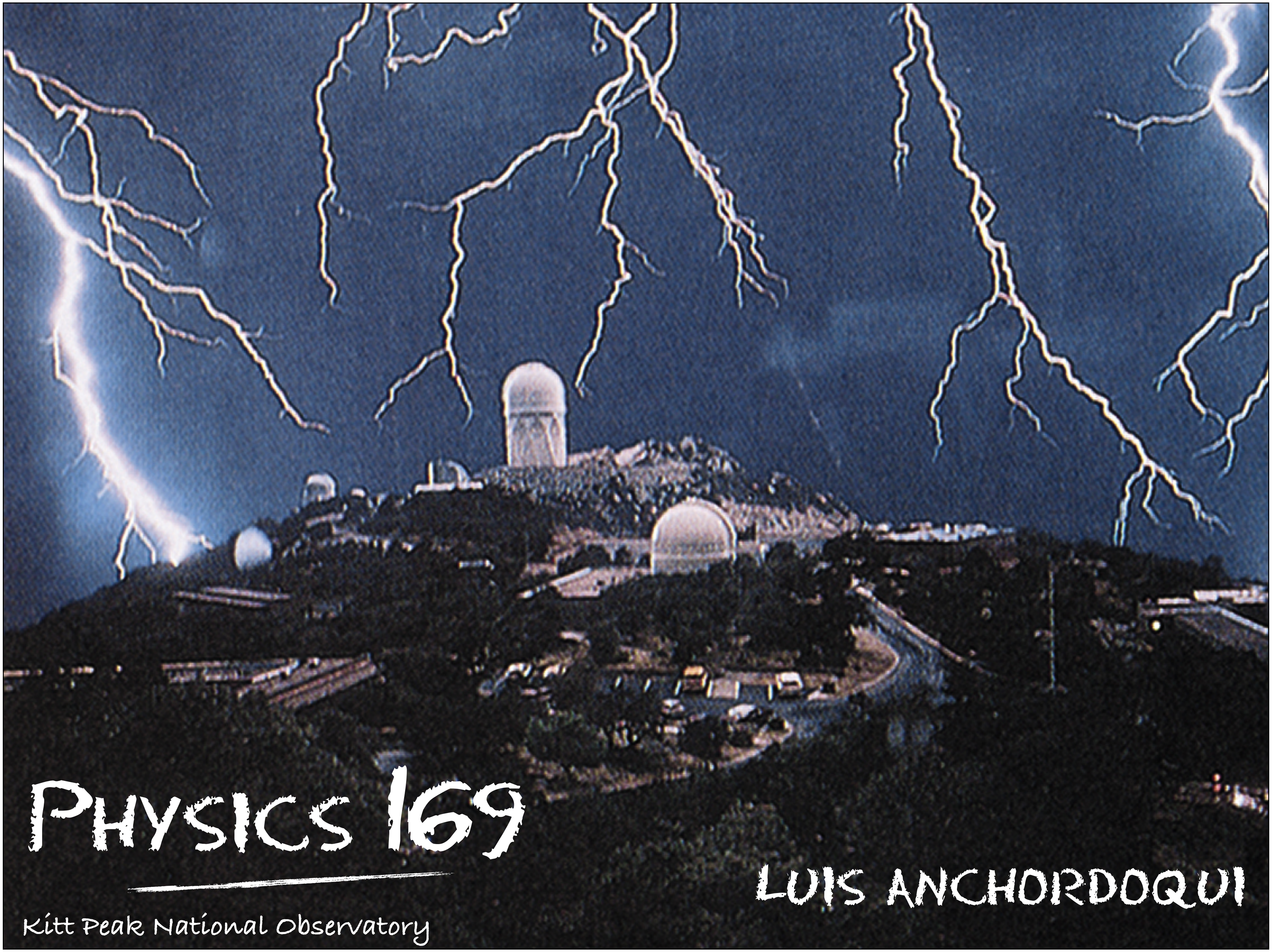
PHYSICS 169
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Kitt Peak National Observatory

2.1 Electric Field Lines

Recall from last class that ☛ to visualize electric field

we can use a graphical tool called electric field lines

Conventions

1. Start on positive charges and end on negative charges

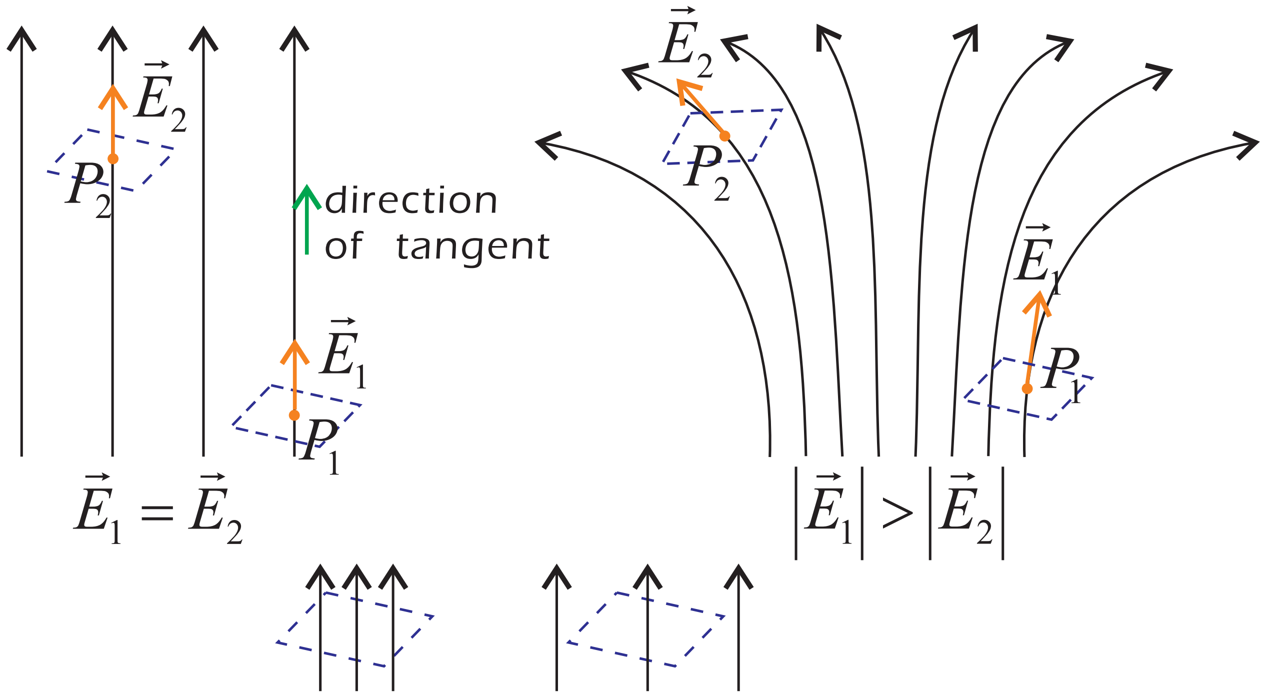
2. Direction of E-field at any point is given by tangent of E-field line

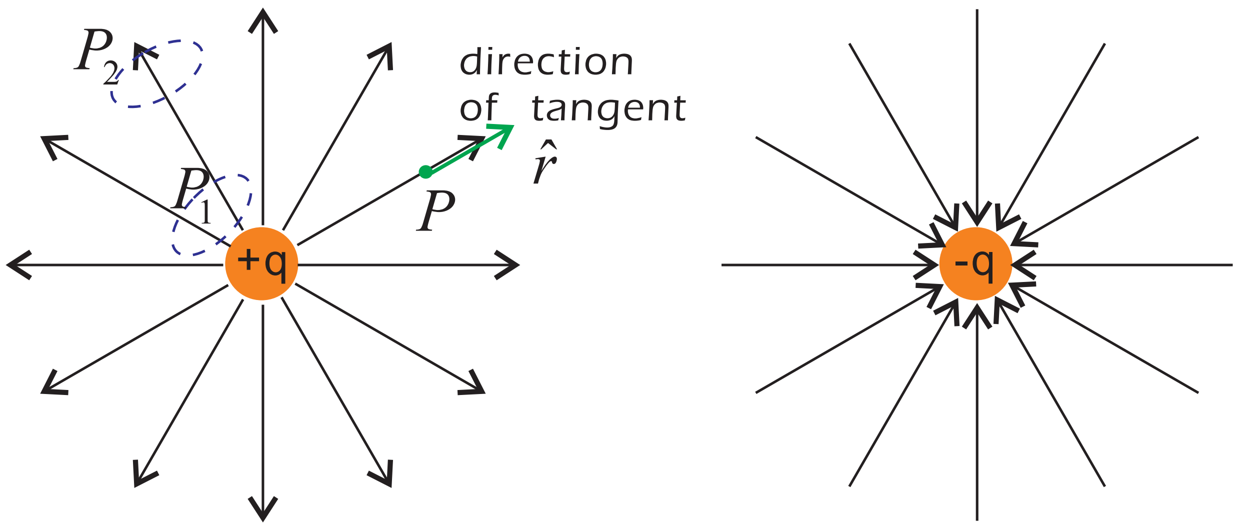
3. Magnitude of E-field at any point

proportional to number of E-field lines per unit area perpendicular to lines

Uniform E-field

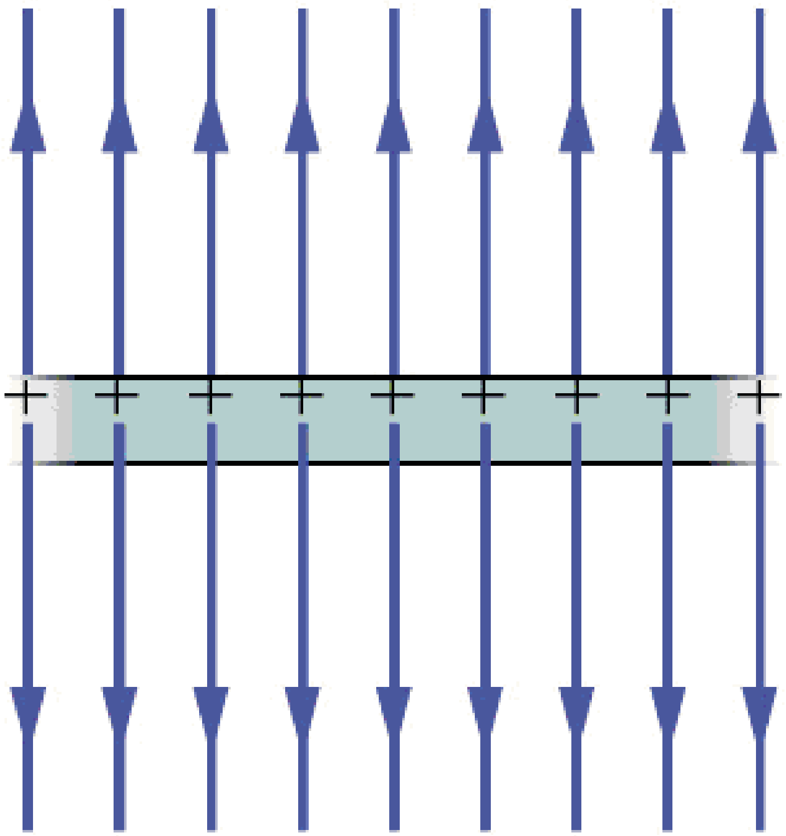
Non-uniform E-field



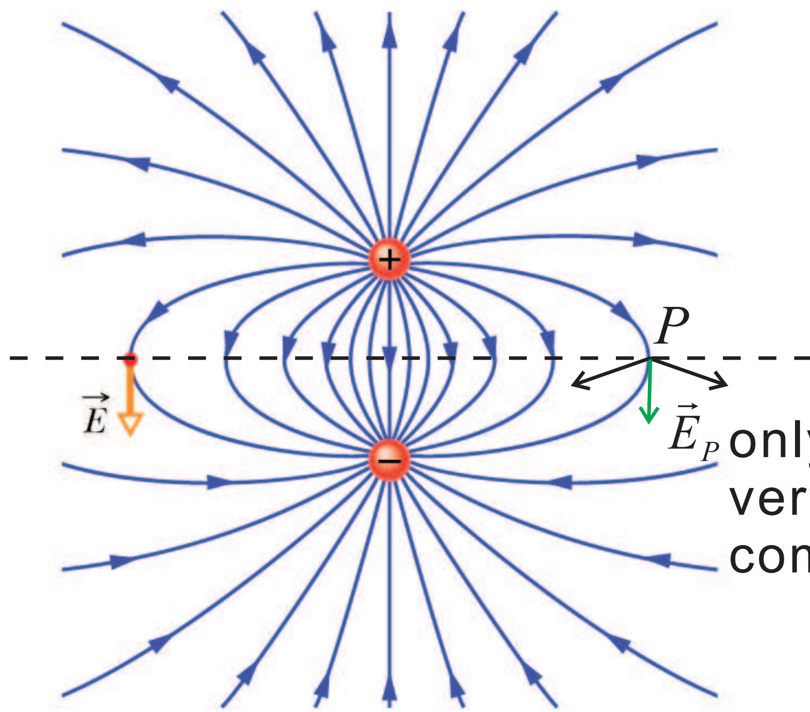


$$\left|\vec{E}_{P_1}\right| > \left|\vec{E}_{P_2}\right| \qquad \vec{E} = \frac{+q}{4\pi\epsilon_0 r^2}\hat{r}$$

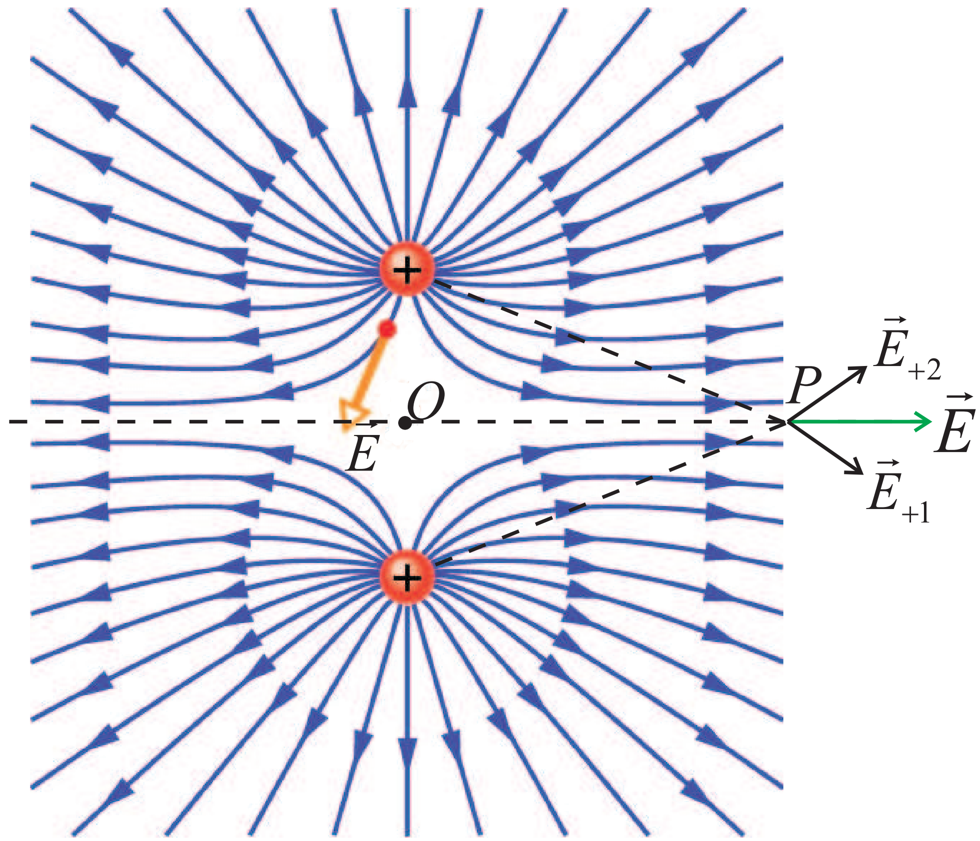
Infinite sheet of charge



$$E = \frac{\sigma}{2\epsilon_0}$$



only has vertical component.



$$\vec{E}_{\text{at point } O} = 0$$

2.2 Dipole in E-field

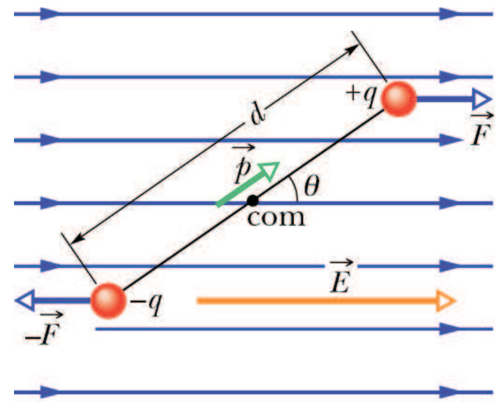
Consider force exerted on dipole in an external E-field

Assumption ☛ E-field from dipole doesn't affect external E-field

- Dipole moment $\vec{p} = q\vec{d}$
- Force due to E-field on $+q$ and $-q$ charge

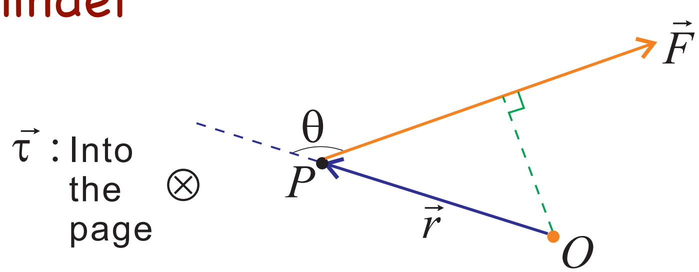
are **equal and opposite in direction**

Total external force on dipole $= 0$



BUT ☛ There is an external **torque** on center of dipole

Reminder

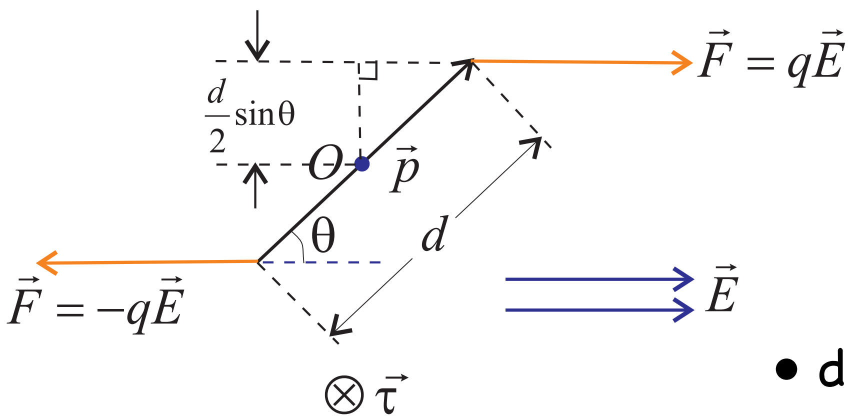


Force $\vec{F}$ exerts at point P

force exerts a torque $\vec{\tau} = \vec{r} \times \vec{F}$

on point P with respect to point O

Direction of torque vector $\vec{\tau}$ is determined from **right-hand rule**



Net torque $\vec{\tau}$

- direction ☛ clockwise torque

- magnitude ☛

$$\tau = \tau_{+q} + \tau_{-q}$$

$$= F \cdot \frac{d}{2} \sin\theta + F \cdot \frac{d}{2} \sin\theta$$

$$= qE \cdot d \sin\theta$$

$$= pE \sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Energy Consideration

When dipole $\vec{p}$ rotates $d\theta$ ☛ E-field does work

Work done by external E-field on dipole

$$dW = -\tau\, d\theta$$

Negative sign here because torque by E-field acts to **decrease** θ

BUT ☛ Because E-field is a **conservative force field**

we can define a **potential energy** (U) for system so that

$$dU = -dW$$

$\therefore$ For dipole in external E-field

$$dU = -dW = pE\,\sin\theta\, d\theta$$

$$\therefore U(\theta) = \int dU = \int pE\,\sin\theta\, d\theta$$

$$= -pE\,\cos\theta + U_0$$

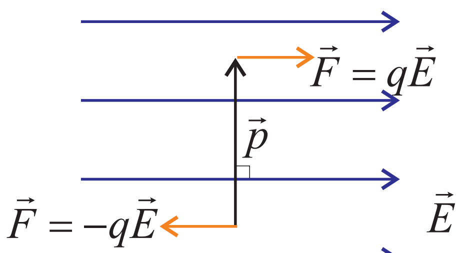
set $U(\theta = 90^\circ) = 0$

$$\therefore\ 0 = -pE \cos 90^\circ + U_0$$

$$\therefore U_0 = 0$$

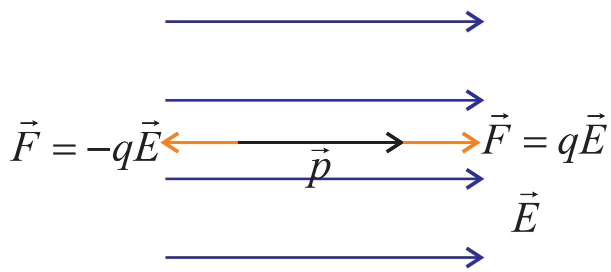
$\therefore$ Potential energy

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$



$\theta = 90^\circ$
Torque $|\vec{\tau}| = pE$

$U = 0$ (define)

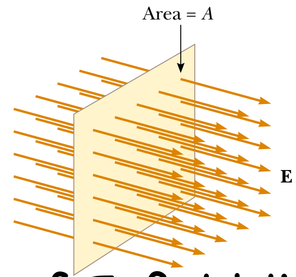


$\theta = 0^\circ$
Torque $|\vec{\tau}| = 0$

$U = -pE$
(based on definition)

Minimum energy configuration

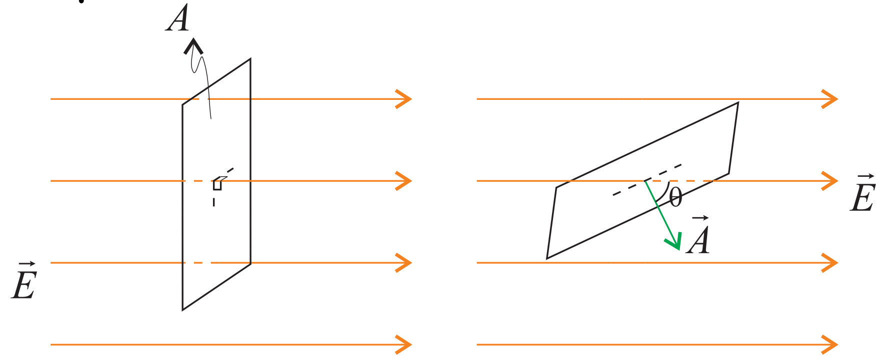
2.3 Electric Flux



Latin ☛ flux = "to flow"

Graphically ☛ Electric flux Φ_E represents number of E-field lines crossing a surface

Mathematically ☛

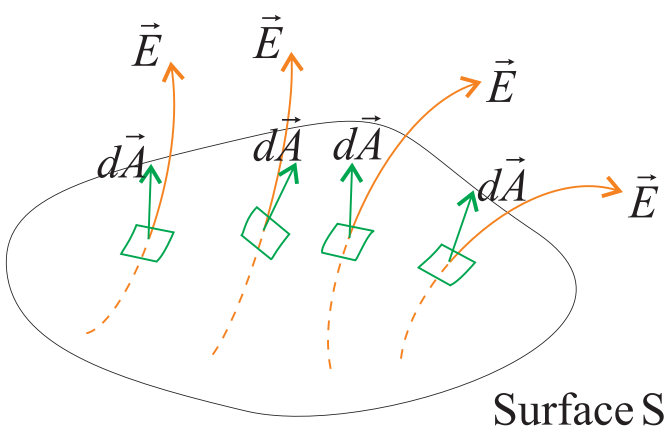


$\Phi_E = EA$ $\Phi_E = \vec{E} \cdot \vec{A}$

Reminder ☛ Vector of area $\vec{A}$ is perpendicular to area A

For non-uniform E-field & surface

direction of area vector $\vec{A}$ is not uniform



$d\vec{A} =$ Area vector for small area element dA

$\therefore$ Electric flux $d\Phi_E = \vec{E} \cdot d\vec{A}$

Electric flux of $\vec{E}$ through surface S ☛ $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$

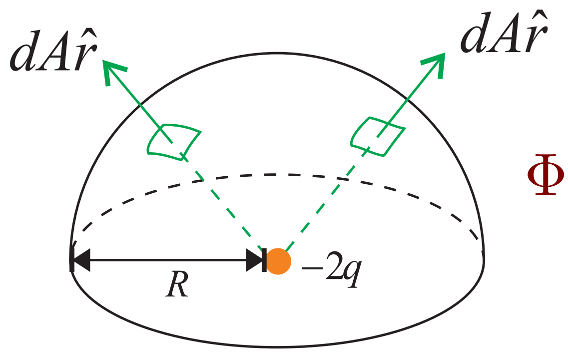
$\int_S$ = Surface integral over surface S

= Integration of integral over all area elements on surface S

Example

S = hemisphere radius R

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{-2q}{r^2}\hat{r} = \frac{-q}{2\pi\epsilon_0 R^2}\hat{r}$$



For a hemisphere, $d\vec{A} = dA\,\hat{r}$

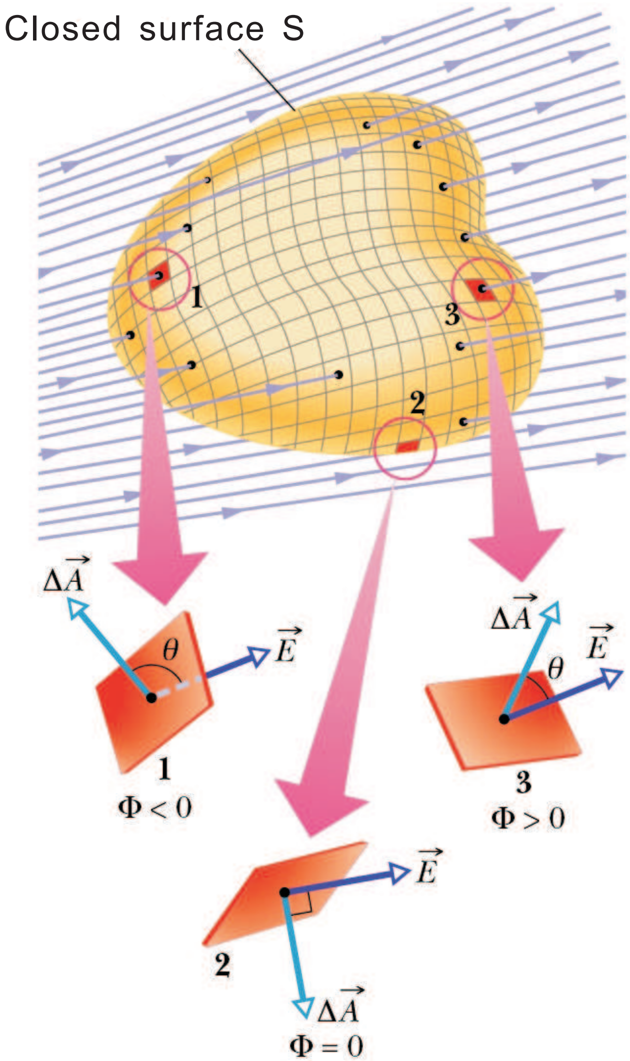
$$\Phi_E = \int_S \frac{-q}{2\pi\epsilon_0 R^2}\hat{r} \cdot (dA\,\hat{r}) \qquad (\because \hat{r} \cdot \hat{r} = 1)$$

$$= -\frac{q}{2\pi\epsilon_0 R^2}\underbrace{\int_S dA}_{2\pi R^2}$$

$$= \frac{-q}{\epsilon_0}$$

$\int_S dA$ = Surface area of S

For a closed surface



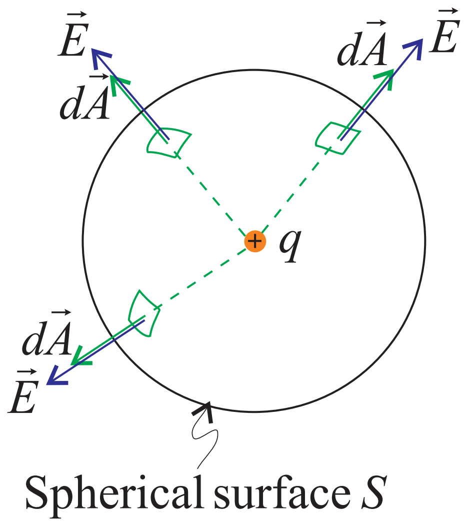
Recall☛ Direction of area vector $d\vec{A}$ goes **from inside to outside** of closed surface S

Electric flux over closed surface S

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$

$\oint_S$ = Surface integral over closed surface S

Example



Electric flux of charge q over closed spherical surface of radius R

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\hat{r} = \frac{q}{4\pi\epsilon_0 R^2}\hat{r} \quad \text{at surface}$$

Again, $d\vec{A} = dA \cdot \hat{r}$

$$\therefore \Phi_E = \oint_S \overbrace{\frac{q}{4\pi\epsilon_0 R^2}\hat{r}}^{\vec{E}} \cdot \overbrace{dA\hat{r}}^{d\vec{A}}$$

$$= \frac{q}{4\pi\epsilon_0 R^2} \underbrace{\oint_S dA}_{\text{Total surface area of S} = 4\pi\text{R}^2}$$

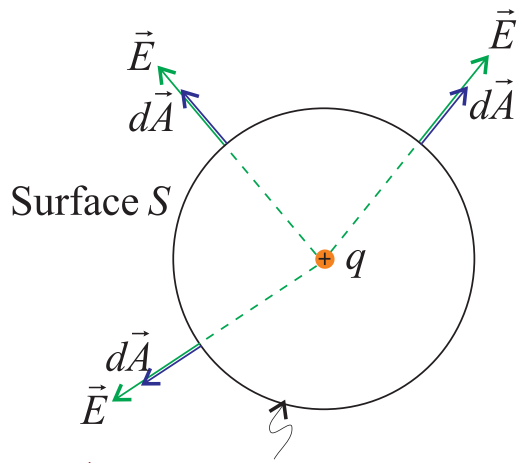
$$\Phi_E = \frac{q}{\epsilon_0}$$

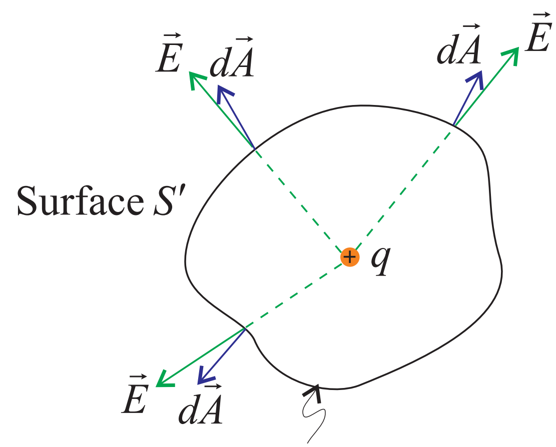
IMPORTANT POINT

If we remove spherical symmetry of closed surface S

total number of E-field lines crossing surface remains same

$\therefore$ electric flux Φ_E





$\vec{E} \parallel d\vec{A} \parallel \hat{r}$ over surface S $\qquad$ $\vec{E}$ is not $\parallel d\vec{A}$ over surface S'

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_{S'} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

2.4 Gauss' Law

The net flux through any closed surface is ➡ $\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$

q represents the net charge inside the surface

$\vec{E}$ represents the electric field at any point on the surface

PROOF

Consider spherical surface of radius r containing area element ΔA

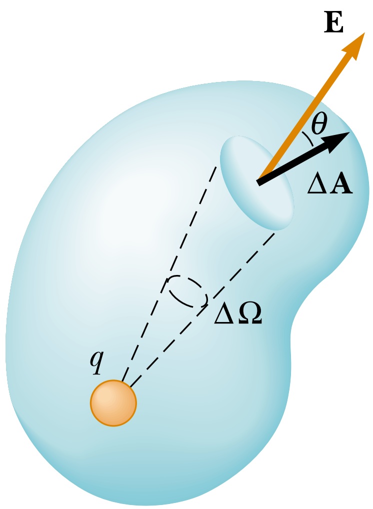
Solid angle subtended at center of the sphere by this element is defined to be

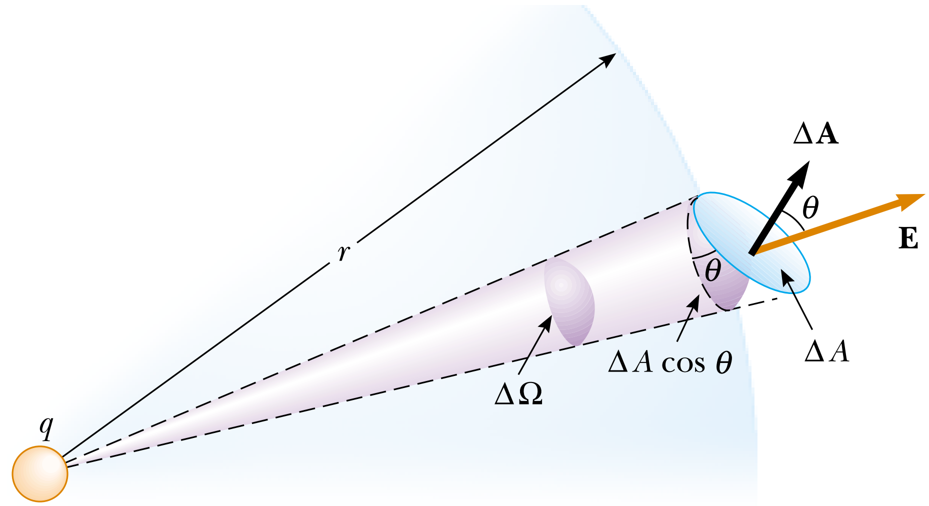
$$\Delta\Omega = \frac{\Delta A}{r^2}$$

Because surface area of sphere is $4\pi r^2$

total solid angle subtended by the sphere is ➡ $\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ sterradians}$

Consider point charge q surrounded by closed surface of arbitrary shape





Total electric flux through this surface can be obtained

by evaluating $\vec{E} \cdot \Delta\vec{A}$ for each small area element ΔA

and summing over all elements

The flux through each element is

$$\Delta\Phi_E = \vec{E} \cdot \Delta\vec{A} = E\Delta A \cos\theta = \frac{q}{4\pi\epsilon_0 r^2}\Delta A \cos\theta$$

r ☛ distance from charge to area element

θ ☛ angle between electric field $\vec{E}$ and $\Delta\vec{A}$ for the element

for point charge ➢ $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

Projection of area element perpendicular to radius vector is ☛ $\Delta A \ \cos\theta$

$$\frac{\Delta A \ \cos\theta}{r^2}$$ ➢ solid angle $\Delta\Omega$ that surface element ΔA subtends at charge q

$\Delta\Omega$ is also solid angle subtended by area element of spherical surface of radius r

Because total solid angle at a point is 4π steradians

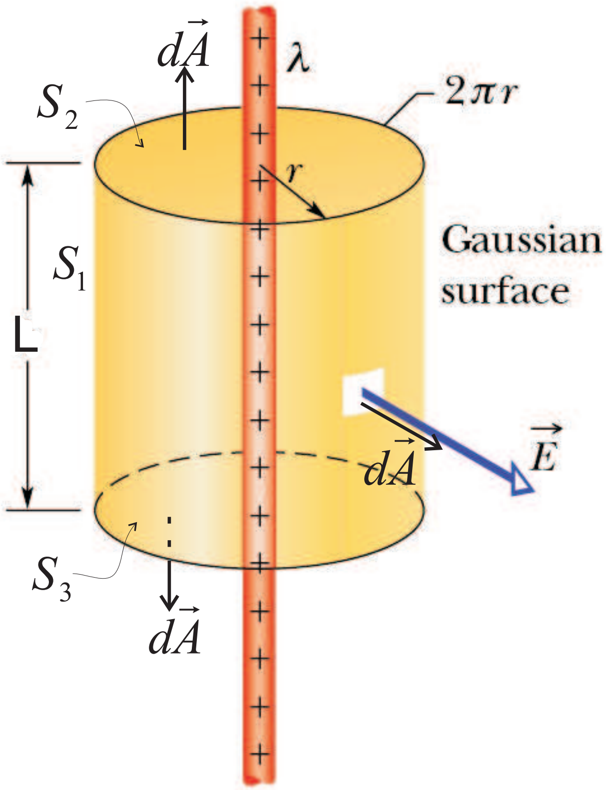
total flux through the closed surface is

$$\Phi_E = \frac{1}{4\pi\epsilon_0} q \oint \frac{dA\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0} \oint d\Omega = \frac{q}{\epsilon_0}$$

Note that this result is independent of the shape of the closed surface and independent of the position of the charge within the surface

2.5 E-field Calculation with Gauss' Law

(A) Infinite line of charge



Linear charge density ☛ λ

Cylindrical symmetry

E -field directs radially outward from rod

Construct Gaussian surface S

in shape of **cylinder**

making up of a curved surface S_1

and top and bottom circles S_2, S_3

Gauss' Law ☛ $$\oint_S \vec{E} \cdot d\vec{A} = \frac{\text{Total charge}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{A} = \underbrace{\int_{S_1} \vec{E}\cdot d\vec{A}}_{\vec{E}\,\parallel\, d\vec{A}} + \underbrace{\int_{S_2} \vec{E}\cdot d\vec{A} + \int_{S_3} \vec{E}\cdot d\vec{A}}_{=0 \because \vec{E}\perp d\vec{A}}$$

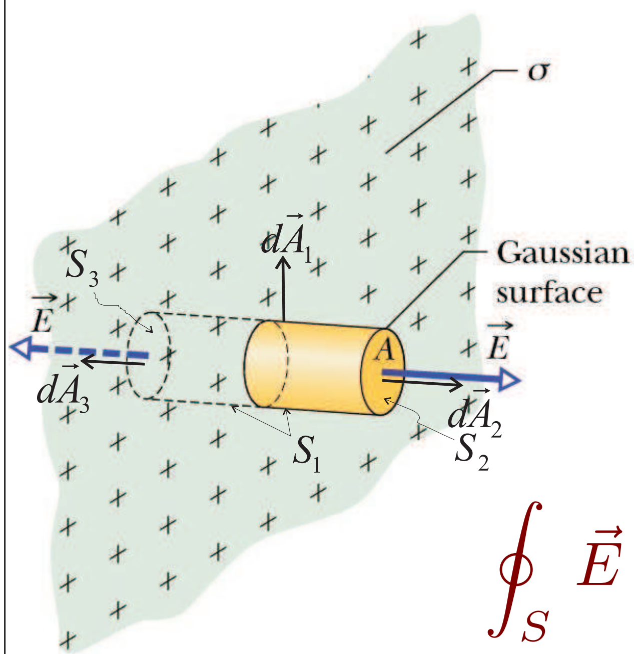
$$\therefore\ E \underbrace{\int_{S_1} dA}_{\text{Total area of surface } \mathrm{S}_1} = \frac{\lambda L}{\epsilon_0}$$

$$E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$

$$\therefore\ E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r}$$

(B) Infinite sheet of charge



Uniform surface charge density ☛ σ

Planar symmetry

E -field directs perpendicular to sheet

Construct a Gaussian surface S

in shape of a **cylinder**

of cross-sectional area A

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{A\sigma}{\epsilon_0}$$

Gauss' Law ☛ $\int_{S_1} \vec{E} \cdot d\vec{A} = 0 \quad \because \vec{E} \perp d\vec{A}$ over whole surface S_1

$$\int_{S_2} \vec{E} \cdot d\vec{A} + \int_{S_3} \vec{E} \cdot d\vec{A} = 2EA \quad (\vec{E} \parallel d\vec{A}_2, \vec{E} \parallel d\vec{A}_3)$$

Note ☛ For S_2 both $\vec{E}$ and $d\vec{A}_2$ point right
For S_3 both $\vec{E}$ and $d\vec{A}_3$ point left

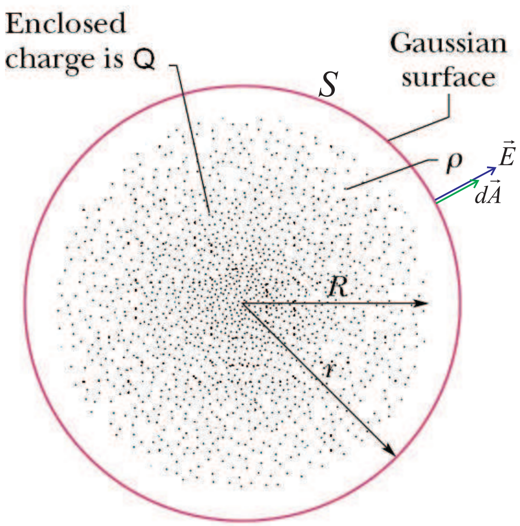
$$\therefore \quad 2EA = \frac{A\sigma}{\epsilon_0} \Rightarrow \quad E = \frac{\sigma}{2\epsilon_0}$$

(C) Uniformly charged sphere **Total charge** ☛ Q

Spherical symmetry

(a) For $r > R$

Consider a spherical Gaussian surface S of radius r



$$\vec{E} \parallel d\vec{A} \parallel \hat{r}$$

Gauss' Law ☛ $$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint_S E \cdot dA = \frac{Q}{\epsilon_0}$$

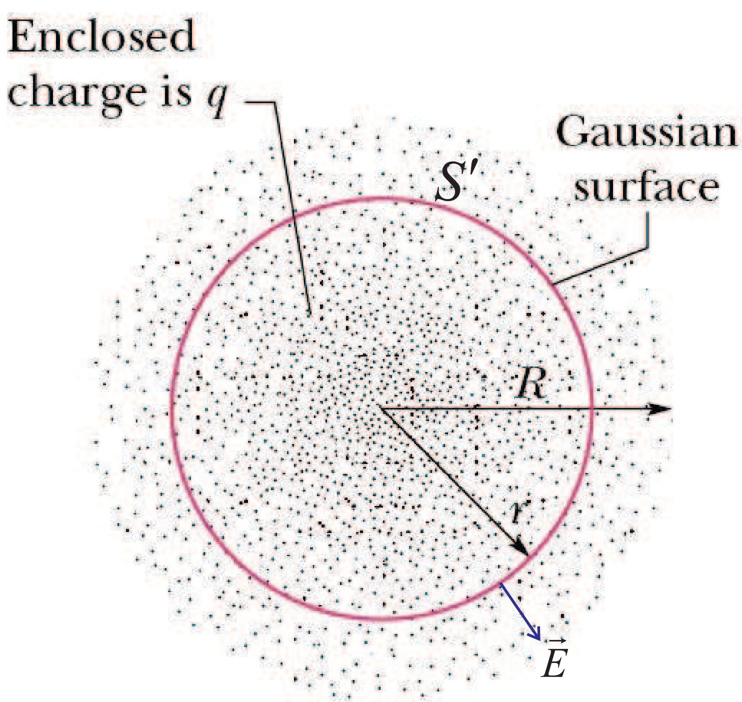
$$E \underbrace{\oint_S dA}_{\text{surface area of } S \ = \ 4\pi r^2} = \frac{Q}{\epsilon_0}$$

$$\therefore \quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r > R$$

(b) For $r < R$

Consider a spherical Gaussian surface S' of radius $r < R$

total charge included q is **proportional to volume included in** S'



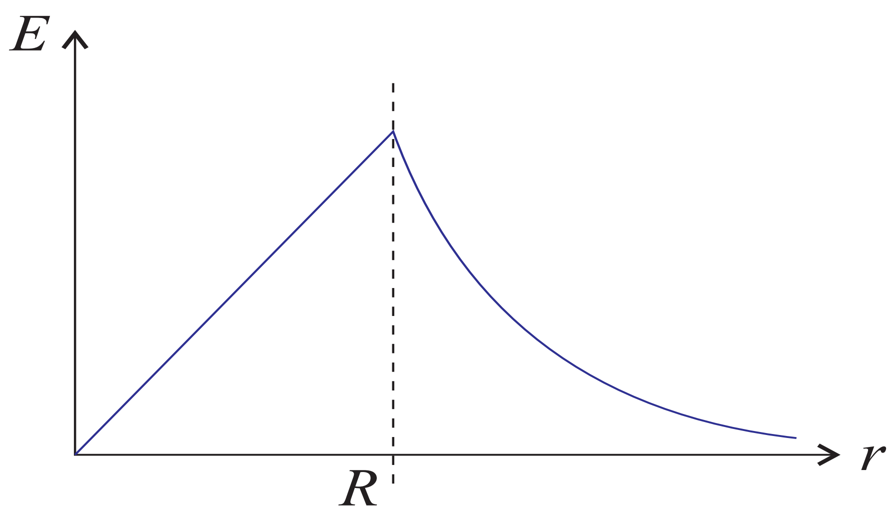
$$\therefore \quad \frac{q}{Q} = \frac{\text{Volume enclosed by } S'}{\text{Total volume of sphere}}$$

$$\frac{q}{Q} = \frac{4/3\pi r^3}{4/3\pi R^3} \quad \Rightarrow \quad q = \frac{r^3}{R^3} Q$$

Gauss' Law ☛ $$\oint_{S'} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$E \underbrace{\oint_{S'} dA}_{\text{surface area of } S' \;=\; 4\pi r^2} = \frac{r^3}{R^3} \frac{1}{\epsilon_0} \cdot Q$$

$$\therefore \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^3} r\,\hat{r} \qquad \text{for } r \leq R$$



2.6 Conductors in electrostatic equilibrium

Electrical conductors contain charges (electrons) that are not bound to any atom and therefore are free to move about within the material

When there is no net motion of charge within a conductor the conductor is in electrostatic equilibrium

A conductor in electrostatic equilibrium has the following properties:

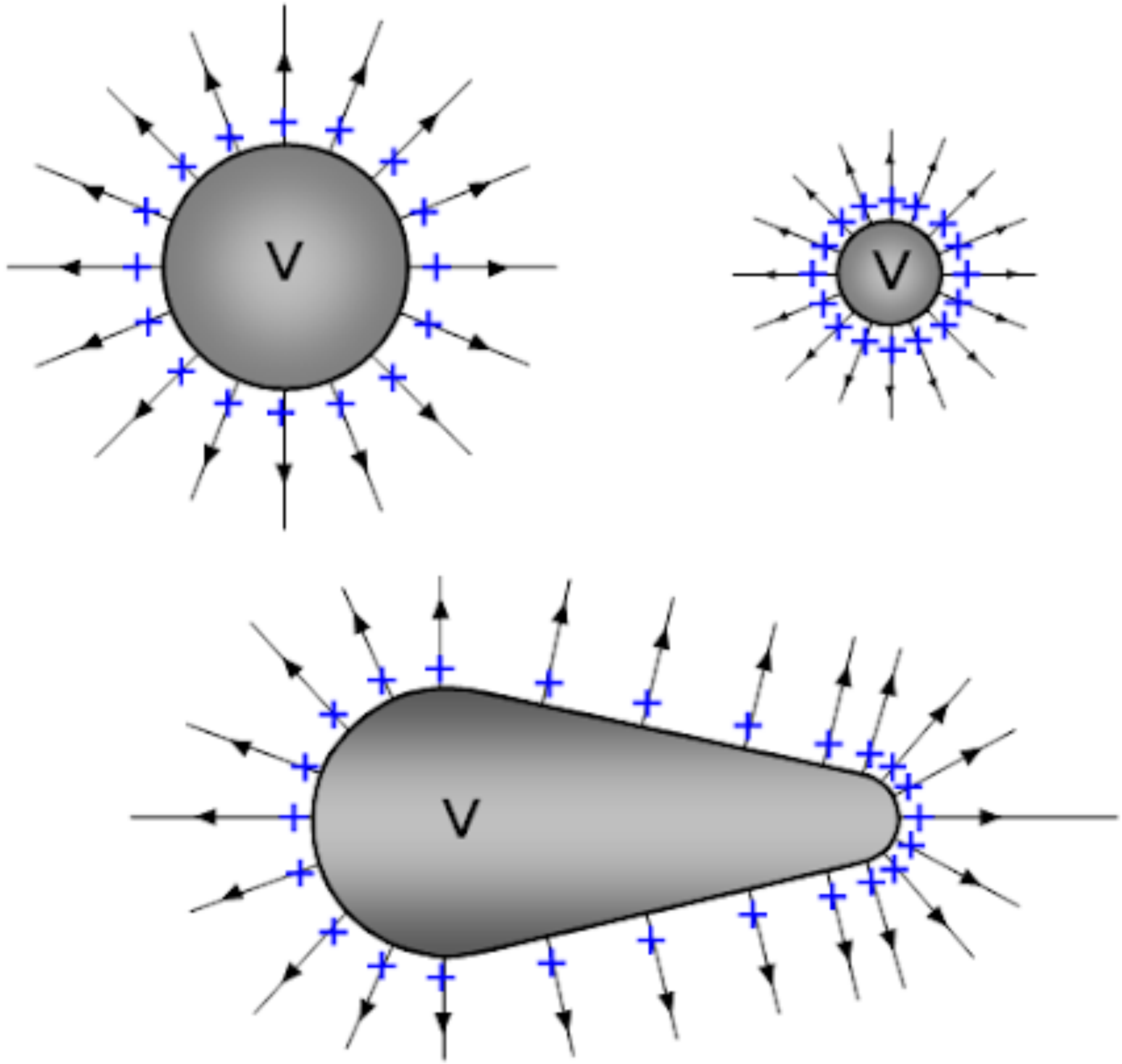
1. The electric field is zero everywhere inside the conductor

2. If an isolated conductor carries a charge the charge resides on its surface

3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ϵ_0

   σ ☛ surface charge density at that point

4. On an irregularly shaped conductor ➢ surface charge density is greatest at locations where the radius of curvature of the surface is smallest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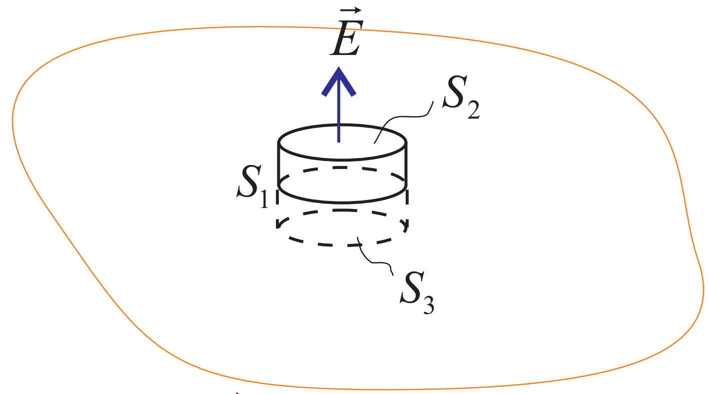
Consider a Gaussian surface S of shape of cylinder



$$\oint_S \vec{E}\cdot d\vec{A} = \frac{\sigma A}{\epsilon_0}$$

$$\int_{S_1} \vec{E}\cdot d\vec{A} = 0 \quad (\because \vec{E} \perp d\vec{A})$$

$$\int_{S_3} \vec{E}\cdot d\vec{A} = 0 \quad (\because \vec{E} = 0\,\mathrm{inside\,conductor})$$

$$\int_{S_2} \vec{E}\cdot d\vec{A} = E \underbrace{\int_{S_2} dA}_{\mathrm{Area\ of\ S_2}} \quad (\because \vec{E} \parallel d\vec{A}) \quad = EA$$

$\therefore$ **Gauss' Law** $\Rightarrow \quad EA = \dfrac{\sigma A}{\epsilon_0}$

$\therefore$ **On conductor surface** $E = \dfrac{\sigma}{\epsilon_0}$

BUT there is no charge inside conductors

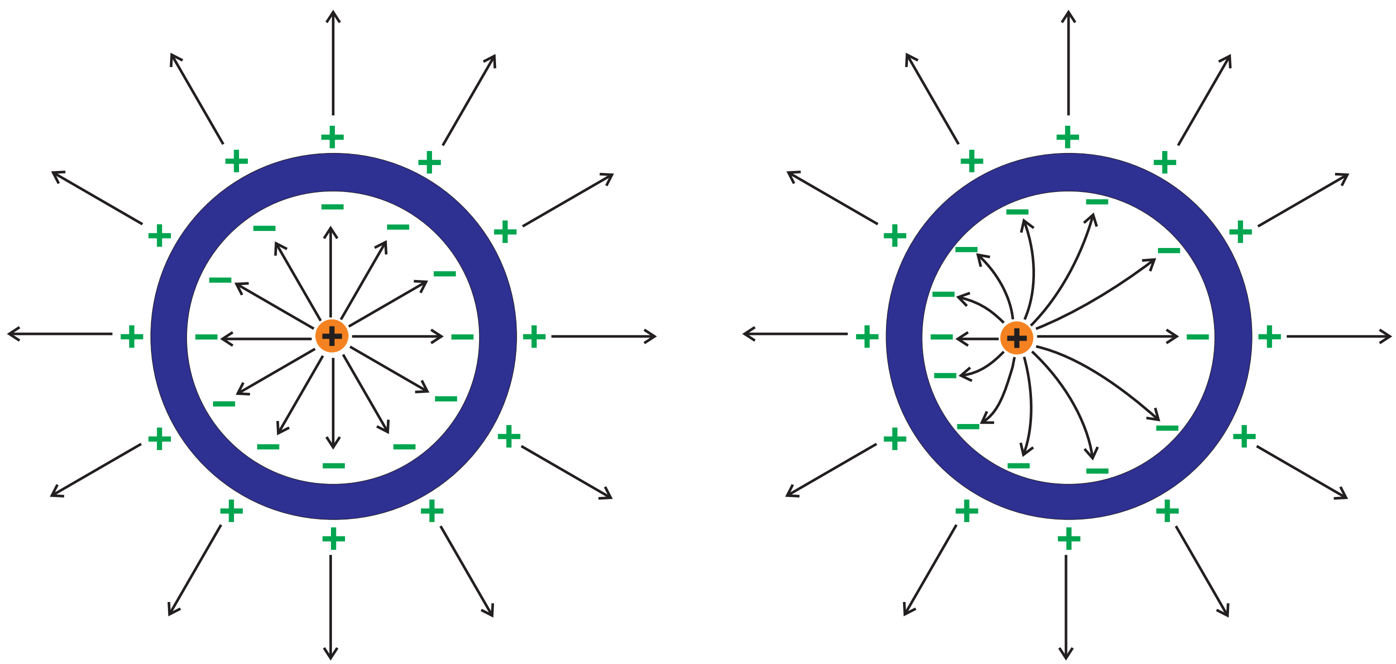
$\therefore$ **Inside conductors** $E = 0$ **Always!**

Notice: Surface charge density on a conductor's surface is **not uniform**

Example

Conductor with a charge inside

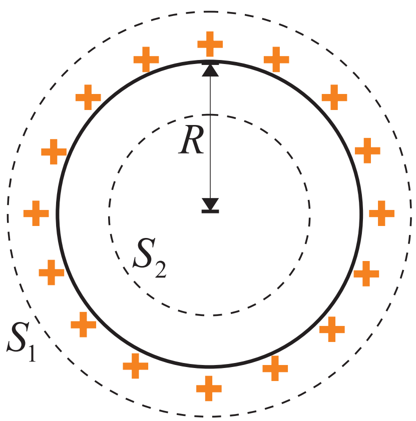
Note ☛ This is **not** an isolated system (because of charge inside)



Note ☛ In **Both** cases $\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\hat{r}$ outside

Example

I. Charge sprayed on a conductor sphere



all charges move to **surface** of conductors

Total charges = Q

(i) For $r < R$ consider Gaussian surface S_2

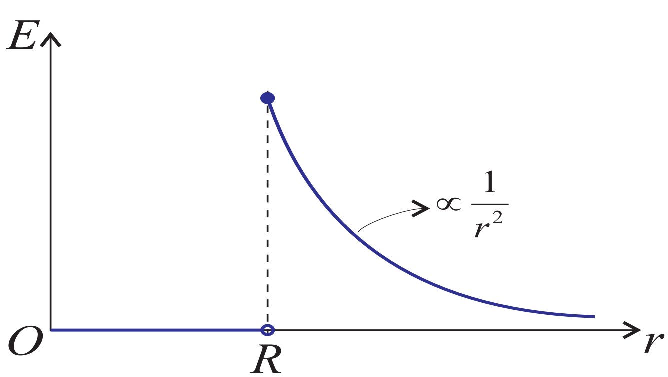
$$\oint_{S_2} \vec{E} \cdot d\vec{A} = 0 \Rightarrow \vec{E} = 0 \text{ everywhere}$$

Consider Gaussian surface S_1

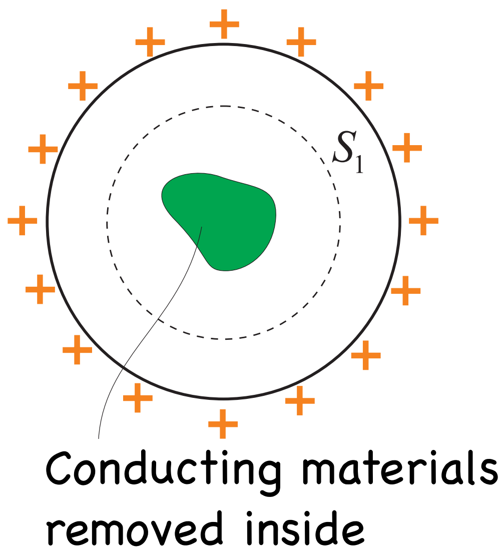
$$\oint_{S_1} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E \underbrace{\oint_{S_1} d\vec{A}}_{4\pi r^2} = \frac{Q}{\epsilon_0} \quad (\overbrace{\vec{E} \parallel d\vec{A}}^{\text{For a conductor}} \underbrace{\parallel \hat{r}}_{\text{Spherically symmetric}})$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$



II. Conductor sphere with hole inside



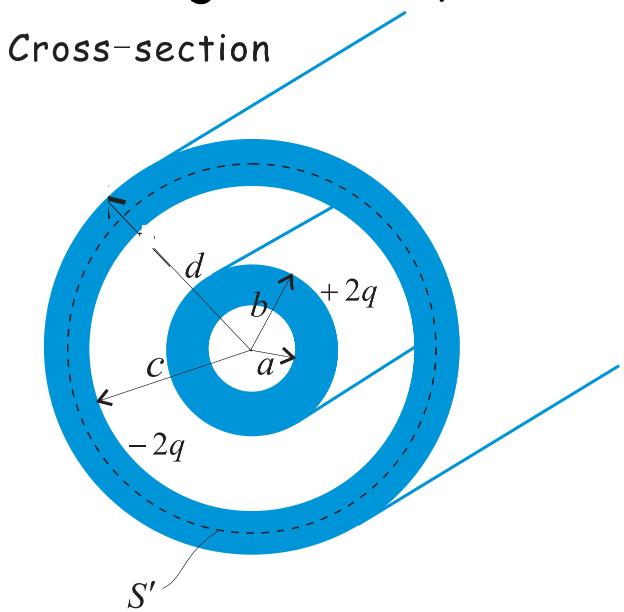
Consider Gaussian surface S_1

Total charge included $= \ 0$

$\therefore$ E-field $= \ 0$ inside

E-field is identical to case of a solid conductor!!

III. A long hollow cylindrical conductor



Example

Inside hollow cylinder $(+2q)$

- Inner radius a
- Outer radius b

Outside hollow cylinder $(-3q)$

- Inner radius c
- Outer radius d

Question

Find charge on each surface of conductor ?

For inside hollow cylinder charges distribute only on surface

$\therefore$ Inner radius a surface ☛ charge $= 0$

Outer radius b surface ☛ charge $= +2q$

For outside hollow cylinder

charges do not distribute only on outside surface

$\because$ It's not an isolated system **(There are charges inside!)**

Consider Gaussian surface S' inside conductor

E-field always $= 0$

Need charge $-2q$ on radius c surface to balance charge of inner cylinder

$\therefore$ charge on radius d surface $= -q$

IV. Large sheets of charge

Total charge Q on sheet of area A

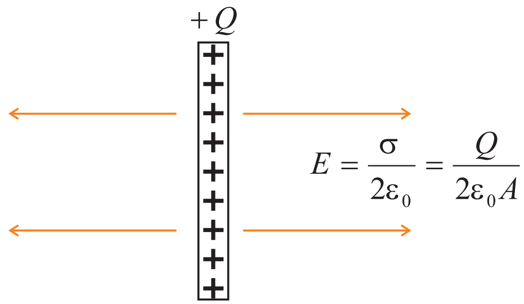
Surface charge density $\sigma = \frac{Q}{A}$

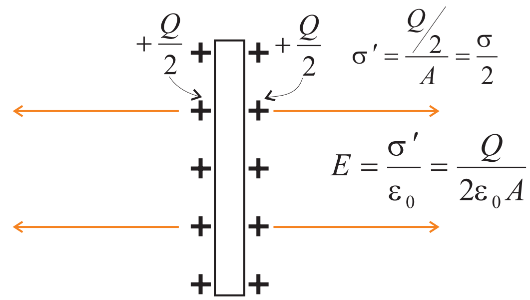
By principle of superposition

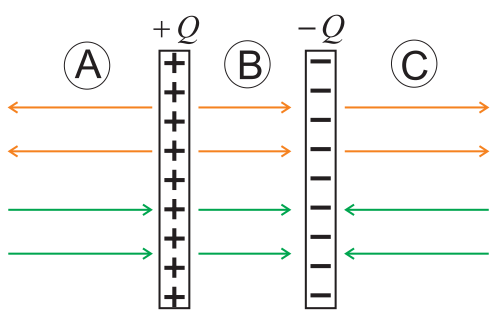
On insulator

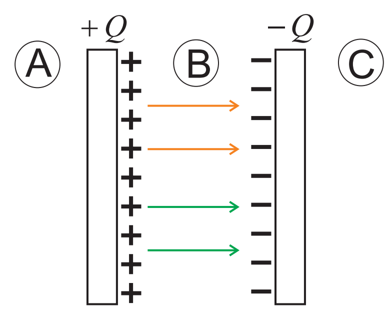
(charge sprayed on insulator)

On conductor









	On insulator	On conductor
Region A	$E = 0$	$E = 0$
Region B	$E = \frac{Q}{\epsilon_0 A}$	$E = \frac{Q}{\epsilon_0 A}$
Region C	$E = 0$	$E = 0$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