Prof. Anchordoqui

Problems set # 6 **Physics 169** March 24, 2015

1. Find the current I in the circuit shown Fig. 1.

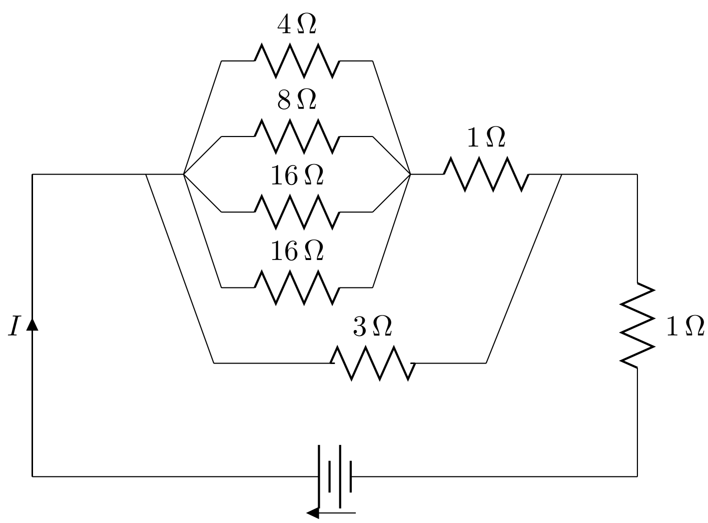
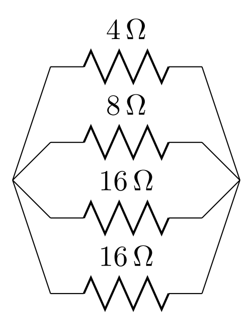


Figure 1: Problem 1.

Solution: The equivalent resistance, $R^1_{\rm eq}$ for



is $\frac{1}{R^1_{eq}} = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}\right)\ \Omega^{-1} = \frac{1}{2}\ \Omega^{-1}$; therefore, $R^1_{\rm eq} = 2\ \Omega$. This is in series with the $1\ \Omega$ resistor. Hence, $R^2_{\rm eq} = R^1_{\rm eq} + 1\ \Omega = 3\ \Omega$. Now, $R^2_{\rm eq}$ is in parallel with the $3\ \Omega$ resistance, $\frac{1}{R^3_{\rm eq}} = \left(\frac{1}{3} + \frac{1}{3}\right)\ \Omega^{-1}$; therefore, $R^3_{\rm eq} = 1.5\ \Omega$ The total equivalent resistance of the circuit is $R_{\rm eq} = (1.5 + 1)\ \Omega = 2.5\ \Omega$ From Ohm's law, $V = IR$, we get $I = \frac{V}{R_{\rm eq}} = \frac{20\ {\rm V}}{2.5\ \Omega} = 8$ A.

2. In the circuit shown in Fig. 2, the power produced by bulb$_1$ and bulb$_2$ is 1 kW and 50 W, respectively. Which light has the higher resistance? (Assume the resitance of the light bulb remains constant with time.)

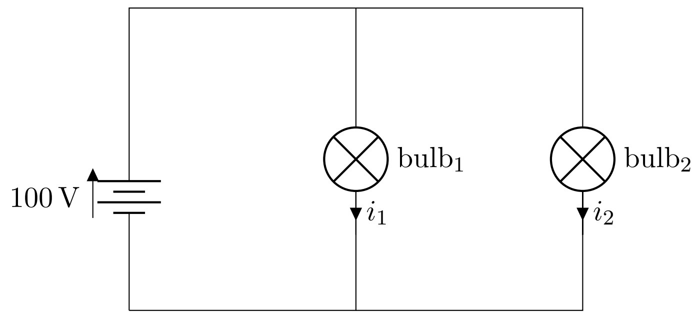


Figure 2: Problem 2.

Solution: The power dissipated by light bulb one and two, respectively, is $P_1 = I_1^2 R_1 = I_1 V$ and $P_2 = I_2^2 R_2 = I_2 V$. Thus, we have $I_1 = P_1/V$ and $I_2 = P_2/V$. This implies that $P_1 = \left(\frac{P_1}{V}\right)^2 R_1 \Rightarrow R_1 = \frac{V^2}{P_1} = 10\ \Omega$. Likewise $R_2 = \frac{V^2}{P_2} = 200\ \Omega$.

3. A regular tetrahedron is a pyramid with a triangular base. Six $R = 10.0\ \Omega$ resistors are placed along its six edges, with junctions at its four vertices, as shown in Fig. 3. A 12.0-V battery is connected to any two of the vertices. Find *(i)* the equivalent resistance of the tetrahedron between these vertices and *(ii)* the current in the battery.

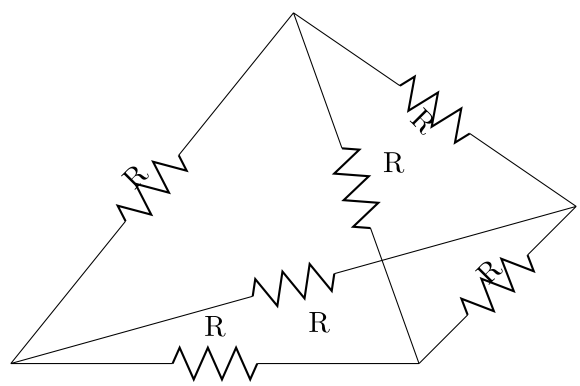


Figure 3: Problem 3.

Solution: *(i)* First let us flatten the circuit on a 2-D plane as shown in Fig. 4; then reorganize it to a format easier to read. Note that the voltage $V_{AB} = 0$ in Fig. 5 and so the middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance $\frac{1}{R_{\rm tot}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} = \frac{2}{R} \Rightarrow R_{\rm eq} = 5\ \Omega$. *(ii)* The current through the battery isn $\frac{\Delta V}{R_{\rm eq}} = \frac{12.0\ {\rm V}}{5\ \Omega} = 2.40$ A.

4. Find the equivalent resistance in the limit $n \to \infty$ for the circuits shown in Figs. 6 and 7.

Solution: *(i)* The equivalent resistance, $R_{\rm eq}$ is given as $\frac{1}{R_{\rm eq}} = \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \frac{1}{2^n}\right)\ \Omega^{-1}$.

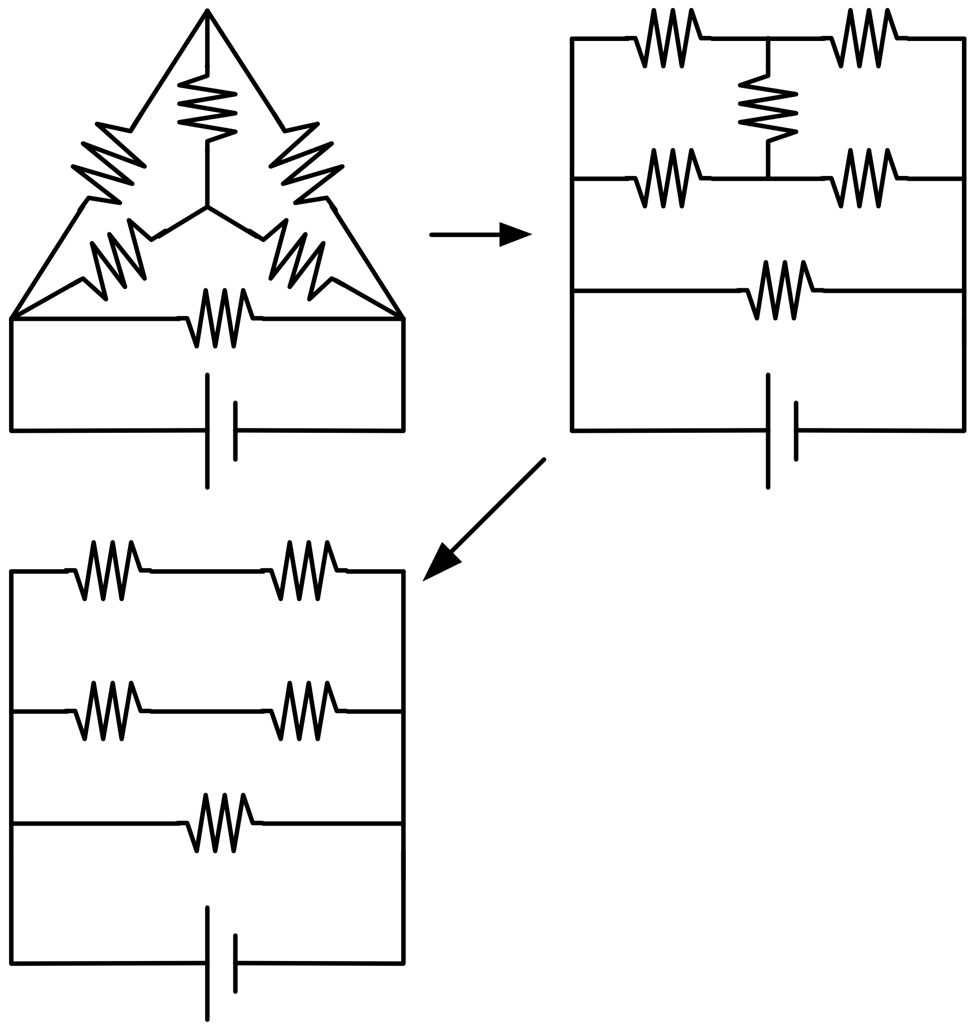


Figure 4: Solution of problem 3.

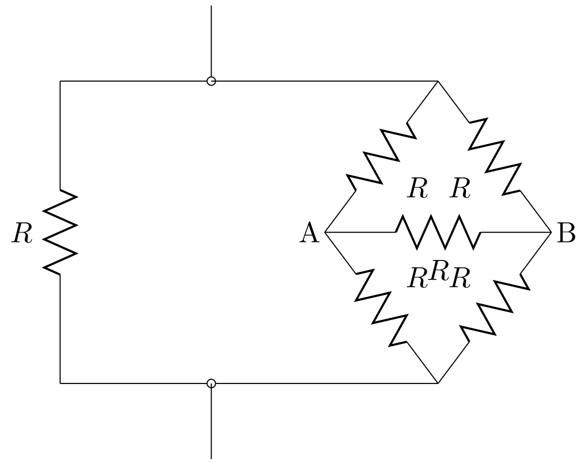


Figure 5: More on the solution of problem 3.

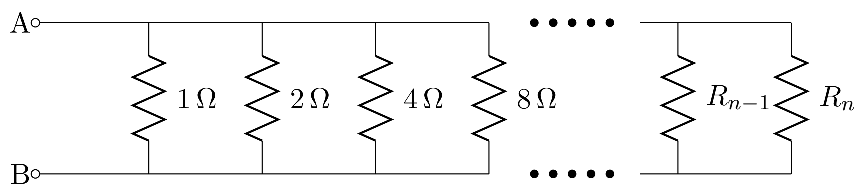


Figure 6: Problem 4 *(i)*.

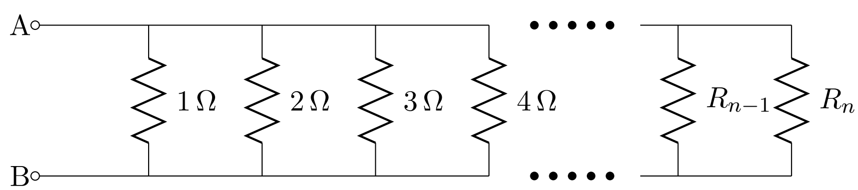


Figure 7: Problem 4 *(ii)*.

The above represents a geometric series with ratio $r = \frac{1}{2}\ \Omega^{-1}$. The sum of n terms is given by $S_n = \sum_{k=0}^{k=n-1} \frac{1-r^n}{1-r}$. In the limit $n \to \infty$, $S_{n\to\infty} = \frac{1}{1-r}$. Hence, $\frac{1}{R_{\rm eq}} = \left(\frac{1}{1-\frac{1}{2}}\right)\ \Omega^{-1}$ and so $R_{\rm eq} = 0.5\ \Omega$. *(ii)* The equivalent resistance, $R_{\rm eq}$ is given as $\frac{1}{R_{\rm eq}} = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n} + \cdots\right)\ \Omega^{-1} = \left(\sum_{j=1}^{\infty} j^{-1}\right)\ \Omega^{-1}$. The above represents a harmonic series with infinite sum. Hence $R_{\rm eq} = 0$.

5. Determine the magnitude and directions of the currents through $R_1 = 22\ \Omega$ and $R_2 = 15\ \Omega$ in the circuit of Fig. 8. The batteries have an internal resistance of $r = 1.2\ \Omega$.

Solution: There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction

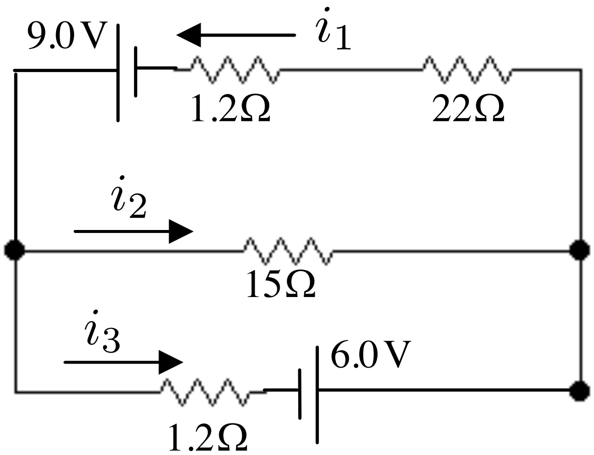


Figure 8: Problem 5.

of the three branches on the left of the circuit: $i_1 = i_2 + i_3$. Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise

$$-i_3(1.2\ \Omega) + 6\ \text{V} - i_1(22\ \Omega) - i_1(1.2\ \Omega) + 9\ \text{V} = 0 \Rightarrow 15 = 23.2 i_1 + 1.2 i_3\ .$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, strating at the lower left corner, and progressing counterclockwise:

$$-i_3(1.2\ \Omega) + 6\ \text{V} + i_2(15\ \Omega) = 0 \Rightarrow 6 = -15 i_2 + 1.2 i_3\,.$$

Substitute $i_1 = i_2 + i_3$ into the loop equation, so that there are two equations with two unknowns

$$15 = 23.2 i_1 + 1.2 i_3 = 23.2(i_2 + i_3) + 1.2 i_3 = 23.2 i_2 + 24.4 i_3$$

and

$$6 = -15 i_2 + 1.2 i_3$$

. Solve the bottom loop equation for i_2 and substitute into the loop equation, resulting in an equation with only one unknown, which can be solved

$$6 = -15 i_2 + 1.2 i_3 \Rightarrow i_2 = \frac{-6 + 1.2 i_3}{15}$$

$$15 = 23.2 i_2 + 24.4 i_3 = 23.2\left(\frac{-6 + 1.2 i_3}{15}\right) + 24.4 i_3 \Rightarrow i_3 = 363/393.84 = 0.917\ \text{A};$$

$$i_2 = \frac{-6 + 1.2 i_3}{15} = -0.33\ \text{A, left.}$$

$$i_1 = i_2 + i_3 = 0.6\ \text{A},\ \text{left.}$$

6. Determine the magnitude and directions of the currents in each resistor shown in Fig. 9. The batteries has emfs of $\varepsilon_1 = 9$ V and $\varepsilon_2 = 12$ V and the resistors have values of $R_1 = 25\ \Omega$,

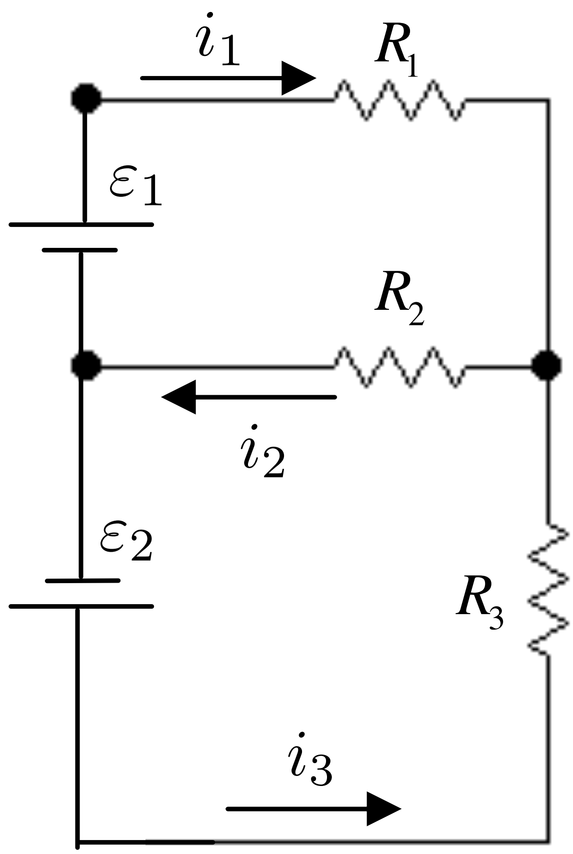


Figure 9: Problem 6.

$R_2 = 18\ \Omega$, and $R_3 = 35\ \Omega$.

Solution: There are three currents involved, and so there must be three independent equations to determine those currents. One comes from Kirchhoff's junction rule applied to the junction on the three branches on the right of the circuit

$$i_2 = i_1 + i_3 \Rightarrow i_1 = i_2 - i_3.$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise

$$\varepsilon_1 - i_1 R_1 - i_2 R_2 = 0 \Rightarrow 9 = 25 i_1 + 18 i_2.$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise

$$\varepsilon_2 - i_3 R_3 - i_2 R_2 = 0 \Rightarrow 12 = 35 i_3 + 18 i_2\,.$$

Substitute $i_1 = i_2 - i_3$ into the loop equation, so that there are two equations with two unknowns:

$$9 = 25 i_1 + 18 i_2 = 25(i_2 - i_3) + 18 i_2 = 43 i_2 - 25 i_3$$

and

$$12 = 35 i_3 + 18 i_2\,.$$

Solve the bottom loop equation for i_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved

$$12 = 35 i_3 + 18 i_2 \Rightarrow i_2 = \frac{12 - 35 i_3}{18}$$

$$9 = 43 i_2 - 25 i_3 = 43\left(\frac{12 - 35 i_3}{18}\right) - 25 i_3 \Rightarrow 162 = 516 - 1505 i_3 - 450 i_3 \Rightarrow i_3 = \frac{354}{1955} = 0.18\ \text{A},\ \text{up}\,;$$

$$i_2 = \frac{12 - 35 i_3}{18} = 0.31\ \text{A},\ \text{left}$$

and

$$i_1 = i_2 - i_3 = 0.13\ \text{A},\ \text{right}.$$

7. For the circuit shown in Fig. 10, calculate *(i)* the current in the 2.00 Ω resistor and *(ii)* the potential difference between points a and b.

(i) We name the currents i_1, i_2, and i_3 as shown in the figure, and so $i_1 = i_2 + i_3$. Going countercolckwise around the loop we get, $12.0\ \text{V} - 2.00\ \Omega\ i_3 - 4.00\ \Omega\ i_1 = 0$. Traversing the bottom loop we have $8\ \text{V} - 6.00\ \Omega\ i_2 + 2.00\ \Omega\ i_3 = 0$. We can solve these last two equations for i_1 and i_2, yielding $i_1 = 3.00\ \text{A} - \frac{1}{2} i_3$ and $i_2 = \frac{4}{3}\ \text{A} + \frac{1}{3} i_3$. This means that $i_3 = 909$ mA. *(ii)* $V_a - 0.909\ \text{A} \cdot 2.00\ \Omega = V_b$, then $V_b - V_a = -1.82$ V.

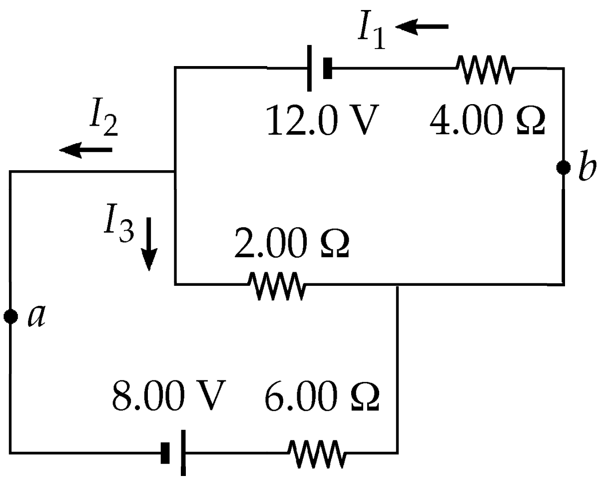


Figure 10: Problem 7.

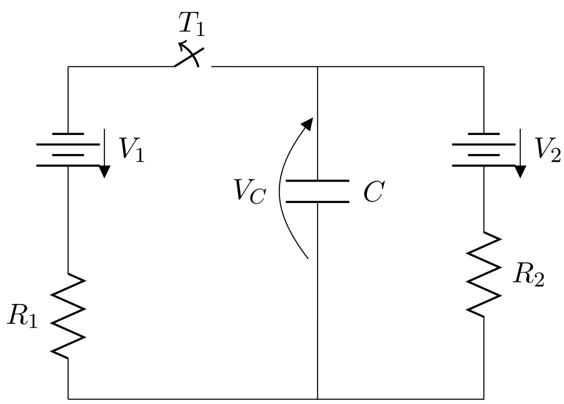


Figure 11: Problem 8.

8. Consider the circuit shown in Fig. 11, with the start up switch T_1 open (for a long time). Now, close the switch and wait for a while. What is the change in the total charge of the capacitor?

Solution: While the switch remains open the charge in the capacitor is $q_i = V_2 C$. If we closed the switch and wait for a while we have $V_1 + IR_1 + IR_2 - V_2 = 0$ and so $I = \frac{V_2 - V_1}{R_1 + R_2}$. The voltage across the capacitor is $V_C = V_1 + IR_1 = V_1 + \left(\frac{V_2 - V_1}{R_1 + R_2}\right) R_1$. Hence, the change in the charge is $\Delta q = C\left(V_1 - V_2 + \frac{V_2 - V_1}{R_1 + R_2} R_1\right)$.

9. The circuit in Fig. 12 has been connected for a long time. *(i)* What is the voltage across the capacitor? *(ii)* If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?

Solution *(i)* Call the potential at the left junction V_L and at the right V_R. After a "long" time,

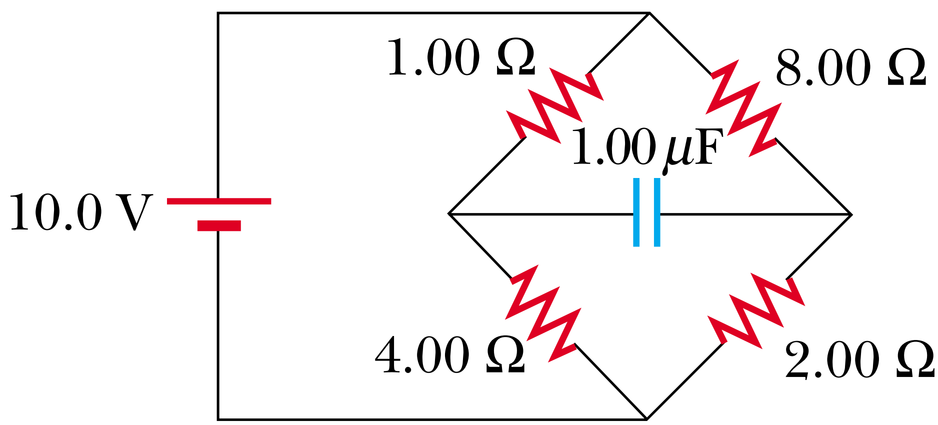


Figure 12: Problem 9.

the capacitor is fully charged. $V_L = 8.00$ V because of voltage divider: $I_L = \frac{10..0 \text{ V}}{5.0\ \Omega} = 2.00$ A, $V_L = 10.0$ V $-$ 2.00 A 1.00 Ω = 8.00 V. Likewise, $V_R = \frac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega} 10.0$ V = 2.00 V, or $I_R = \frac{10.0 \text{ V}}{10.0\ \Omega} = 1.00$ A, $V_R = 10.0$ V $-$ 8.00 Ω 1.00 A = 2.00 V. Therefore $\Delta V = V_L - V_R = 8.00 - 2.00 = 6.00$ V. *(ii)* Redraw the circuit $R = \frac{1}{1/9\ \Omega + 1/6\ \Omega} = 3.60\ \Omega$, so $RC = 3.60 \times 10^{-6}$ s and $e^{-t/RC} = \frac{1}{10}$, so $t = RC \ln 10 = 8.29\ \mu$s.

10. Find the voltage across A and B (i.e. V_{AB}) as a function of time in the circuit shown in Fig. 13

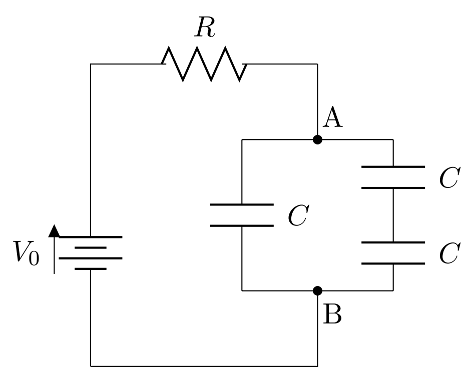


Figure 13: Problem 10.

Solution It is easily seen that for an RC circuit $V(t) = V_0 \left[1 - \exp\left(-\frac{t}{RC_{\text{tot}}}\right)\right]$, where for the case at hand $C_{\text{tot}} = C + C/2 = 3C/2$. Therefore, $V(t) = V_0 \left[1 - \exp\left(-\frac{2t}{3RC}\right)\right]$.

11. The switch in Fig. 14*(a)* closes when $\Delta V_c > 2\Delta V/3$ and opens when $\Delta V_c < \Delta V/3$. The voltmeter reads a voltage as plotted in Fig. 14*(b)*. What is the period T of the waveform in terms of R_1, R_2, and C?

<u>Solution</u> Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards zero V with a time constant R_2C, $\Delta V_C(t) = \frac{2}{3}\Delta V e^{-t/(R_2C)}$. We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$. Therefore, $\frac{1}{3}\Delta V = \frac{2}{3}\Delta V e^{-t/(R_2C)}$, or $e^{-t/(R_2C)} = \frac{1}{2}$, or $t_1 = R_2 C \ln 2$. After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1+R_2)C$, hence $\Delta V_C(t) = \Delta V - \frac{2}{3}\Delta V e^{-t/[(R_1+R_2)C]}$. For $\Delta V_C(t) = \frac{2}{3}\Delta V$, we have $\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/[(R_1+R_2)C]}$, or $e^{-t/[(R_1+R_2)C]} = \frac{1}{2}$. Therefore, $t_2 = (R_1 + R_2)C \ln 2$ and $T = t_1 + t_2 = (R_1 + 2R_2)C \ln 2$.

12. This problem illustrates how a digital voltmeter affects the voltage across a capacitor in an RC circuit. A digital voltmeter of internal resistance r is used to measure the voltage across a capacitor after the switch in Fig. 15 is closed. Because the meter has finite resistance, part of the current supplied by the battery passes through the meter. *(i)* Apply Kirchhoff's rules to this circuit, and use the fact that $i_C = dq/dt$ to show that this leads to the differential equation

$$R_{\rm eq}\frac{dq}{dt} + \frac{q}{C} = \frac{r}{r+R}\mathcal{E},$$

where $R_{\rm eq} = rR/(r+R)$. *(ii)* Show that the solution to this differential equation is

$$q = \frac{r}{r+R}C\mathcal{E}\left(1 - e^{-t/(R_{\rm eq}C)}\right)$$

and that the voltage across the capacitor as a function of time is

$$V_C = \frac{r}{r+R}\mathcal{E}(1 - e^{-t/(R_{\rm eq}C)}).$$

(iii) If the capacitor is fully charged, and the switch is then opened, how does the voltage across the capacitor behave in this case?

<u>Solution</u> Let i represent the current in the battery and i_c the current charging the capacitor. Then $i - i_c$ is the current in the voltmeter. The loop rule applied to the inner loop is $\mathcal{E} - iR - \frac{q}{C} = 0$. The loop rule for the outer perimeter is $\mathcal{E} - iR - (i - i_c)r = 0$. With $i_c = \frac{dq}{dt}$, this becomes $\mathcal{E} - iR - ir + \frac{dq}{dt}r = 0$. Between the two loop equations we eliminate $i = \frac{\mathcal{E}}{R} - \frac{q}{RC}$ by substitution to obtain $\mathcal{E} - (R+r)\left(\frac{\mathcal{E}}{R} - \frac{q}{RC}\right) + \frac{dq}{dt}r = 0$. Rearranging terms $\mathcal{E} - \frac{R+r}{R}\mathcal{E} + \frac{R+r}{RC}q + \frac{dq}{dt}r = 0$, or equivalently $-\frac{r}{R+r}\mathcal{E} + \frac{q}{C} + \frac{Rr}{R+r}\frac{dq}{dt} = 0$. This is the differential equation required. *(ii)* To solve we follow the same steps as on the lecture: $\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{R+r}{RrC}q = -\frac{R+r}{RrC}\left(q - \frac{\mathcal{E}rC}{R+r}\right)$; integration $\int_0^q \frac{dq}{q - \mathcal{E}rC/(R+r)} = -\frac{R+r}{RrC}\int_0^t dt$ leads to $\ln\left(q - \frac{\mathcal{E}rC}{R+r}\right)\Big|_0^q = -\frac{R+r}{RrC}t\Big|_0^t$, yielding $\ln\left[\frac{q - \mathcal{E}rC/(R+r)}{-\mathcal{E}rC/(R+r)}\right] = -\frac{R+r}{RrC}t$, or equivalently $q - \frac{\mathcal{E}rC}{R+r} = -\frac{\mathcal{E}rC}{R+r}e^{[-(R+r)/(RrC)]t}$. Rearranging terms $q = \frac{r}{r+R}C\mathcal{E}\left(1 - e^{-t/(R_{\rm eq}C)}\right)$, where $R_{\rm eq} = \frac{Rr}{R+r}$. The voltage across the capacitor is $V_C = \frac{q}{C} = \frac{r}{r+R}\mathcal{E}\left(1 - e^{-t/(R_{\rm eq}C)}\right)$. *(iii)* As $t \to \infty$ the capacitor voltage approaches $\frac{r}{r+R}\mathcal{E}(1-0) = \frac{r\mathcal{E}}{r+R}$. If the switch is then opened, the capacitor discharges through the voltmeter. Its voltage decays exponentially according to $\frac{r\mathcal{E}}{r+R}e^{-t/(rC)}$.

13. When two slabs of n-type and p-type semiconductors are put in contact, the relative affinities of the materials cause electrons to migrate out of the n-type material across the junction to the

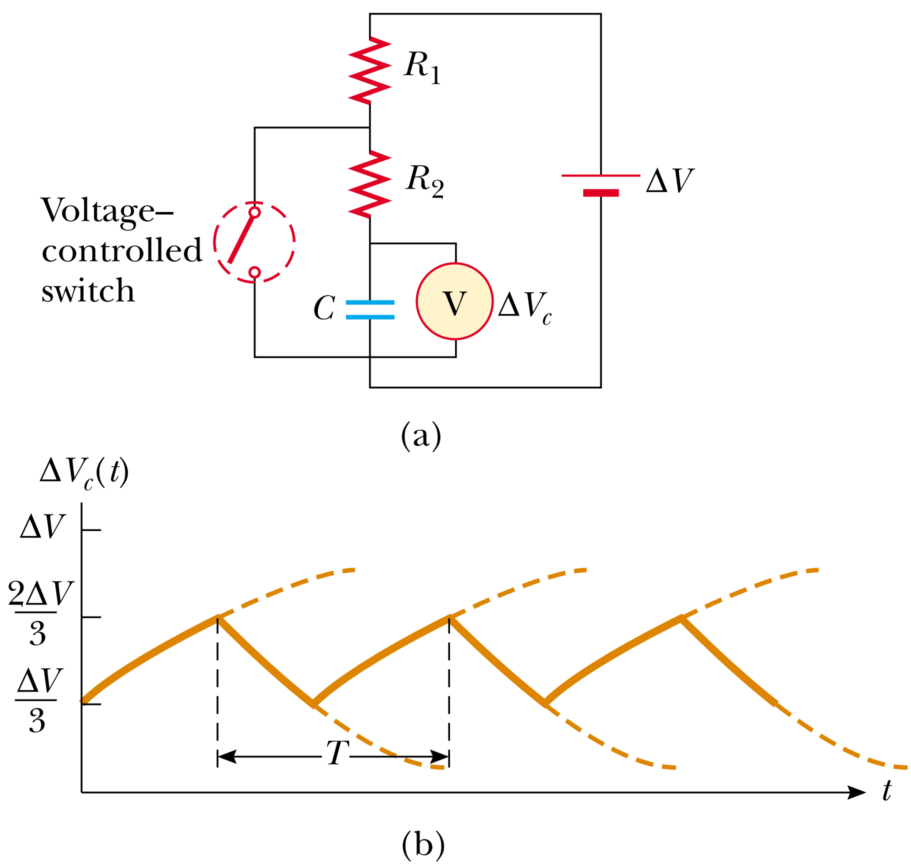


Figure 14: Problem 11.

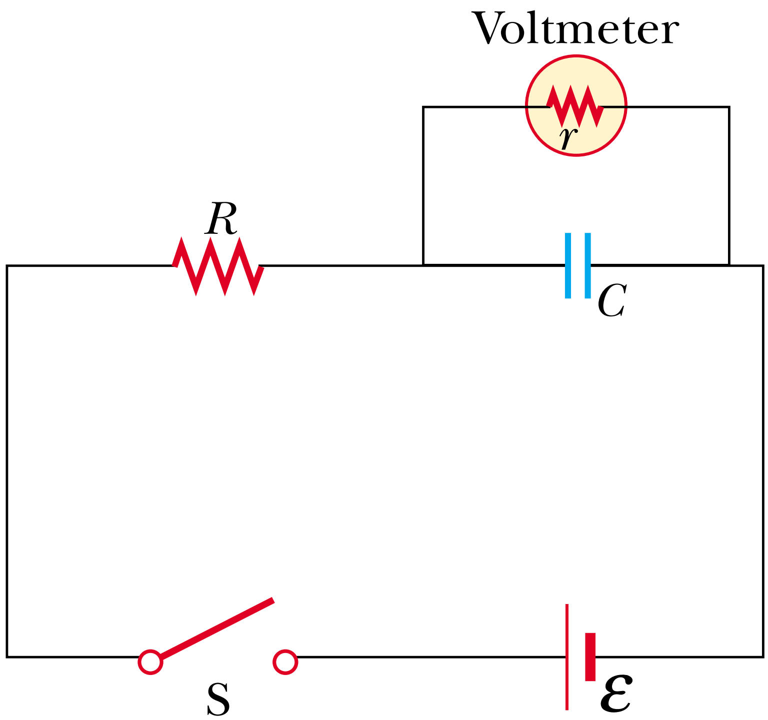


Figure 15: Problem 12.

p-type material. This leaves behind a volume in the n-type material that is positively charged and creates a negatively charged volume in the p-type material. Let us model this as two infinite slabs of charge, both of thickness a with the junction lying on the plane $z = 0$. The n-type material lies in the range $0 < z < a$ and has uniform charge density $+\rho_0$. The adjacent p-type material lies in the range $-a < z < 0$ and has uniform charge density $-\rho_0$; see Fig. 16. Hence:

$$\rho(x, y, z) = \rho(z) = \begin{cases} +\rho_0 & 0 < z < a \\ -\rho_0 & -a < z < 0 \\ 0 & |z| > a \end{cases} .$$

(i) Find the electric field everywhere. *(ii)* Find the potential difference between the points P_1 and P_2. The point P_1 is located on a plane parallel to the slab a distance $z_1 > a$ from the center of the slab. The point P_2 is located on plane parallel to the slab a distance $z_2 < -a$ from the center of the slab.

Solution In this problem, the electric field is a superposition of two slabs of opposite charge density. Outside both slabs, the field of a positive slab $\vec{E}_P$ (due to the p-type semi-conductor) is constant and points away and the field of a negative slab $\vec{E}_N$ (due to the n-type semi-conductor) is also constant and points towards the slab, so when we add both contributions we find that the electric field is zero outside the slabs. The fields $\vec{E}_P$ and $\vec{E}_N$ are shown on Fig. 17. The superposition of these fields $\vec{E}_T$ is shown on the top line in the figure. The electric field can be

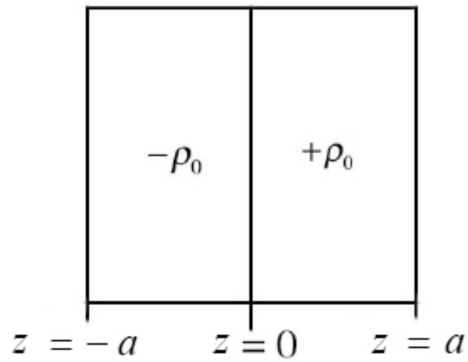


Figure 16: Problem 13.

described by

$$\vec{E}_T(z) = \begin{cases} \vec{0} & z < -a \\ \vec{E}_2 & -a < z < 0 \\ \vec{E}_1 & 0 < z < a \\ \vec{0} & z > a \end{cases}.$$

We shall now calculate the electric field in each region using Gauss law: For region $-a < z < 0$, the Gaussian surface is shown on the left hand side of Fig 17. Notice that the field is zero outside. Gauss law states that $\oint_{\substack{\text{closed}\\\text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$. So for our choice of Gaussian surface, on the cap inside the slab the unit normal for the area vector points in the positive z-direction, thus $\hat{n} = +\hat{k}$. Consequently the dot product becomes $\vec{E}_2\cdot\hat{n}dA = E_{2,z}\hat{k}\cdot\hat{k}dA = E_{2,z}dA$. Therefore the flux is $\oint_{\substack{\text{closed}\\\text{surface}}} \vec{E}\cdot d\vec{A} = E_{2,z}A_{\text{cap}}$. The charge enclosed is $\frac{Q_{\text{enc}}}{\epsilon_0} = -\frac{\rho_0 A_{\text{cap}}(a+z)}{\epsilon_0}$ where the length of the Gaussian cylinder is $a+z$ since $z < 0$. Substituting these two results into Gauss law yields $E_{2,z}A_{\text{cap}} = -\frac{\rho_0 A_{\text{cap}}(a+z)}{\epsilon_0}$. Hence the electric field in the n-type is given by $E_{2,z} = -\frac{\rho_0(a+z)}{\epsilon_0}$. The negative sign means that the electric field point in the $-z$-direction so the electric field is $\vec{E}_2 = -\frac{\rho_0(a+z)}{\epsilon_0}\hat{k}$. Note that when $z = -a$ then $\vec{E}_2 = 0$ and when $z = 0$, $\vec{E}_2 = -\frac{\rho_0 a}{\epsilon_0}\hat{k}$. We make a similar calculation for the electric field in the p-type noting that the charge density has changed sign and the expression for the length of the Gaussian cylinder is $a - z$ since $z > 0$. Also the unit normal now points in the negative z-direction. So the dot product becomes $\vec{E}_1 \cdot \hat{n}dA = E_{1,z}(-\hat{k}) \cdot \hat{k}dA = -E_{1,z}dA$. Thus the Gauss law becomes $-E_{1,z}A_{\text{cap}} = \frac{\rho_0 A_{\text{cap}}(a-z)}{\epsilon_0}$. So the electric field is $E_{1,z} = -\frac{\rho_0(a-z)}{\epsilon_0}$. The vector description is then $\vec{E}_1 = -\frac{\rho_0(a-z)}{\epsilon_0}\hat{k}$. Note that when $z = a$ then $\vec{E}_1 = \vec{0}$ and when $z = 0$, $\vec{E}_1 = -\frac{\rho_0 a}{\epsilon_0}\hat{k}$. So the resulting field is

$$\vec{E}_T(z) = \begin{cases} \vec{0} & z < -a \\ \vec{E}_2 = -\frac{\rho_0(a+z)}{\epsilon_0}\hat{k} & -a < z < 0 \\ \vec{E}_1 = -\frac{\rho_0(a-z)}{\epsilon_0}\hat{k} & 0 < z < a \\ \vec{0} & z > a \end{cases}.$$

The graph of the electric field is shown in Fig. 17. *(ii)* The electric potential difference is given by the integral $V(P_2) - V(P_1) = -\int_{P_1}^{P_2} \vec{E}_T\cdot d\vec{r}$. We first break this line integral into four pieces covering each region $V(z_2) - V(z_1) = -\left(\int_{z=z_1}^{z=a} \vec{E}_t \cdot d\vec{r} + \int_{z=a}^{z=0} \vec{E}_T \cdot d\vec{r} + \int_{z=0}^{z=-a} \vec{E}_T \cdot d\vec{r} + \int_{z=-a}^{z=z_2} \vec{E}_T \cdot d\vec{r}\right)$. Since the

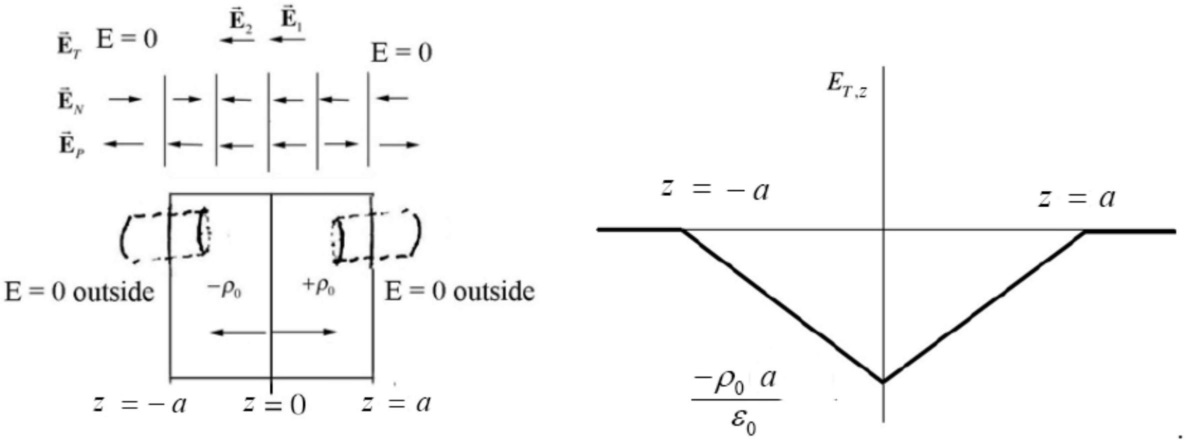


Figure 17: Solution of problem 13.

fields are zero outside the slab, the only non-zero pieces are $V(z_2) - V(z_1) = -\int_{z=a}^{z=0} \vec{E}_T \cdot d\vec{r} - \int_{z=0}^{z=-a} \vec{E}_T \cdot d\vec{r}$. We now use our explicit result for the electric field in each region and that $d\vec{r} = dz\hat{k}$, $V(z_2) - V(z_1) = -\left(\int_{z=a}^{z=0} E_{1,z}\hat{k} \cdot dz\hat{k} + \int_{z=0}^{z=-a} E_{2,z}\hat{k} \cdot dz\hat{k}\right) = \int_{z=a}^{z=0} \rho_0 \frac{a-z}{\epsilon_0} dz + \int_{z=0}^{z=-a} \rho_0 \frac{a+z}{\epsilon_0} dz$. We now calculate the two integrals $V(z_2) - V(z_1) = \frac{\rho_0(za - z^2/2)}{\epsilon_0}\Big|_{z=a}^{z=0} + \frac{\rho_0(za + z^2/2)}{\epsilon_0}\Big|_{z=0}^{z=-a} = -\frac{\rho_0(a^2 - a^2/2}{\epsilon_0} + \frac{\rho_0[(-a)a + (-a)^2/2]}{\epsilon_0} = -\frac{\rho_0(a^2/2)}{\epsilon_0} - \frac{\rho_0(a^2/2)}{\epsilon_0} = -\frac{\rho_0 a^2}{\epsilon_0}$. The potential difference is negative because we are moving along the direction of the field. So the type *pn*-semiconductor slab established a small potential difference across the slab.