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Problems set # 8 **Physics 169** April 14, 2015

1. The unit of magnetic flux is named for Wilhelm Weber. The practical-size unit of magnetic field is named for Johann Karl Friedrich Gauss. Both were scientists at Gottingen, Germany. Along with their individual accomplishments, together they built a telegraph in 1833. It consisted of a battery and switch, at one end of a transmission line 3 km long, operating an electromagnet at the other end. (André Ampéré suggested electrical signaling in 1821; Samuel Morse built a telegraph line between Baltimore and Washington in 1844.) Suppose that Weber and Gausss transmission line was as diagrammed in Fig. 1. Two long, parallel wires, each having a mass per unit length of 40.0 g/m, are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current I, the wires repel each other so that the angle θ between the supporting strings is 16.0°. *(i)* Are the currents in the same direction or in opposite directions? *(ii)* Find the magnitude of the current.

Solution The separation between the wires is a $a = 2 \cdot 6.00cm \cdot \sin 8.00^\circ = 1.67$ cm. *(i)* Because the wires repel, the currents are in opposite directions. *(ii)* Because the magnetic force acts horizontally, $\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a m g} = \tan 8.00^\circ$, yielding $I^2 = \frac{mg 2\pi a}{\mu_0 \ell} \tan 8.00^\circ$ and so $I = 67.8$ A.

2. Figure 2 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1.00 A out of the page and the current in the outer conductor is 3.00 A into the page. Determine the magnitude and direction of the magnetic field at points a and b.

Solution From Ampere's law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so $B_a = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\cdot 1\ \text{A}}{2\pi\cdot 1.00\times10^{-3}\ \text{m}} = 200\ \mu$T toward top of the page. Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00$ A, or $I_b = 2.00$ A into the page. Therefore, $B_b = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\cdot 2\ \text{A}}{2\pi\cdot 3.00\times10^{-3}\ \text{m}} = 133\ \mu$T toward the bottom of the page.

3. A long cylindrical conductor of radius R carries a current I as shown in Fig. 3. The current density J, however, is not uniform over the cross section of the conductor but is a function of the radius according to $J = br$, where b is a constant. Find an expression for the magnetic field B *(i)* at a distance $r_1 < R$ and *(ii)* at a distance $r_2 > R$, measured from the axis.

Solution Use Ampérè's law, $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$. For current density $\vec{J}$, this becomes $\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$. *(i)* For $r_1 < R$, this gives $B 2\pi r_1 = \mu_0 \int_0^{r_1} br 2\pi r dr$ and $B = \frac{\mu_0 b r_1^2}{3}$ (for $r_1 < R$ or inside the cylinder. *(ii)* When $r_2 > R$, Ampérè's law yields $2\pi r_2) B = \mu_0 \int_0^R br 2\pi r dr = \frac{2\pi \mu_0 b R^3}{3}$ or $B = \frac{\mu_0 b R^3}{3 r_2}$ (for $r_2 > R$or outside the cylinder).

4. A toroid with a mean radius of 20.0 cm and 630 turns is filled with powdered steel whose magnetic susceptibility χ is 100. The current in the windings is 3.00 A. Find B (assumed uniform) inside the toroid.

Solution: Assuming a uniform B inside the toroid is equivalent to assuming $r \ll R$ (see Fig. 4); then $B_0 = \mu_0 H \approx \mu_0 N I$ as for a tightly wound solenoid. This leads to $B_0 = \mu_0 \frac{630 \cdot 3.00}{2\pi \cdot 0.200} = 0.00189$ T. With the steel, $B = (1+\chi)\mu_0 H = 101 \cdot 0.00189 \text{ T} = 0.191$ T.

5. In Bohrs 1913 model of the hydrogen atom, the electron is in a circular orbit of radius 5.29×10^{-11} m and its speed is 2.19×10^6 m/s. *(i)* What is the magnitude of the magnetic moment due to the electrons motion? *(ii)* If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of this magnetic moment vector?

Solution The current induced by the electron is $I = \frac{ev}{2\pi r}$, so it is straightforward to see that the Bohr model predicts the correct magnetic moment $\mu = IA = \frac{ev}{2\pi r}\pi r^2 = 9.27 \times 10^{-24}\ \text{A}\cdot\text{m}^2$. *(ii)* Because the electron has a negative charge, its [conventional] current is clockwise, as seen from above, and $\vec{\mu}$ points downward.

6. The magnetic moment of the Earth is approximately $8.00 \times 10^{22}\ \text{A}\cdot\text{m}^2$. *(i)* If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? *(ii)* At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? [*Hint:* Iron has a density of $7,900\ \text{kg/m}^3$, and approximately 8.50×10^{28} iron atoms/m^3.]

Solution *(i)* Number of unpaired electrons $= \frac{8.00\times10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = 8.63 \times 10^{45}$. Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2} 8.63 \times 10^{45}$. *(ii)* Mass $= \frac{4.31\times10^{45}\ \text{atoms}\cdot 7{,}900\ \text{kg/m}^3}{8.50\times10^{28}\ \text{atoms/m}^3} = 4.01 \times 10^{20}$ kg.

7. Find the magnetic field on the axis at a distance a above a disk of radius R with charge density σ rotating at an angular speed (clockwise when viewed from the point P).

Solution Consider a thin ring between r and $r + dr$ as shown in Fig. 5. The charge per length on the ring is $q = \sigma dr$ and each point on it is moving at a speed $v = \omega \vec{r}$, so the current is $I = vq = \sigma\omega r dr$. By symmetry, the magnetic field is in the z direction. Using the result from the lectures, the contribution from the ring is $dB_z = \frac{\mu_0(\sigma\omega r\ dr)}{2}\frac{r^2}{(r^2+z^2)^{3/2}}$. The magnetic field for the entire disk is $B_z = \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3}{(r^2+z^2)^{3/2}}dr$. Let $u = r^2$, so $du = 2rdr$. This substitution (dont forget to change limits) gives $B_z = \frac{\mu_0\sigma\omega}{4}\int_0^{R^2}\frac{u}{(u+z^2)^{3/2}}du = \frac{\mu_0\sigma\omega}{4}\left.\frac{2(u+2z^2)}{(u+z^2)^{1/2}}\right|_0^{R^2} = \frac{\mu_0\sigma\omega}{2}\left[\frac{R^2+2z^2}{(R^2+z^2)^{1/2}} - 2z\right]$.

8. A thin uniform ring of radius R and mass M carrying a charge $+Q$ rotates about its axis with constant angular speed ω. Find the ratio of the magnitudes of its magnetic dipole moment to

its angular momentum. (This is called the gyromagnetic ratio.)

Solution: The current in the ring shown in Fig. 6 is $i = \frac{Q}{T} = \frac{Q\omega}{2\pi}$. The magnetic moment is $\vec{\mu} = Ai\hat{k} = \pi R^2 \frac{Q\omega}{2\pi}\hat{k} = \frac{Q\omega R^2}{2}\hat{k}$. The angular momentum is $\vec{L} = I\omega\hat{k} = MR^2\omega\hat{k}$. So the gyromagnetic ratio is $\frac{|\vec{\mu}|}{|\vec{L}|} = \frac{Q\omega R^2/2}{MR^2\omega} = \frac{Q}{2M}$.

9. A wire ring lying in the xy-plane with its center at the origin carries a counterclockwise I. There is a uniform magnetic field $\vec{B} = B\hat{\imath}$ in the $+x$-direction. The magnetic moment vector μ is perpendicular to the plane of the loop and has magnitude $\mu = IA$ and the direction is given by right-hand-rule with respect to the direction of the current. What is the torque on the loop?

Solution: The torque on a current loop in a uniform field is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\mu = IA$ and the vector $\hat{\imath}$ is perpendicular to the plane of the loop and right-handed with respect to the direction of current flow. The magnetic dipole moment is given by $\vec{\mu} = I\vec{A} = \pi IR^2\hat{k}$. Therefore, $\vec{\tau} = \vec{\mu} \times \vec{B} = \pi IR^2\hat{k} \times B\hat{\imath} = \pi IR^2 B\hat{\jmath}$. Instead of using the above formula, we can calculate the torque directly as follows. Choose a small section of the loop of length $ds = Rd\theta$. Then the vector describing the current-carrying element is given by $Id\vec{s} = IRd\theta(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})$. The force $d\vec{F}$ that acts on this current element is $d\vec{F} = Id\vec{s} \times \vec{B} = IRd\theta(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}) \times B\hat{\imath} = -IRB\cos\theta d\theta\hat{k}$. The force acting on the loop can be found by integrating the above expression, $\vec{F} = \oint d\vec{F} = \int_0^{2\pi}(-IRB\cos\theta)d\theta\hat{k} = -IRB\sin\theta|_0^{2\pi}\hat{k} = 0$. We expect this because the magnetic field is uniform and the force on a current loop in a uniform magnetic field is zero. Therefore we can choose any point to calculate the torque about. Let $\vec{r}$ be the vector from the center of the loop to the element $Id\vec{s}$. That is, $\vec{r} = R(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})$. The torque $d\vec{\tau} = \vec{r} \times d\vec{F}$ acting on the current element is then $d\vec{\tau} = \vec{r} \times d\vec{F} = R(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}) \times (-IRBd\theta\cos\theta\hat{k}) = -IR^2Bd\theta\cos\theta(\sin\theta\hat{\imath} - \cos\theta\hat{\jmath})$. Finally, integrate $d\vec{\tau}$ over the loop to find the total torque, $\vec{\tau} = \oint d\vec{\tau} = -\int_0^{2\pi} IR^2Bd\theta\cos\theta(\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) = \pi IR^2B\hat{\jmath}$. This agrees with our result above.

10. A square coil with sides equal to 25.0 cm carries a current of 2.00 A. It lies in the $z = 0$ plane in a magnetic field $\vec{B} = 0.40\hat{\imath} + 0.30\hat{k}$ T with the current counter-clockwise when viewed from a point on the positive z-axis. If the coil has 6 turns what is *(i)* the torque acting on the coil, and *(ii)* the potential energy of the coil/field system?

Solution If the current is counter-clockwise when viewed from a point on the positive z-axis, then the magnetic moment of the coil is in the positive z direction (i.e., along $\hat{k}$). It follows that $\vec{\mu} = niA\hat{k} = 6 \times (2.00 \text{ A}) \times (0.25 \text{ m})^2\hat{k} = (0.75 \text{ A}\cdot\text{m}^2)\hat{k}$. *(i)* The torque is $\vec{\tau} = \vec{\mu} \times \vec{B} = (0.75 \text{ A}\cdot\text{m}^2)\hat{k} \times (0.40\hat{\imath} + 0.30\hat{k}) \text{ T} = (0.30 \text{ N}\cdot\text{m})\hat{\jmath}$. *(ii)* The potential energy is $U = -\vec{\mu}\cdot\vec{B} = (0.75 \text{ A}\cdot\text{m}^2)\hat{k} \cdot (0.40\hat{\imath} + 0.30\hat{k}) \text{ T} = 0.225$ J.

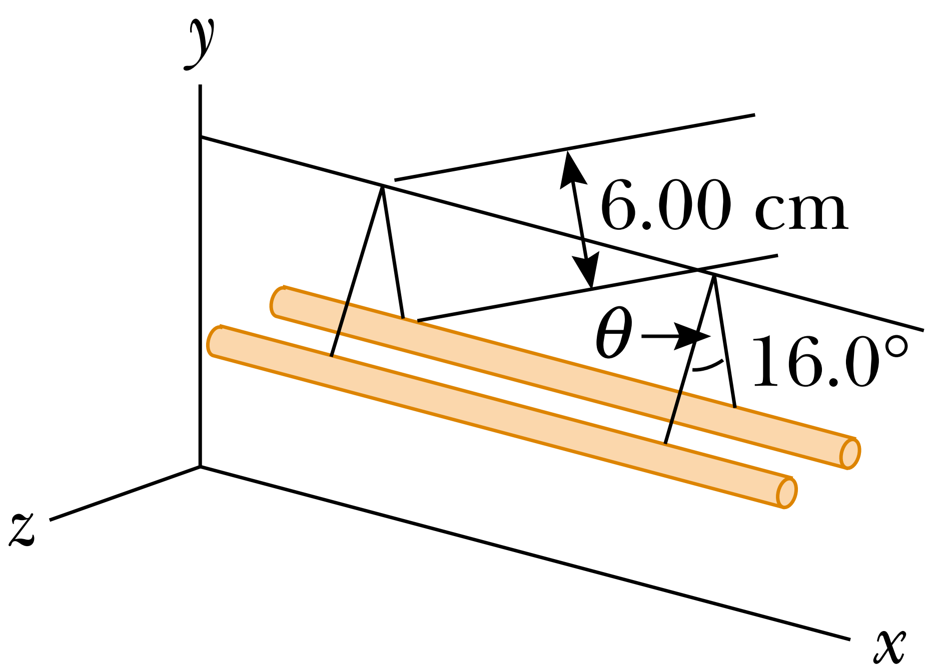

Figure 1: Problem 1.

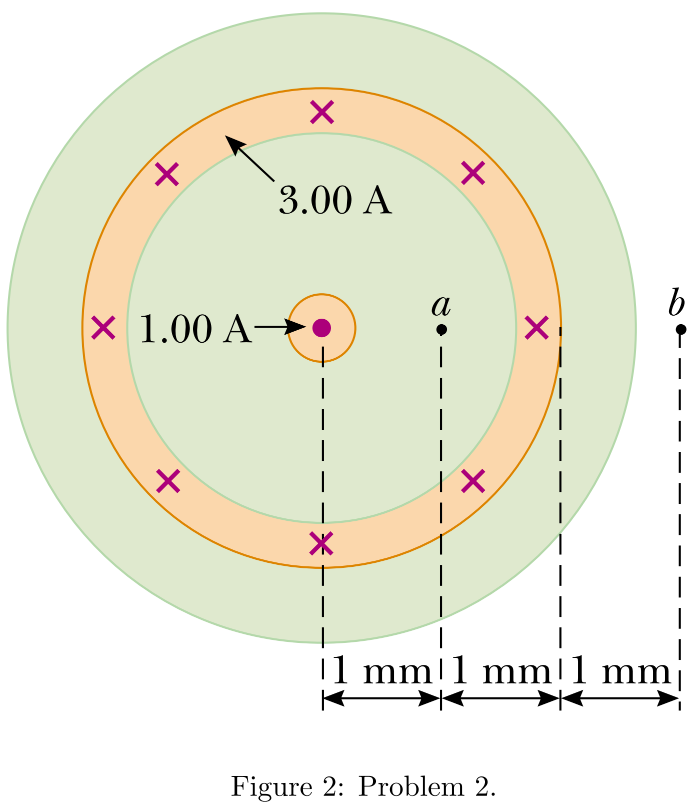

Figure 2: Problem 2.

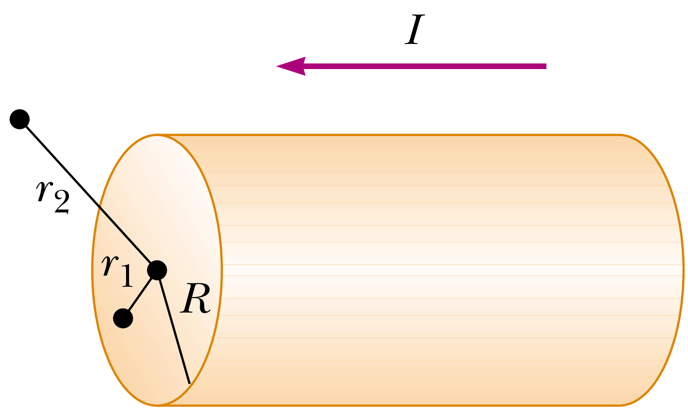

Figure 3: Problem 3.

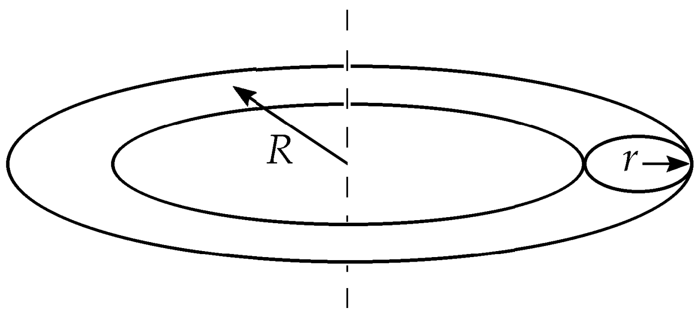

Figure 4: Problem 4.

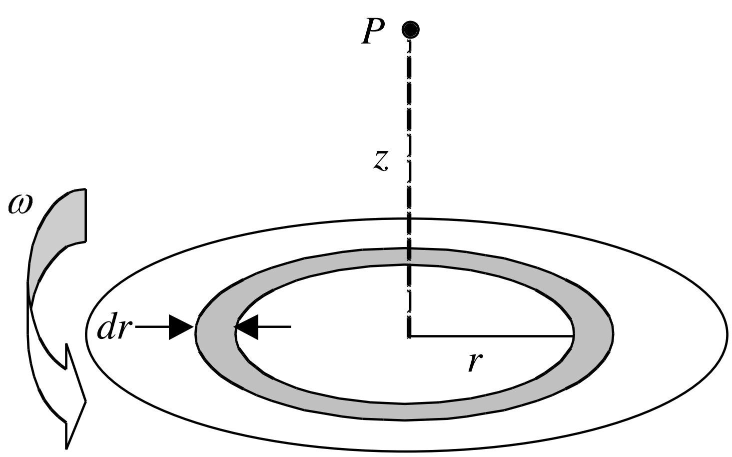

Figure 5: Problem 7.

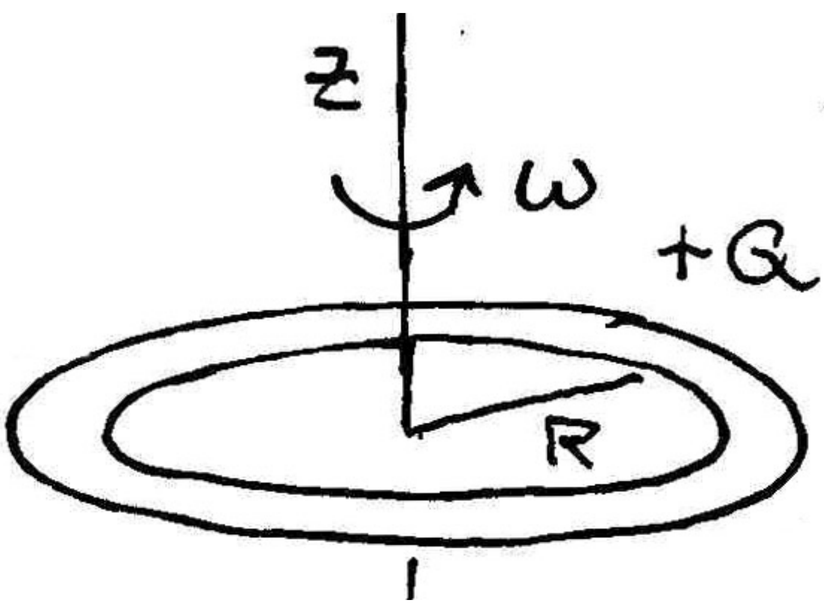

Figure 6: Problem 8.