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Problems set # 9	**Physics 169**	April 21, 2015

1. Consider the hemispherical closed surface in Fig. 1. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through *(i)* the flat surface S_1 and *(ii)* the hemispherical surface S_2.

Solution *(i)* $\Phi_B|_{\rm flat} = \vec{B}\cdot\vec{A} = B\pi R^2\cos(\pi-\theta) = -B\pi R^2\cos\theta$. *(ii)* The net flux out of the closed surface is zero: $\Phi_B|_{\rm flat} + \Phi_B|_{\rm curved} = 0$, hence $\Phi_B|_{\rm curved} = B\pi R^2\cos\theta$.

2. A cube of edge length $\ell = 2.50$ cm is positioned as shown in Fig. 2. A uniform magnetic field given by $\vec{B} = (5\hat{\imath} + 4\hat{\jmath} + \hat{3}\hat{k})$ T exists throughout the region. *(i)* Calculate the flux through the shaded face. *(ii)* What is the total flux through the six faces?

Solution *(i)* $\Phi_B = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = (5\hat{\imath}+4\hat{\jmath}+3\hat{k})\ {\rm T}\cdot(2.50\times10^{-2}\ {\rm m})^2\hat{\imath} = 3.12\times10^{-3}\ {\rm T\cdot m^2} = 3.12\times10^{-3}\ {\rm Wb} = 3.12$ mWb. *(ii)* $\Phi_B^{\rm total} = \oiint_S \vec{B}\cdot d\vec{A} = 0$ for any closed surface (Gauss law for magnetism).

3. A solenoid 2.50 cm in diameter and 30.0 cm long has 300 turns and carries 12.0 A. *(i)* Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as shown in Fig. 3 (a). *(ii)* Figure 3 (b) shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is defined by an annulus that has an inner radius of 0.400 cm and outer radius of 0.800 cm.

Solution $\Phi_B = \vec{B}\cdot\vec{A} = BA$, where A is the cross-sectional area of the solenoid, $\Phi_B = \frac{\mu_0 NI}{L}\,\pi r^2 = 7.40\ \mu$Wb. *(ii)* $\Phi_B = \vec{B}\cdot\vec{A} = BA = \frac{\mu_0 NI}{L}\,\pi\,(r_2^2 - r_1^2) = 2.27\ \mu$Wb.

4. The rectangular loop shown in Fig. 4 is coplanar with the long, straight wire carrying the current $I = 20$ A. Determine the magnetic flux through the loop.

Solution The field due to the long wire is, $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$. In the plane of the loop, $\hat{\phi}$ becomes $-\hat{\imath}$ and r becomes y, $\vec{B} = -\frac{\mu_0 I}{2\pi y}\hat{\imath}$. The flux through the loop is along $-\hat{\imath}$, and the magnitude of the flux is $\Phi = \oiint_S \vec{B}\cdot d\vec{A} = \frac{\mu_0 I}{2\pi}\int_{5\ {\rm cm}}^{20\ {\rm cm}} \frac{1}{y}(-\hat{\imath})\cdot(-\hat{\imath})\ 30\ {\rm cm}\,dy = \frac{\mu_0 I}{2\pi}0.3\int_{0.05}^{0.2}\frac{dy}{y} = \frac{0.3\mu_0}{2\pi}\cdot 20\cdot\ln(0.2/0.05) = 1.66\times10^{-6}$ Wb.

5. Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00-Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails (Fig. 5). The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00-Ω resistor.

Solution Name the currents as shown in Fig. 5: left loop, $+Bdv_2 - I_2R_2 - I_1R_1 = 0$; right loop, $+Bdv_3 - I_3R_3 + I_1R_1 = 0$; and at the junction, $I_2 = I_1 + I_3$ Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$, with $I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$. Hence $Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_iR_1R_2}{R_3} = 0$, yielding $I_1 = Bd\,\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3} = 145\ \mu$A upward.

6. A helicopter has blades of length 3.00 m, rotating at 2.00 rev/s about a central hub as shown in Fig. 6. If the vertical component of the Earths magnetic field is 50.0 μT, what is the emf induced between the blade tip and the center hub? *(ii)* What is the emf induced between any two blade tips?

Solution The magnitude of the angular velocity is $\omega = 2$ rev/s $\cdot\, 2\pi$ rev$^{-1} = 4\pi$ s^{-1}. A blade, rotating with magnitude of angular velocity ω sweeps an angle $d\phi$ proportional to the time dt, i.e. $d\phi = \omega dt$, which corresponds to an area of $dA = \frac{1}{2}R^2 d\phi = \frac{1}{2}R^2\omega dt$. In a uniform magnetic field, the magnetic flux swept by the blade is therefore $d\Phi_B = \vec{B}\cdot d\vec{A} = BdA\cos\theta = \pm B_\perp dA = \pm\frac{1}{2}B_\perp R^2\omega dt$, where $B_\perp$ is the vertical component of the magnetic field. Hence the electromotive force induced in the blade is $\mathcal{E} = -\frac{d\Phi_B}{dt} = \mp\frac{1}{2}B_\perp R^2\omega = \mp\frac{1}{2}5.0\times 10^{-5}\ \text{T}\cdot 4\pi(3.0\ \text{m})^2 = \mp 2.8\times 10^{-3}$ V. *(ii)* Zero (This is like two batteries head to head).

7. A loop of area 0.1 m^2 is rotating at 60 rev/s with the axis of rotation perpendicular to a 0.2 T magnetic field; see Fig. 7. *(i)* If there are 1000 turns on the loop, what is the maximum voltage induced in the loop? *(ii)* When the maximum induced voltage occurs, what is the orientation of the loop with respect to the magnetic field?

Solution As the loop rotates, the angle between the direction of the magnetic field vector $\vec{B}$ and the vector $\vec{A}$, normal to the loop with magnitude equal to the area of the loop, is a linear function of time. The magnetic flux Φ over the surface of the loop is therefore a time dependent function, and at instant t its value is: $\Phi(t) = \int_{\text{loop}} \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\omega t$. The electromotive force in the loop depends on the number of loops and the rate of change in the magnetic flux over the loop (or any other surface bound to the loop): $\mathcal{E}(t) = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}[BA\cos(\omega t)] = NAB\omega\sin(\omega t)$. The rest of the problem involves an analysis of the above function. *(i)* There is no mention of polarity, therefore the maximum voltage (electromotive force) occurs for the extreme values of the trigonometric function. The absolute value of this voltage is: $V_{\text{max}} = NAB\omega = 1000\cdot 0.1\ \text{m}^2\cdot 0.2\ \text{T}\cdot 2\pi\cdot 60\ \text{s}^{-1} = 7,540$ V. *(ii)* To answer this part we must analyze and interpret for what values of the argument the function assumes the extreme values. It happens when the argument of the trigonometric function is $\omega t = \frac{\pi}{2} + n\pi$. Recall that ωt represents the angle between the magnetic field and the normal to the loop. Therefore, the voltage assumes maximum values when the magnetic field is parallel to the plane of the loop.

8. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Fig. 8. The magnitude of B inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?

Solution In the loop on the left, the induced emf is $|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.10\ \text{m})^2\,100\ \text{T/s} = \pi$ V and it attempts to produce a counterclockwise current in this loop. In the loop on the right, the induced emf is $|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.15 \text{ m})^2\, 100 \text{ T/s} = 2.25\ \pi$ V and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00 Ω resistor, I_2 flows down through the 5.00 Ω resistor, and that I_3 flows up through the 3.00 Ω resistor. From Kirchhoffs junction rule: $I_3 = I_1 + I_2$. Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$. Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ Solving these three equations simultaneously, $I_1 = 0.062$ A, $I_2 = 0.860$ A, and $I_3 = 0.923$ A.

9. The square loop shown in Fig. 9 is coplanar with a long, straight wire carrying a current $I(t) = 5\ \cos(2\pi \times 10^4 t)$ A. *(i)* Determine the emf induced across a small gap created in the loop. *(ii)* Determine the direction and magnitude of the current that would flow through a 4 Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

Solution The magnetic field due to the wire is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi} = -\frac{\mu_0 I}{2\pi y}\hat{\imath}$ where in the plane of the loop, $\hat{\phi} = -\hat{\imath}$ and $r = y$. The flux passing through the loop is $\Phi = \int_S \vec{B} \cdot d\vec{s} = \int_{5 \text{ cm}}^{15 \text{ cm}} \left(-\frac{\mu_o I}{2\pi y}\hat{\imath}\right) \cdot (-10 \text{ cm}\,\hat{\imath})\, dy = \frac{\mu_0 I \times 10^{-1} \text{ m}}{2\pi} \ln\frac{15}{5} = \frac{4\pi\times 10^{-7} \text{ N}\cdot\text{A}^{-2}\times 5\cos(2\pi\times 10^4 t) \text{ A}\times 10^{-1} \text{ m}}{2\pi} \times 1.1 = 1.1\times 10^{-7}\cos(2\pi \times 10^4 t)$ Wb, and so $V_{\text{emf}} = -\frac{d\Phi}{dt} = 1.1\times 2\pi\times 10^4 \sin(2\pi\times 10^4 t)\times 10^{-7} \text{ V} = 6.9\times 10^{-3}\sin(2\pi\times 10^4 t)$ V. *(ii)* $I_{\text{ind}} = \frac{V_{\text{emf}}}{R_{\text{tot}}} = \frac{6.9\times 10^{-3}}{5\ \Omega}\ \sin(2\pi \times 10^4 t) \text{ V} = 1.38\sin(2\pi \times 10^4 t)$ mA. At $t = 0$, $\vec{B}$ is a maximum, it points in $-x$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be counterclockwise when looking down on the loop, as shown in Fig. 9.

10. The loop shown in Fig. 10 moves away from a wire carrying a current $I_1 = 10$ A at a constant velocity $u = 7.5$ m/s. If $R = 10\ \Omega$ and the direction of I_2 is as defined in Fig. 10, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

Solution Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by $V_{\text{emf}} = \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{l}$. The magnetic field B is created by the wire carrying I_1. Choosing $\hat{k}$ to coincide with the direction of I_1, the external magnetic field due to the long wire is $\vec{B} = \frac{\mu_0 I_1}{2\pi r}\hat{\phi}$. For positive values of y_0 in the y-z plane, $\hat{\jmath} = \hat{r}$, so $\vec{u} \times \vec{B} = |\vec{u}|\hat{\jmath} \times \vec{B} = \hat{r}|u| \times \hat{\phi}\frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1 u}{2\pi r}\hat{k}$. Integrating around the four sides of the loop with $d\vec{l} = dz\hat{k}$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to V_{emf}, we have $V_{\text{emf}} = \int_0^{0.2} \left(\hat{k}\frac{\mu_0 I_1 u}{2\pi r}\right)\Big|_{r=y_0/\text{m}} \cdot \hat{k}\, dz + \int_{0.2}^{0} \left(\hat{k}\frac{\mu_0 I_1 u}{2\pi r}\right)\Big|_{r=y_0/\text{m}+0.1} \cdot \hat{k}\, dz = \frac{4\pi\times 10^{-7} \text{ N}\cdot\text{A}^{-2}\times 10 \text{ A}\times 7.5 \text{ m/s}\times 0.2}{2\pi}\left(\frac{1}{y_0/\text{m}} - \frac{1}{y_0/\text{m}+0.1}\right) = 3 \times 10^{-6}\left(\frac{1}{y_0/\text{m}} - \frac{1}{y_0/\text{m}+0.1}\right)$ V, and therefore $I_2 = \frac{V_{\text{emf}}}{2R} = 150\left(\frac{1}{y_0/\text{m}} - \frac{1}{y_0/\text{m}+0.1}\right)$ nA.

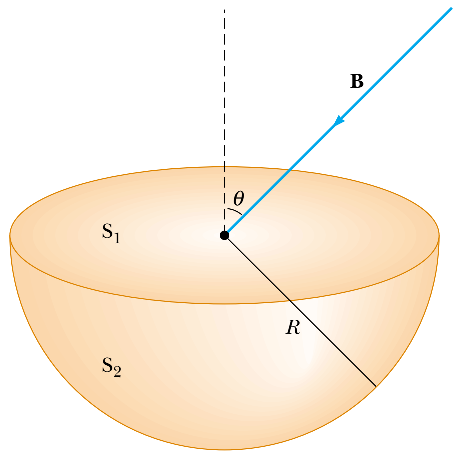


Figure 1: Problem 1.

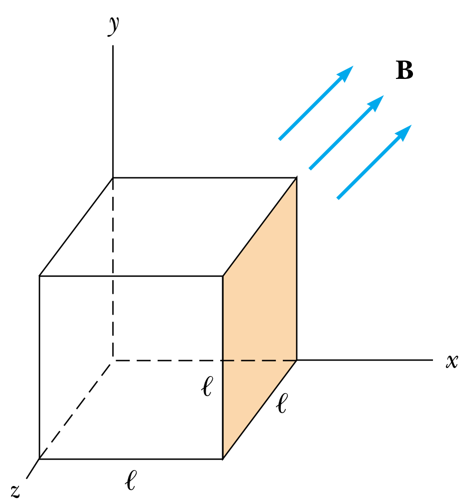
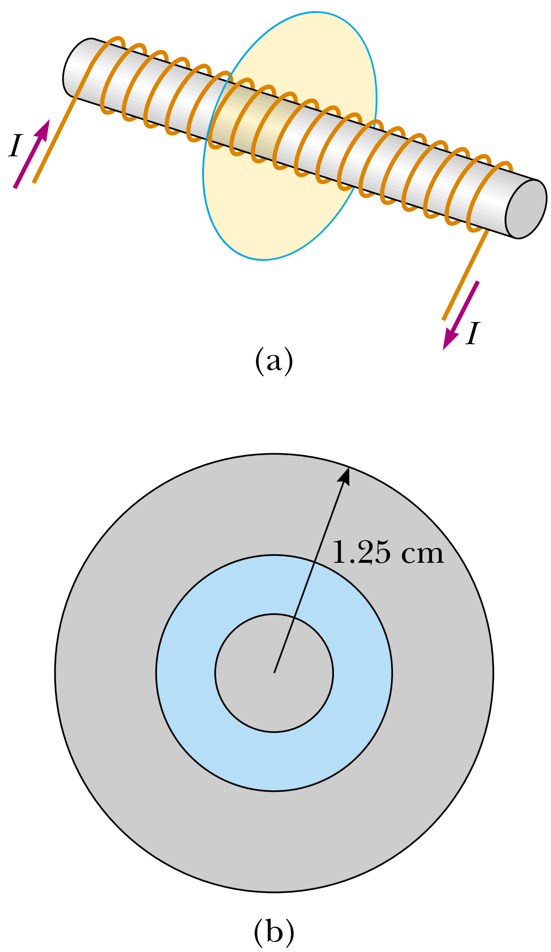


Figure 2: Problem 2.



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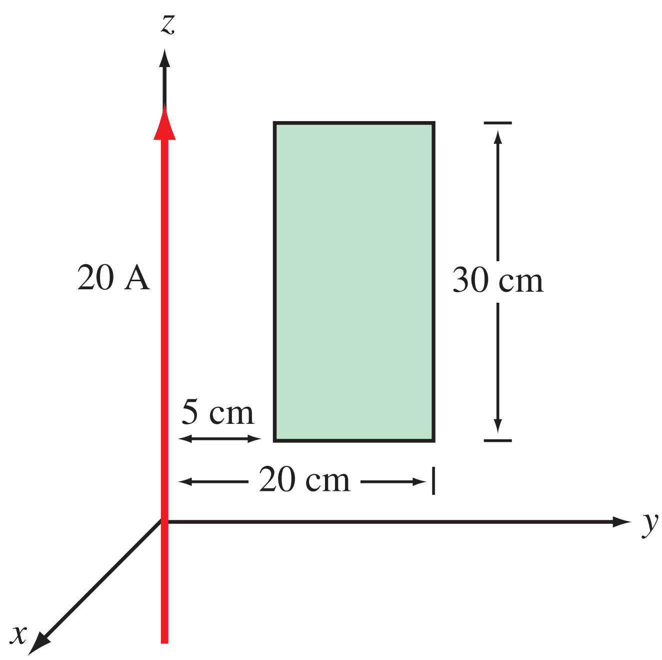
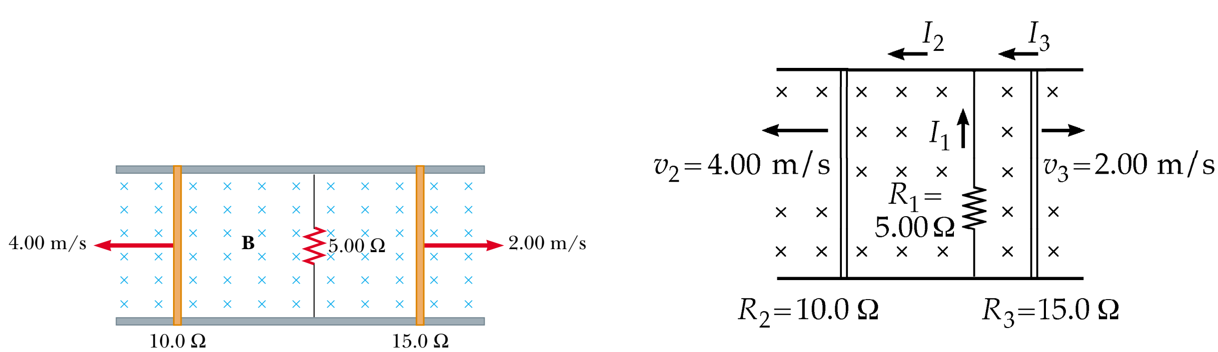


Figure 4: Problem 4.



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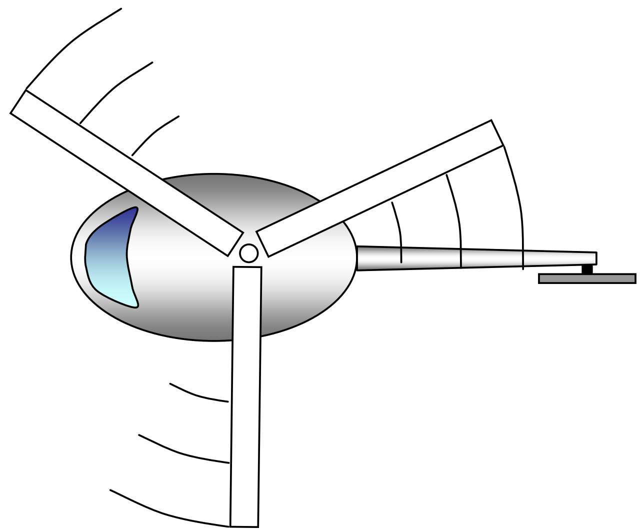

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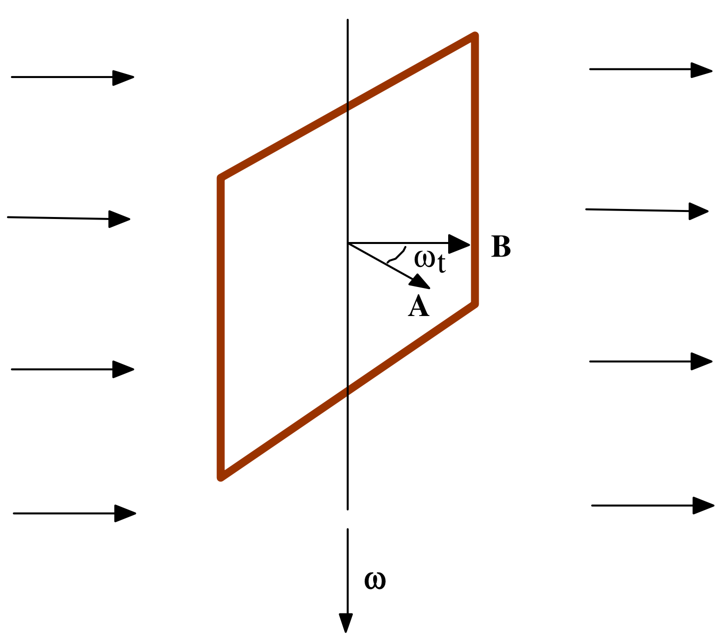

Figure 7: Problem 7.

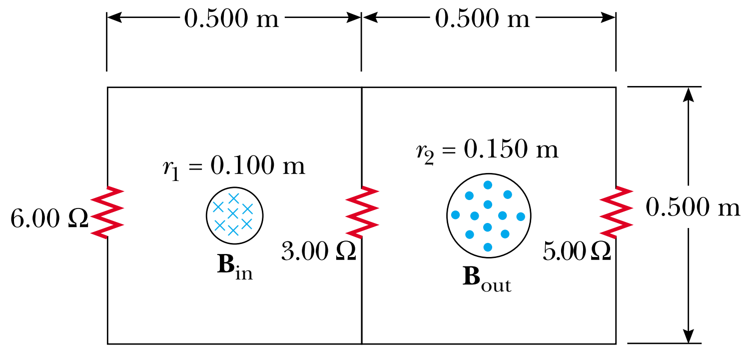

Figure 8: Problem 8.

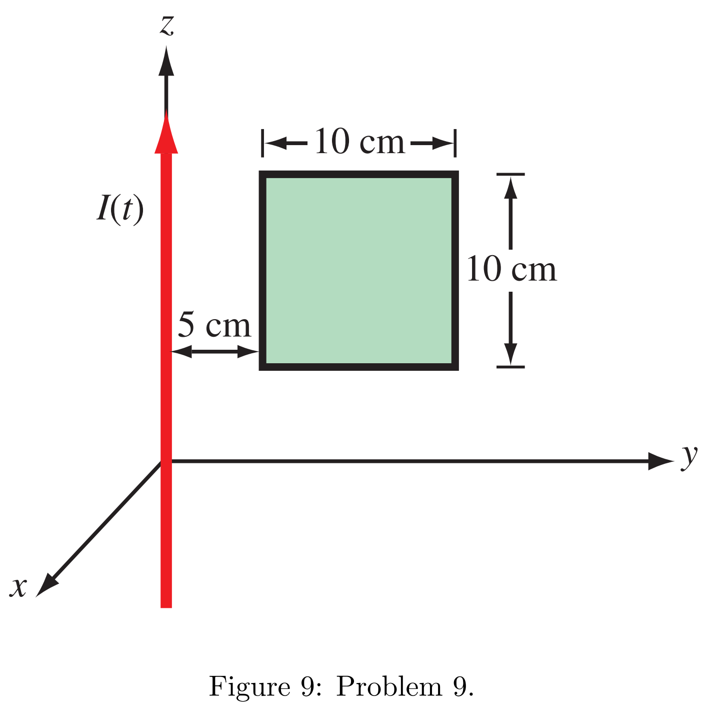

Figure 9: Problem 9.

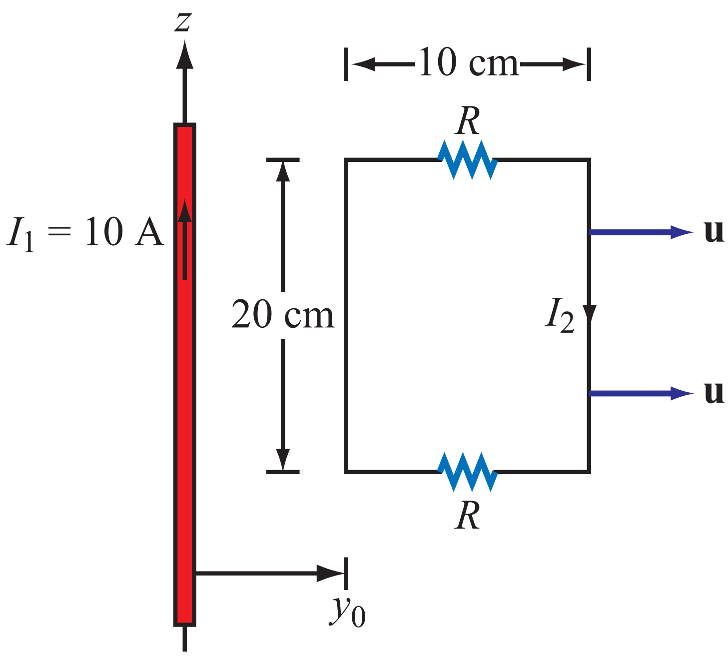

Figure 10: Problem 10.