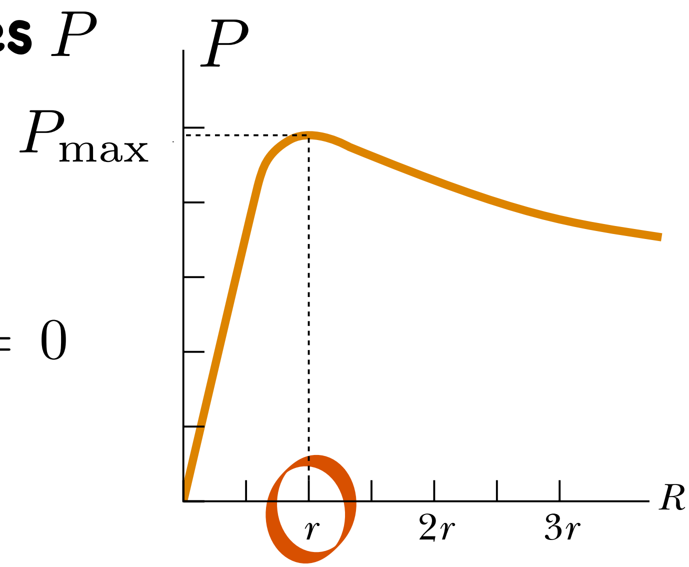
Answer ☛ We want to find R that **maximizes** P

$$\frac{dP}{dR} = \frac{\mathcal{E}^2}{(R + r)^2} - \frac{\mathcal{E}^2 2R}{(R + r)^3}$$

Setting $\frac{dP}{dR} = 0 \Rightarrow \frac{\mathcal{E}^2}{(R + r)^3}[(R + r) - 2R] = 0$

$$\Rightarrow r - R = 0$$

$$\Rightarrow R = r$$



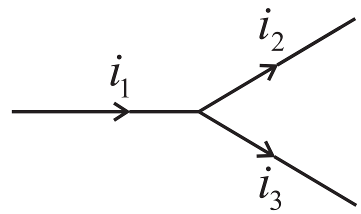
ANALYSIS OF COMPLEX CIRCUITS

KIRCHOFF'S LAWS

① First Law (Junction Rule):

Total current entering a junction equal to total current leaving junction

Conservation of charge



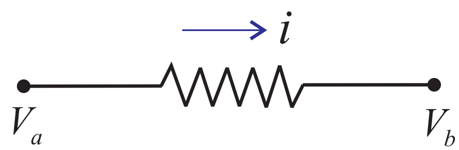
$$\Rightarrow i_1 = i_2 + i_3$$

② Second Law (Loop Rule):

The sum of potential differences around a complete circuit loop is zero

Convention

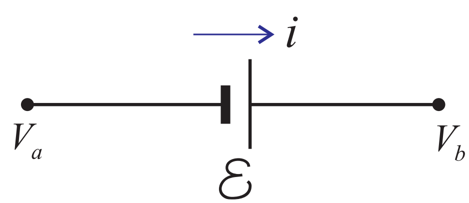
(i)



$$V_a > V_b \quad \Rightarrow \text{Potential difference} = -iR$$

i.e. Potential **drops** across resistors

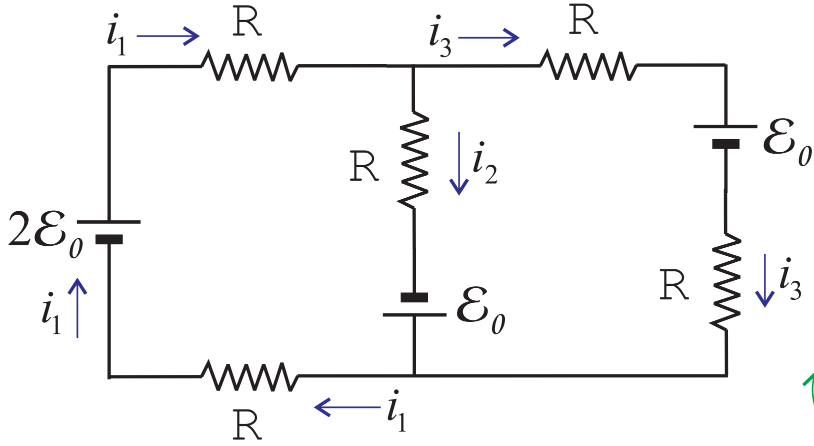
(ii)



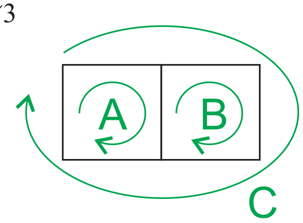
$$V_b > V_a \quad \Rightarrow \text{Potential difference} = +\mathcal{E}$$

i.e. Potential **rises across** negative plate of battery

Example



Find i_1, i_2, i_3



By junction rule:

$$i_1 = i_2 + i_3 \qquad \textbf{(6.1)}$$

By loop rule:

$$\text{Loop A} \quad \Rightarrow \quad 2\mathcal{E}_0 - i_1 R - i_2 R + \mathcal{E}_0 - i_1 R = 0 \qquad \textbf{(6.2)}$$

$$\text{Loop B} \quad \Rightarrow \quad -i_3 R - \mathcal{E}_0 - i_3 R - \mathcal{E}_0 + i_2 R = 0 \qquad \textbf{(6.3)}$$

$$\text{Loop C} \quad \Rightarrow \quad 2\mathcal{E}_0 - i_1 R - i_3 R - \mathcal{E}_0 - i_3 R - i_1 R = 0 \qquad \textbf{(6.4)}$$

BUT ☛ (6.4) = (6.2) + (6.3)

General rule Need only 3 equations for 3 current

$$i_1 = i_2 + i_3 \quad \textbf{(6.1)}$$

$$3\mathcal{E}_0 - 2i_1R - i_2R = 0 \quad \textbf{(6.2)}$$

$$-2\mathcal{E}_0 + i_2R - 2i_3R = 0 \quad \textbf{(6.3)}$$

Substitute **(6.1)** into **(6.2)**

$$3\mathcal{E}_0 - 2(i_2 + i_3)R - i_2R = 0 \quad \textbf{(6.4)}$$

$$\Rightarrow \quad 3\mathcal{E}_0 - 3i_2R - 2i_3R = 0$$

Subtract **(6.3)** from **(6.4)**, i.e. **(6.4)-(6.3)**

$$3\mathcal{E}_0 - (-2\mathcal{E}_0) - 3i_2R - i_2R = 0$$

$$\Rightarrow \quad i_2 = \frac{5}{4} \cdot \frac{\mathcal{E}_0}{R}$$

Substitute i_2 into **(6.3)**

$$-2\mathcal{E}_0 + \left(\frac{5}{4} \cdot \frac{\mathcal{E}_0}{R}\right)R - 2i_3R = 0$$

$$\Rightarrow \quad i_3 = -\frac{3}{8} \cdot \frac{\mathcal{E}_0}{R}$$

Substitute i_2, i_3 into **(6.1)**

$$i_1 = \left(\frac{5}{4} - \frac{3}{8}\right)\frac{\mathcal{E}_0}{R} = \frac{7}{8} \cdot \frac{\mathcal{E}_0}{R}$$

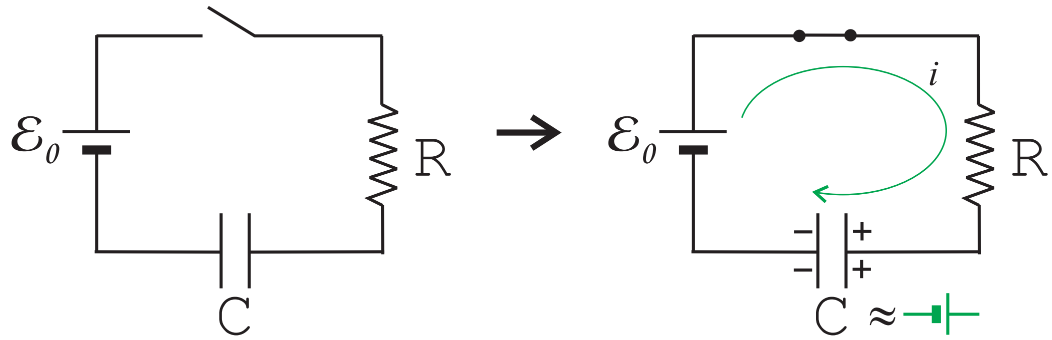
Note

A **negative** current means that

it is flowing in **opposite direction** from one assumed

5.3 RC Circuits

(A) Charging a capacitor with battery



Using loop rule

$$+\mathcal{E}_0 \quad - \underbrace{iR}_{\text{P.D across } R} \quad - \underbrace{\frac{Q}{C}}_{\text{P.D across } C} = 0$$

Note ☛

Direction of i is chosen so that current represents rate at which charge on capacitor is **increasing**

$$\therefore \quad \mathcal{E} = R\overbrace{\frac{dQ}{dt}}^{i} + \frac{Q}{C}$$

1st order differential eq.

$$\Rightarrow \quad \frac{dQ}{\mathcal{E}C - Q} = \frac{dt}{RC}$$

Integrate both sides and use initial condition

$$t = 0, \quad Q \text{ on capacitor } = 0$$

$$\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC}$$

$$-\ln(\mathcal{E}C - Q)\Big|_0^Q = \frac{t}{RC}\Big|_0^t$$

$$\Rightarrow \quad -\ln(\mathcal{E}C - Q) + \ln(\mathcal{E}C) = \frac{t}{RC}$$

$$\Rightarrow \quad \ln\left(\frac{1}{1 - \frac{Q}{\mathcal{E}C}}\right) = \frac{t}{RC}$$

$$\Rightarrow \quad \frac{1}{1 - \frac{Q}{\mathcal{E}C}} = e^{t/RC}$$

$$\Rightarrow \quad \frac{Q}{\mathcal{E}C} = 1 - e^{-t/RC}$$

$$\Rightarrow \quad Q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)$$

Note ☛

① At $t = 0$, $Q(t = 0) = \mathcal{E}C\,(1 - 1) = 0$

② As $t \to \infty$, $Q(t \to \infty) = \mathcal{E}C\,(1 - 0) = \mathcal{E}C$

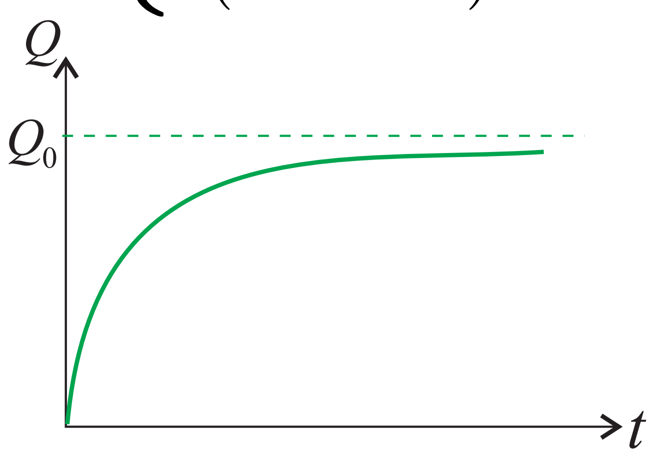
③ Current

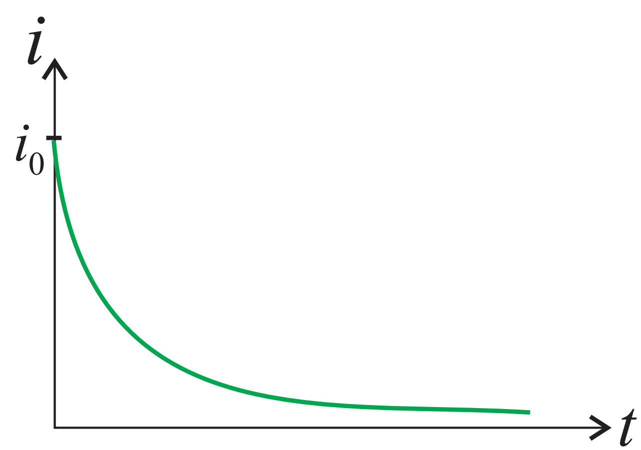
$$i = \frac{dQ}{dt}$$

$$= \mathcal{E}C\left(\frac{1}{RC}\right)e^{-t/RC}$$

$$i(t) = \frac{\mathcal{E}}{R}\,e^{-t/RC}$$

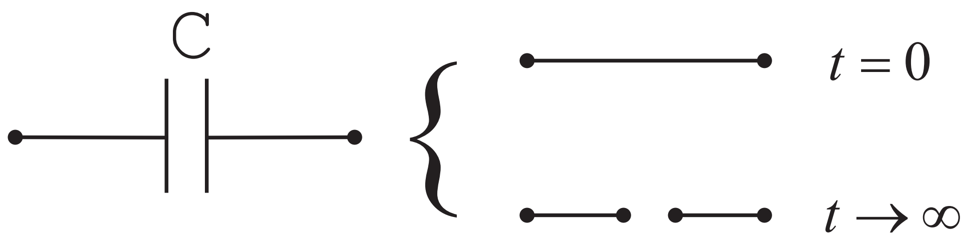
$$\begin{cases} i(t = 0) & \dfrac{\mathcal{E}}{R} = \text{Initial current} = i_0 \\ i(t \to \infty) & = 0 \end{cases}$$





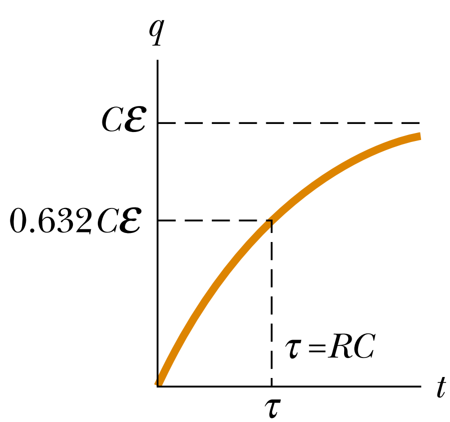
④ At time $= 0$ ☛ capacitor acts like **short circuit** when there is **zero charge on capacitor**

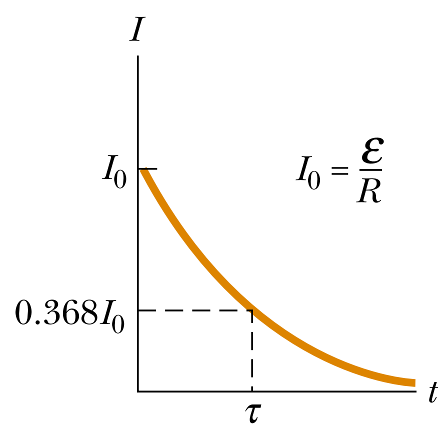
⑤ As time $\rightarrow \infty$ ☛ capacitor is **fully charged** and current $= 0$ it acts like a **open circuit**



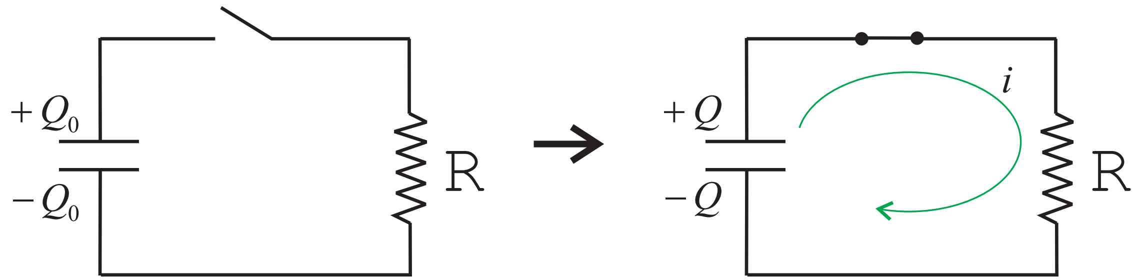
⑥ $\tau_c = RC$ is called time constant

It is time it takes for charge to reach $\left(1 - \frac{1}{e}\right) Q_0 \simeq 0.63\, Q_0$





(B) Discharging a charged capacitor



Note ☛

Direction of i is chosen so that

current represents rate at which charge on capacitor is **decreasing**

$$\therefore \quad i = -\frac{dQ}{dt}$$

Loop Rule

$$V_c - iR = 0$$

$$\Rightarrow \quad \frac{Q}{C} + \frac{dQ}{dt}R = 0$$

$$\Rightarrow \quad \frac{dQ}{dt} = -\frac{1}{RC}Q$$

Integrate both sides and use initial condition

$t_0, \quad Q$ on capacitor $= Q_0$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC} \int_0^t dt$$

$$\Rightarrow \quad \ln Q - \ln Q_0 = -\frac{t}{RC}$$

$$\Rightarrow \quad \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

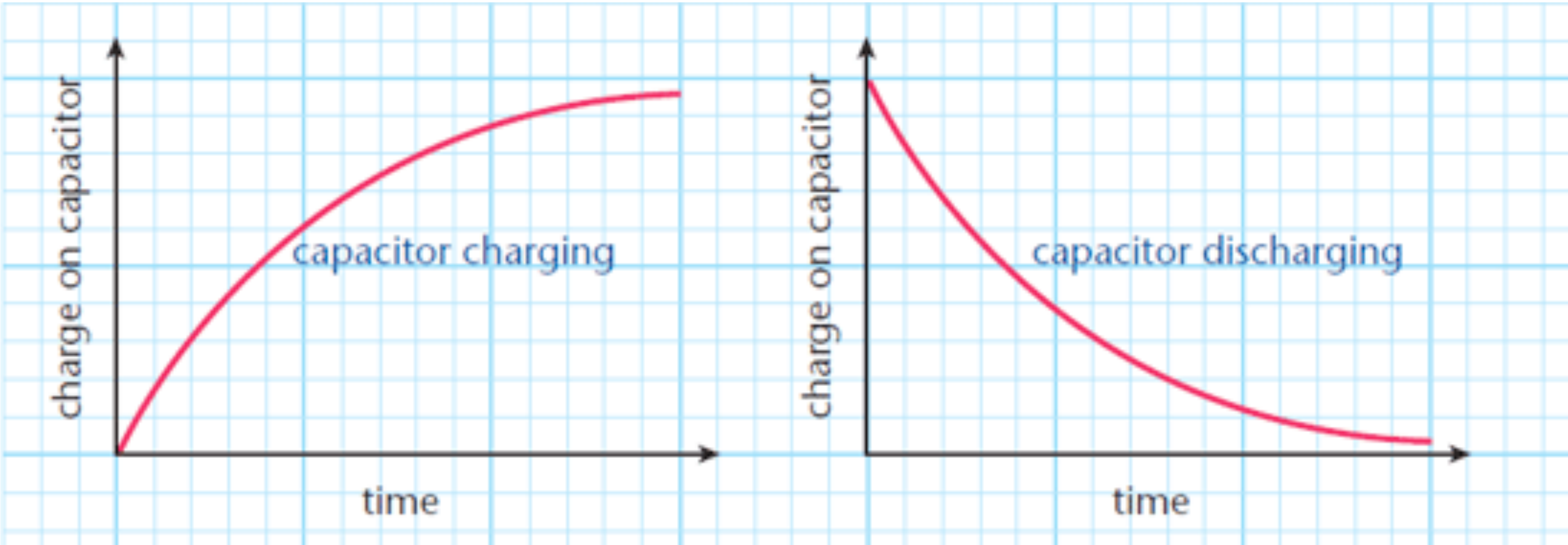
$$\Rightarrow \quad \frac{Q}{Q_0} = e^{-t/RC}$$

$$\Rightarrow \quad Q(t) = Q_0\, e^{-t/RC}$$

$$\left(i = -\frac{dQ}{dt}\right) \quad \Rightarrow \quad i(t) = \frac{Q_0}{RC}\, e^{-t/RC}$$

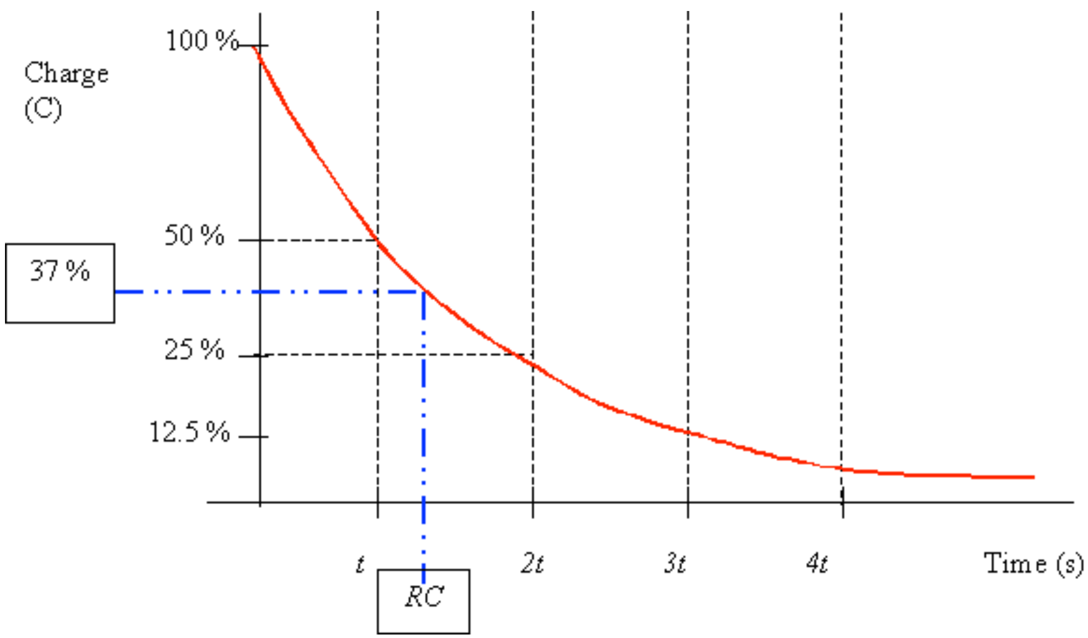
$$(\text{At } t = 0) \quad \Rightarrow \quad i(t = 0) = \frac{1}{R} \cdot \underbrace{\frac{Q_0}{C}}_{\text{Initial P.D. across capacitor}}$$

$$i_0 = \frac{V_0}{R}$$



The charge on a capacitor during charging and discharging

At $t = RC = \tau$ $$Q(t = RC) = \frac{1}{e} Q_0 \simeq 0.37\, Q_0$$



5.4 Semiconductors

A semiconductor is a substance that can conduct electricity under some conditions but not others making it a good medium for control of electrical current

Specific properties of a semiconductor depend on impurities added to it (or dopants)

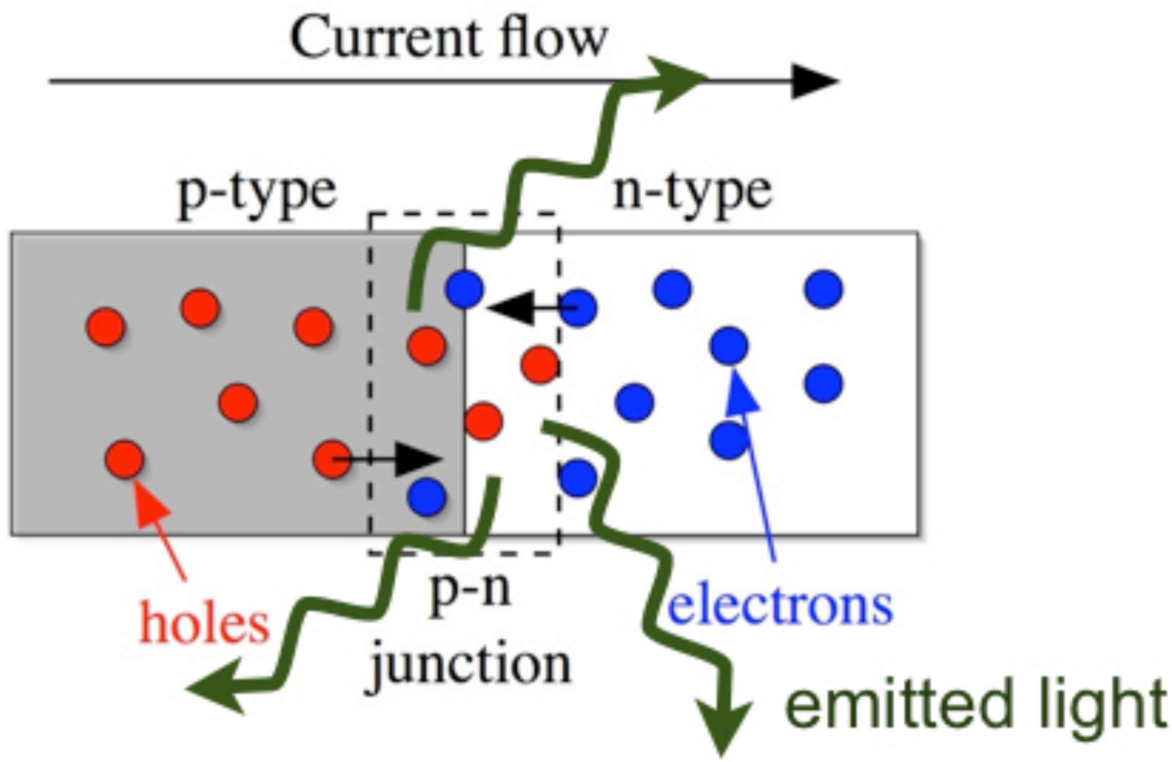
N-type semiconductor carries current in form of negatively-charged e^- in a manner similar to conduction of current in a wire

P-type semiconductor carries current predominantly as e^- deficiencies called **holes**

A hole has a positive electric charge equal and opposite to charge on e^-

In a semiconductor material flow of holes occurs in a direction opposite to flow of electrons

Acronym LED stands for Light Emitting Diode



A diode is a combination of two semiconductors

one of which is doped n-type and other p-type

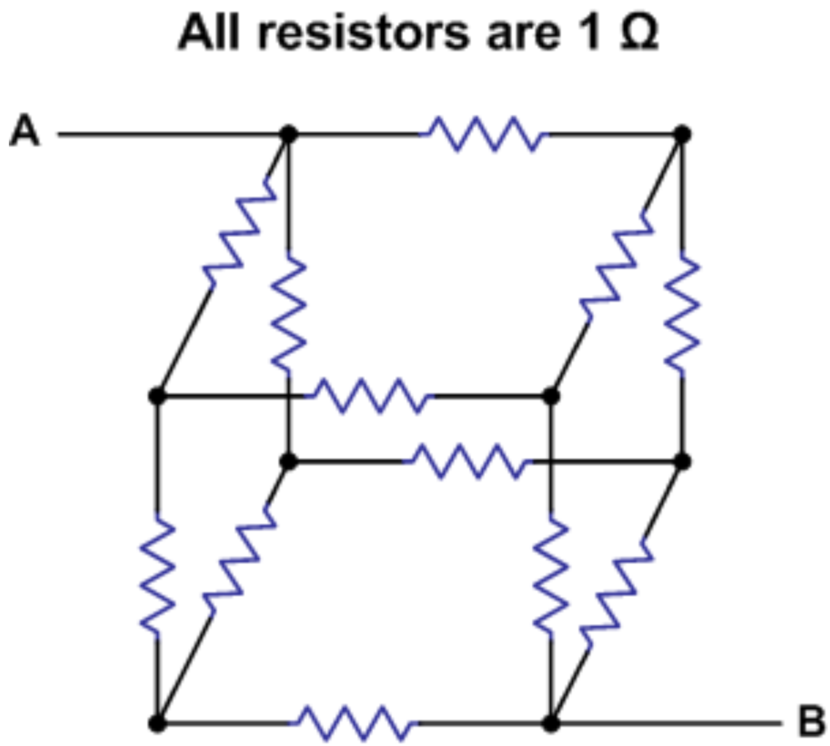
(region between n-type and p-type material)

When electrons and holes meet in junction

they can recombine to liberate energy

5.5 Intuition

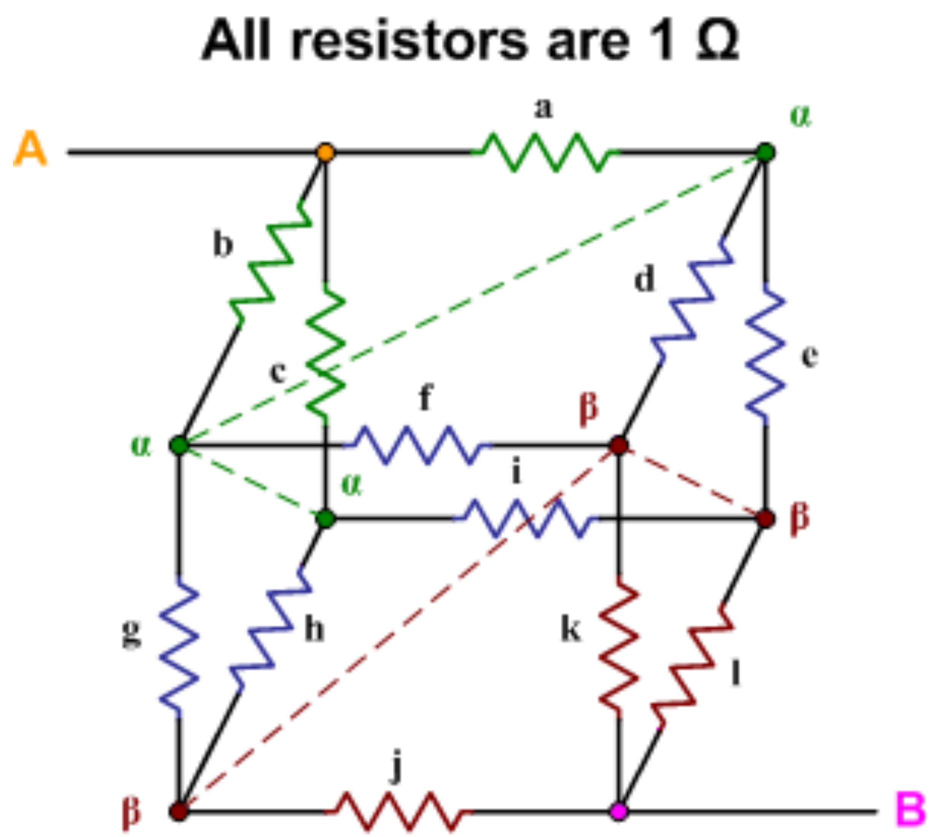
Each of the 12 edges of a cube contain a 1 Ω resistor



Calculate the equivalent resistance between two opposing corners

Here is where the intuition comes into play
Color coding is used to help keep track of the resistors and associated nodes
Due to symmetry P.D. at the three nodes labeled "α" are equal

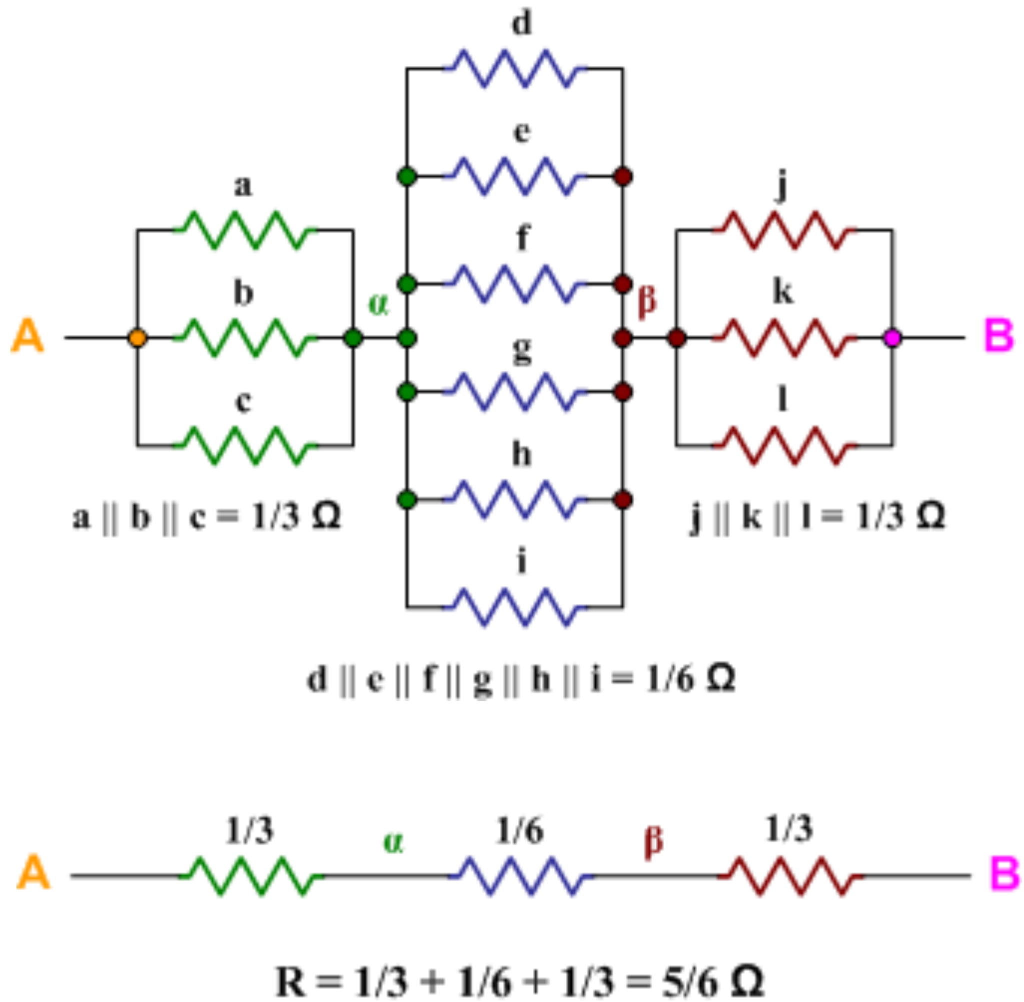
Since no current flows between nodes with a potential difference of zero V, they can be shorted together without affecting the circuit's integrity
The same can be done for the nodes labeled "β"



Once you short those nodes ☛ you obtain the following equivalent circuit

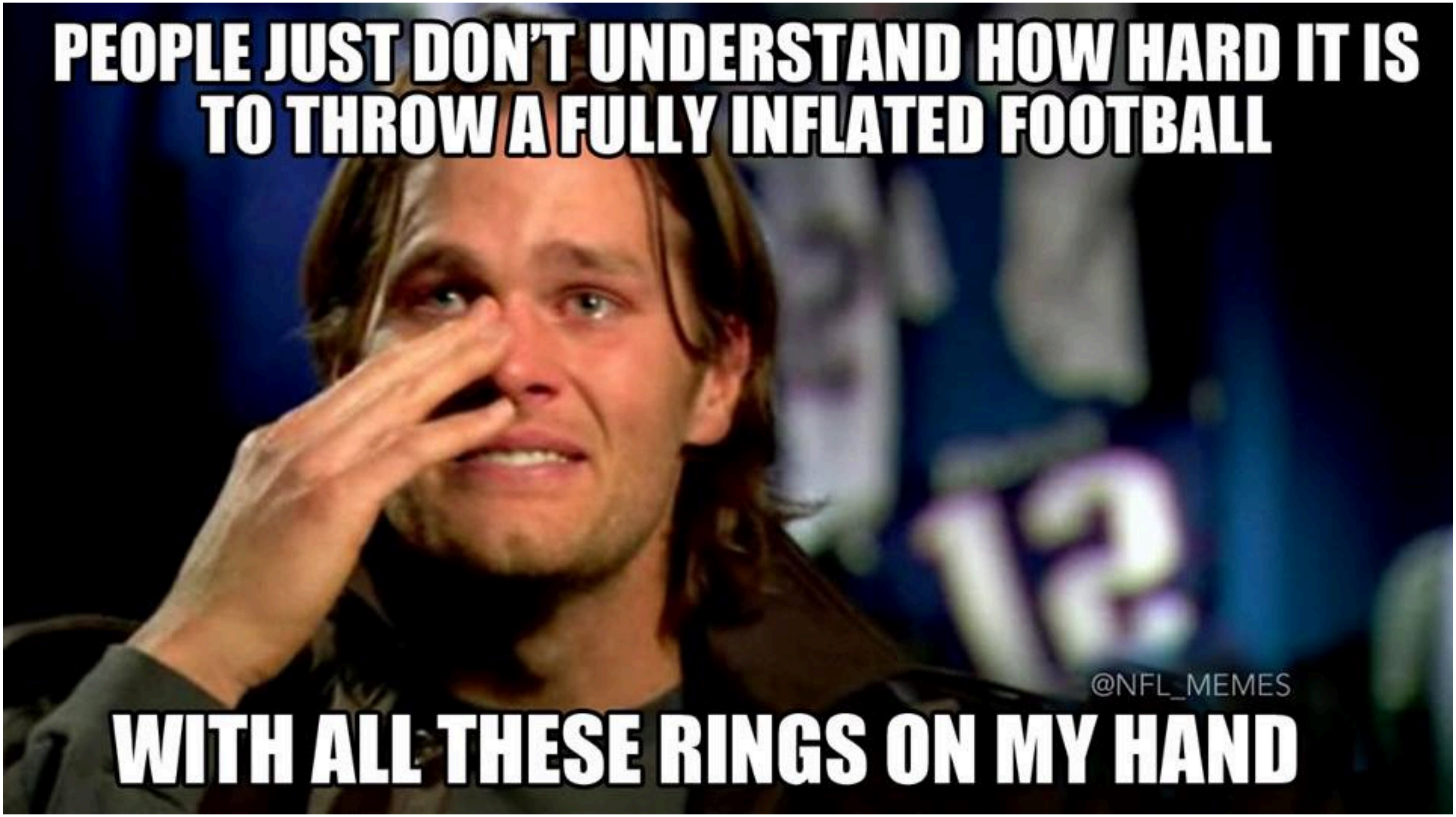
There are two sets of three resistors in parallel

in series with one set of six resistors in parallel



So ☛ you have 1/3 Ω in series with 1/6 Ω in series with 1/3 Ω ➡ which equals 5/6 Ω


HOW MANY RINGS
AVRIL
TOM BRADY GOT?


PEOPLE JUST DON'T UNDERSTAND HOW HARD IT IS TO THROW A FULLY INFLATED FOOTBALL
@NFL_MEMES
WITH ALL THESE RINGS ON MY HAND