Prof. Anchordoqui

Problems set # 7 **Physics 169** March 31, 2015

1. A particle of charge $-e$ is moving with an initial velocity v when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in Fig. 1. You may ignore effects of the gravitational force. *(i)* Is the trajectory of the particle deflected upward or downward? *(ii)* What is the magnitude of the velocity of the particle if it just strikes the end of the plate?

2. The entire $x-y$ plane to the right of the origin O is filled with a uniform magnetic field of magnitude B pointing out of the page, as shown in Fig. 2. Two charged particles travel along the negative x axis in the positive x direction, each with velocity $\vec{v}$, and enter the magnetic field at the origin O. The two particles have the same mass m, but have different charges, q_1 and q_2. When propagate thorugh the magnetic field, their trajectories both curve in the same direction (see sketch in Fig. 2), but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly twice as big as the radius of the semi-circle traced out by particle 1. *(i)* Are the charges of these particles positive or negative? Explain your reasoning. *(ii)* What is the ratio q_2/q_1?

3. Shown in Fig. 3 are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass m and charge $q = +e$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, S_1, in which there is an adjustable magnetic field $\vec{B}_1$, pointing out of the page and a deflecting electric field $\vec{E}$, pointing from positive to negative plate. Only particles of a uniform velocity $\vec{v}$ leave the selector. The emerging particles at S_2, enter a second magnetic field B_2, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance x from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{E}|$, e, x, m, $B_2 \equiv |\vec{B}_2|$, and ΔV. *(i)* What magnetic field B_1 in the selector chamber is needed to insure that the particle travels straight through? *(ii)* Find an expression for the mass of the particle after it has hit the electronic sensor at a distance x from the entry slit.

4. Electrons in a beam are accelerated from rest through a potential difference V. The beam enters an experimental chamber through a small hole. As shown in Fig. 4, the electron velocity vectors lie within a narrow cone of half angle ϕ oriented along the beam axis. We wish to use a uniform magnetic field directed parallel to the axis to focus the beam, so that all of the electrons can pass through a small exit port on the opposite side of the chamber after they travel the length d of the chamber. What is the required magnitude of the magnetic field? [*Hint:* Because every electron passes through the same potential difference and the angle ϕ is small, they all require the same time interval to travel the axial distance d.

5. Find the magnetic field at point P due to the current distribution shown in Fig. 5.

6. A nonconducting sphere has mass 80.0 g and radius 20.0 cm. A flat compact coil of wire with 5 turns is wrapped tightly around it, with each turn concentric with the sphere. As shown in Fig. 6, the sphere is placed on an inclined plane that slopes downward to the left, making an angle θ with the horizontal, so that the coil is parallel to the inclined plane. A uniform magnetic field of

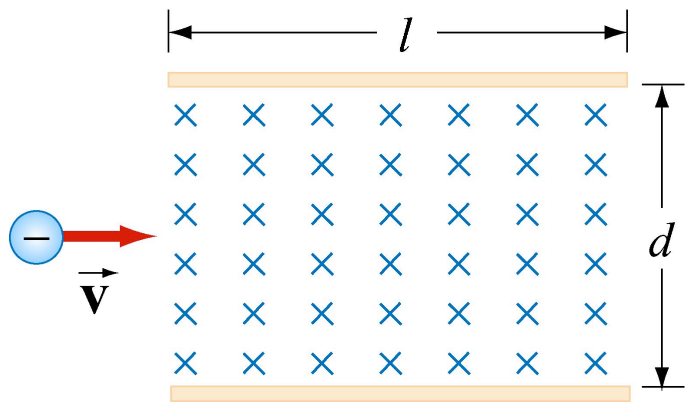

Figure 1: Problem 1.

0.350 T vertically upward exists in the region of the sphere. What current in the coil will enable the sphere to rest in equilibrium on the inclined plane? Show that the result does not depend on the value of θ.

7. Two circular coils of radius a, each with N turns, are perpendicular to a common axis. Each coil carries a steady current I in the same direction, as shown in Fig. 7 *(i)* Find the total magnetic field on the axis $B_z^{\rm tot}$ as a function of z, using the midway point as the origin ($z = 0$). *(ii)* Show that $\frac{dB_z^{\rm tot}}{dz} = 0$ at the midpoint. This means that, functionally, at least at that one location, the field is not changing. *(iii)* If the separation b is picked correctly, then $\frac{d^2 B_z^{\rm tot}}{dz^2} = 0$ at the midpoint. This configuration is known as Helmholtz coils. Determine the appropriate value of b. *(iv)* Show that the magnetic field on the axis near the origin, as an expansion in powers of z (up to z^4 inclusive) is given by

$$B_z^{\rm tot} = \frac{\mu_0 N I a^2}{d^3}\left[1 + \frac{3(b^2 - a^2)z^2}{2d^4} + \frac{15(b^4 - 6b^2a^2 + 2a^4)z^4}{16d^8} + \cdots\right],$$

where $d^2 = a^2 + b^2/4$. *(v)* Show that the magnetic field on the z axis for large $|z|$ is given by the expansion in inverse odd powers of $|z|$ obtained from the small z expansion of part (iv) by the formal substitution $d \to |z|$. *(vi)* What is the maximum permitted value of $|z|/a$ if the field is to be uniform to one part in 10^4 (or one part in 10^2) in the Helmholtz configuration?

8. Let us treat the motion of an electron (charge $-e$, mass m) in a hydrogen atom classically. Suppose that an electron follows a circular orbit of radius r around a proton. What is the angular frequency ω_0 of the orbital motion? Suppose now that a small magnetic field $\vec{B}$ perpendicular to the plane of the orbit is switched on. Assuming that the radius of the orbit does not change, calculate the shift in the angular frequency $\Delta\omega = \omega_f - \omega_0$ of the orbital motion in terms of the magnitude B of the magnetic field, charge $-e$, mass m, and radius r. This is known as the “Zeeman effect.”

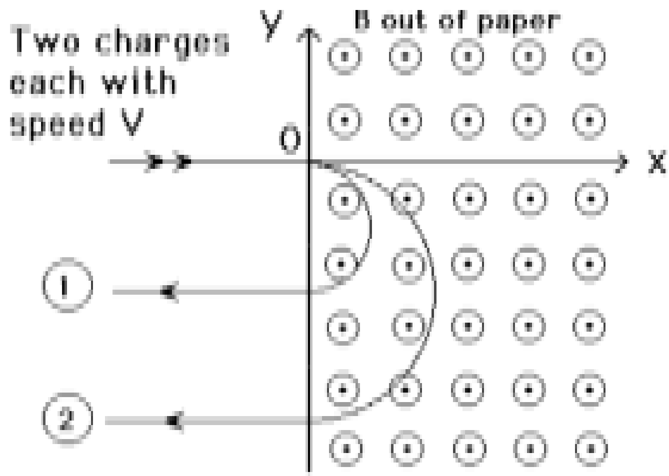

Figure 2: Problem 2.

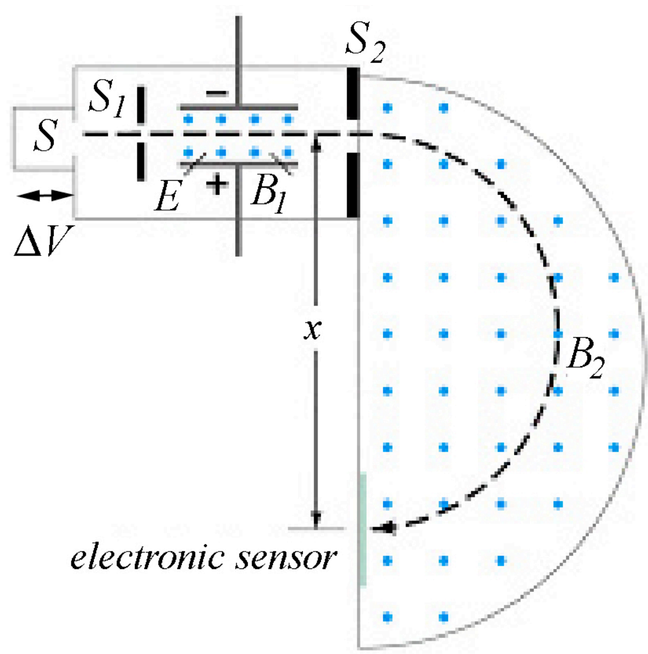

Figure 3: Problem 3.

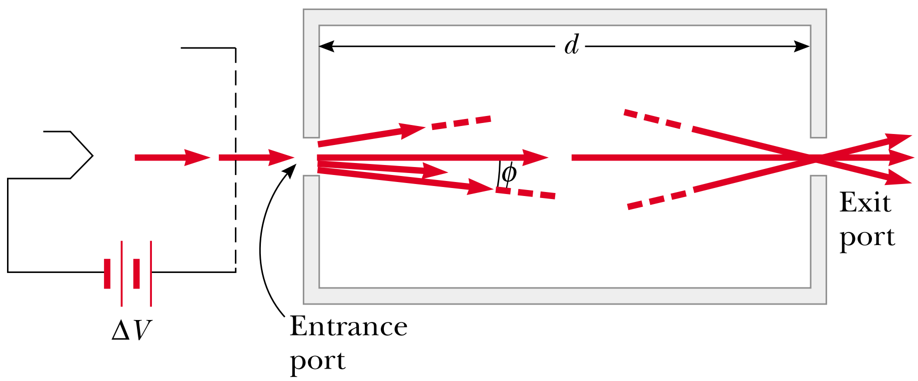

Figure 4: Problem 4.

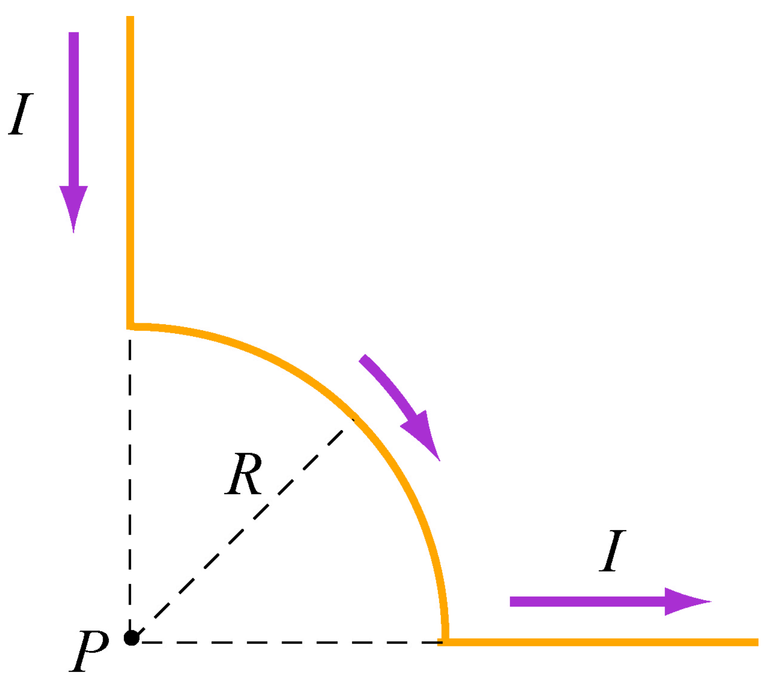

Figure 5: Problem 5.

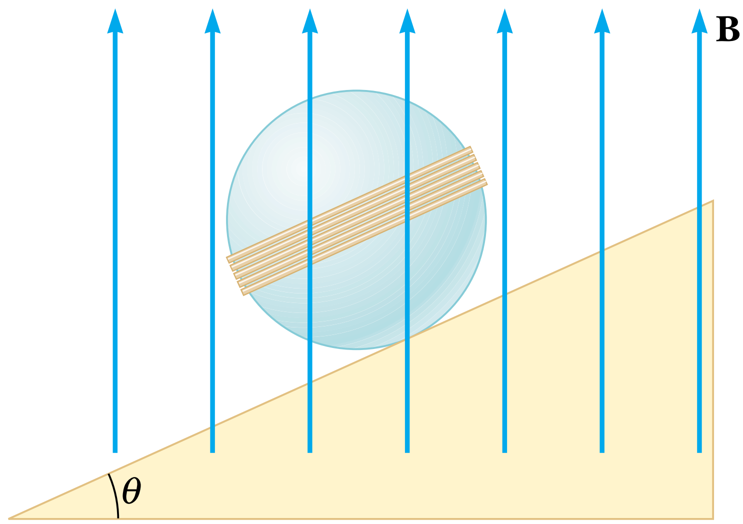

Figure 6: Problem 6.

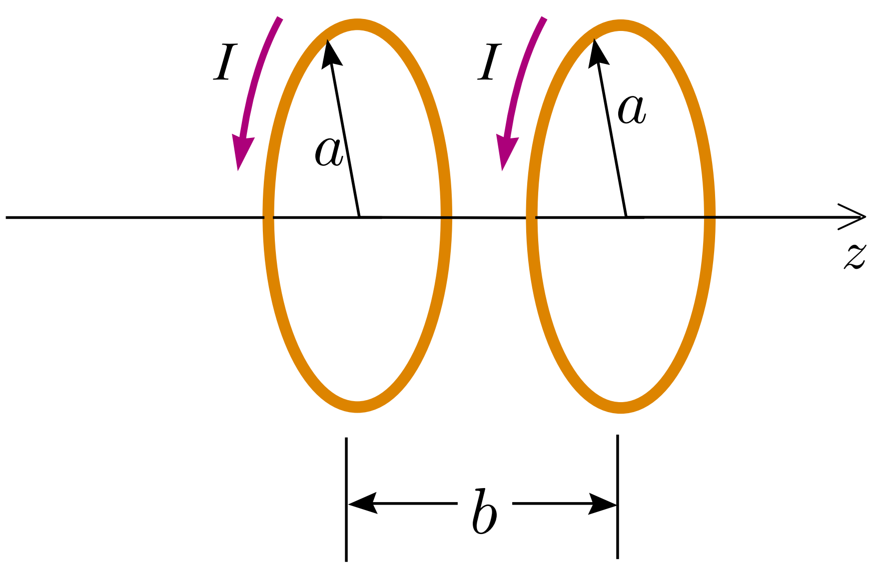

Figure 7: Problem 7.