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Problems set # 5 **Physics 169** March 17, 2015

1. *(i)* Three capacitors are connected to a 12.0 V battery as shown in Fig. 1 Their capacitances are $C_1 = 3.00\ \mu$F, $C_2 = 4.00\ \mu$F, and $C_3 = 2\mu$F. Find the equivalent capacitance of this set of capacitors. *(ii)* Find the charge on and the potential difference across each.

2. A dielectric rectangular slab has length s, width w, thickness d, and dielectric constant κ. The slab is inserted on the right hand side of a parallel-plate capacitor consisting of two conducting plates of width w, length L, and thickness d. The left hand side of the capacitor of length $L - s$ is empty, see Fig. 2. The capacitor is charged up such that the left hand side has surface charge densities $\pm\sigma_L$ on the facing surfaces of the top and bottom plates respectively and the right hand side has surface charge densities $\pm\sigma_R$ on the facing surfaces of the top and bottom plates respectively. The total charge on the entire top and bottom plates is $+Q$ and $-Q$ respectively. The charging battery is then removed from the circuit. Neglect all edge effects. *(i)* Find an expression for the magnitude of the electric field E_L on the left hand side in terms of σ_L, σ_R, κ, s, w, L, ϵ_0, and d as needed. *(ii)* Find an expression for the magnitude of the electric field E_R on the right hand side in terms of σ_L, σ_R, κ, s, w, L, ϵ_0, and d as needed. *(iii)* Find an expression that relates the surface charge densities σ_L and σ_R in terms of κ, s, w, L, ϵ_0, and d as needed. *(iv)* What is the total charge $+Q$ on the entire top plate? Express your answer in terms of σ_L, σ_R, κ, s, w, L, ϵ_0, and d as needed. *(v)* What is the capacitance of this system? Express your answer in terms of κ, s, w, L, ϵ_0, and d as needed. *(vi)* Suppose the dielectric is removed. What is the change in the stored potential energy of the capacitor? Express your answer in terms of Q, κ, s, w, L, ϵ_0, and d as needed.

3. *(i)* Consider a plane-parallel capacitor completely filled with a dielectric material of dielectric constant κ. What is the capacitance of this system? *(ii)* A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Fig. 3. You may assume that $l \gg d$. Find an expression for the capacitance of the device in terms of the plate area A, d, κ_1, κ_2, and κ_3.

4. A model of a red blood cell portrays the cell as a spherical capacitor – a positively charged liquid sphere of surface area A, separated by a membrane of thickness t from the surrounding negatively charged fluid. Tiny electrodes introduced into the interior of the cell show a potential difference of 100 mV across the membrane. The membranes thickness is estimated to be 100 nm and its dielectric constant to be 5.00. *(i)* If an average red blood cell has a mass of 1.00×10^{-12} kg, estimate the volume of the cell and thus find its surface area. The density of blood is $1,100\ \text{kg/m}^3$. *(ii)* Estimate the capacitance of the cell. *(iii)* Calculate the charge on the surface of the membrane. How many electronic charges does this represent?

5. A capacitor consists of two concentric spherical shells. The outer radius of the inner shell is $a = 0.1$ m and the inner radius of the outer shell is $b = 0.2$ m. *(i)* What is the capacitance C of this capacitor? *(ii)* Suppose the maximum possible electric field at the outer surface of the inner shell before the air starts to ionize is $E_{\max}(a) = 3.0 \times 10^6\ \text{V} \cdot \text{m}^{-1}$. What is the maximum possible charge on the inner capacitor? *(iii)* What is the maximum amount of energy stored in this capacitor? *(iv)* What is the potential difference between the shells when $E(a) = 3.0 \times 10^6\ \text{V} \cdot \text{m}^{-1}$?

6. A parallel plate capacitor has capacitance C. It is connected to a battery until is fully charged,

and then disconnected. The plates are then pulled apart an extra distance d, during which the measured potential difference between them changed by a factor of 4. What is the volume of the dielectric necessary to fill the region between the plates? (Make sure that you give your answer only in terms of variables defined in the statement of this problem, fundamental constants and numbers).

7. Consider two nested cylindrical conductors of height h and radii a and b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder). See Fig. 4. You may ignore edge effects. *(i)* Calculate the electric field between the two cylinders ($a < r < b$). *(ii)* Calculate the potential difference between the two cylinders. *(iii)* Calculate the capacitance of this system, $C = Q/\Delta V$ *(iii)* Numerically evaluate the capacitance, given: $h \simeq 15$ cm, $a \simeq 4.75$ cm and $b \simeq 7.25$ cm. *(iv)* Find the electric field energy density at any point between the conducting cylinders. How much energy resides in a cylindrical shell between the conductors of radius r (with $a < r < b$), height h, thickness dr, and volume $2\pi rhdr$? Integrate your expression to find the total energy stored in the capacitor and compare your result with that obtained using $U_E = \frac{1}{2}C(\Delta V)^2$.

8. A capacitor is made of three sets of parallel plates of area A, with the two outer plates on the left and the right connected together by a conducting wire as shown in Fig. 5. The outer plates are separated by a distance d. The distance from the middle plate to the left plate is z. The distance from the inner plate to the right plate is $d - z$. You may assume all three plates are very thin compared to the distances d and z . Neglect edge effects. *(i)* The positive terminal of a battery is connected to the outer plates. The negative terminal is connected to the middle plate. The potential difference between the outer plates and inner plate is $\Delta V = V(z = 0) - V(z)$. Find the capacitance of this system. *(ii)* Find the total energy stored in this system.

9. Two flat, square metal plates have sides of length L, and thickness $s/2$, are arranged parallel to each other with a separation of s, where $s \ll L$ so you may ignore fringing fields. A charge Q is moved from the upper plate to the lower plate. Now a force is applied to a third uncharged conducting plate of the same thickness $s/2$ so that it lies between the other two plates to a depth x, maintaining the same spacing $s/4$ between its surface and the surfaces of the other two. The configuration is shown in Fig. 6. *(i)* What is the capacitance of this system? *(ii)* How much energy is stored in the electric field? *(iii)* If the middle plate is released, it starts to move. Will it move to the right or left? [*Hint*: If the middle plate moves to the left by a small positive amount Δx, the change in potential energy is approximately $\Delta U = (dU/dx)\Delta x$. Will the stored potential energy increase or decrease? *(iv)*) For a small displacement Δx in the direction you determined in part *(iii)*, find the horizontal force exerted by the charge distribution on the outer plates acting on the charges on the middle plate that cause it to move.

10. A flat conducting sheet A is suspended by an insulating thread between the surfaces formed by the bent conducting sheet B as shown in Fig. 7. The sheets are oppositely charged, the difference in potential, in volts, is ΔV. This causes a force F, in addition to the weight of A, pulling A downward. *(i)* What is the capacitance of this arrangement of conductors as a function of y, the distance that plate A is inserted between the sides of plate B ? *(ii)* How much energy is needed to increase the inserted distance by Δy? *(iii)* Find an expression for the difference in potential ΔV in terms of F and relevant dimensions shown in the figure.

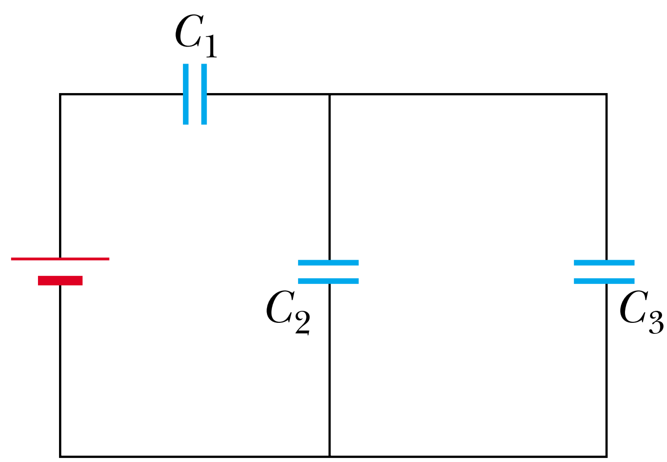

Figure 1: Problem 1.

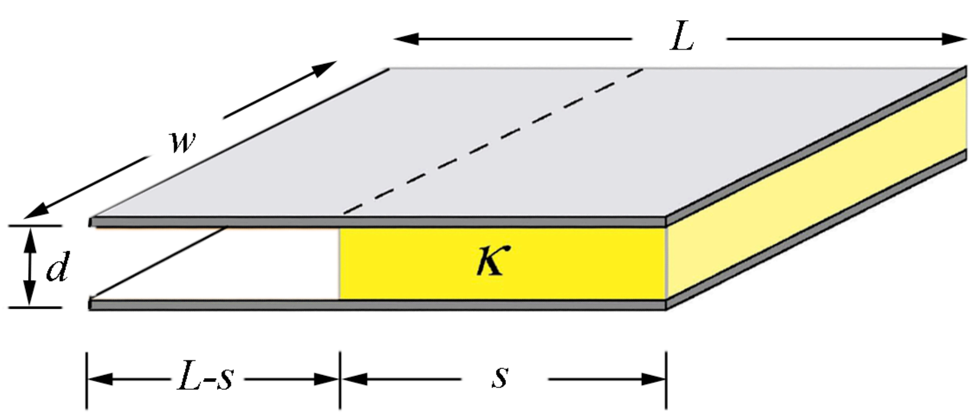

Figure 2: Problem 2.

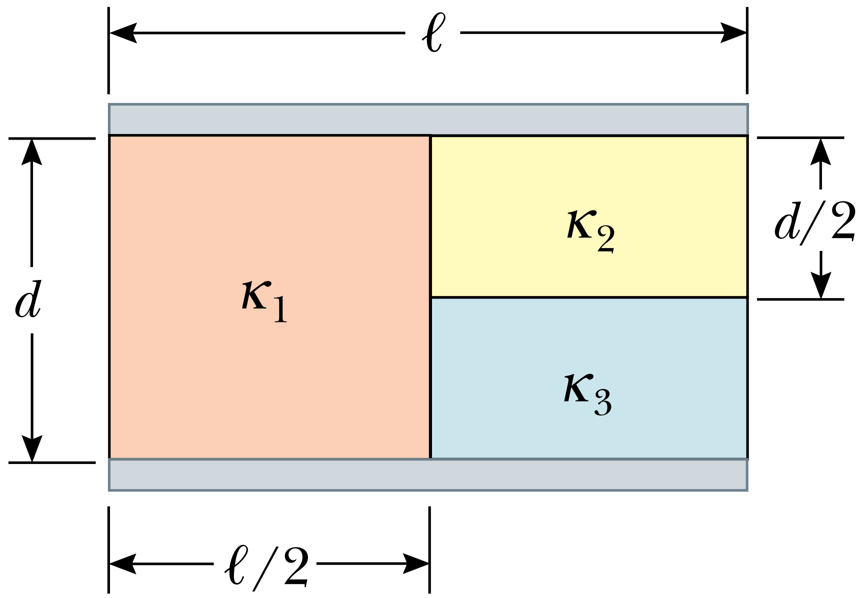

Figure 3: Problem 3.

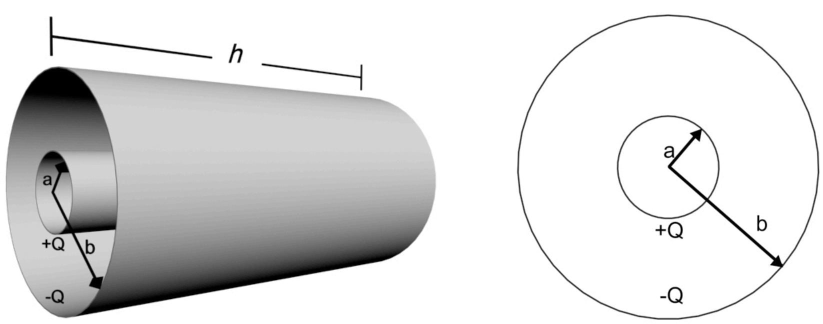

Figure 4: Problem 7.

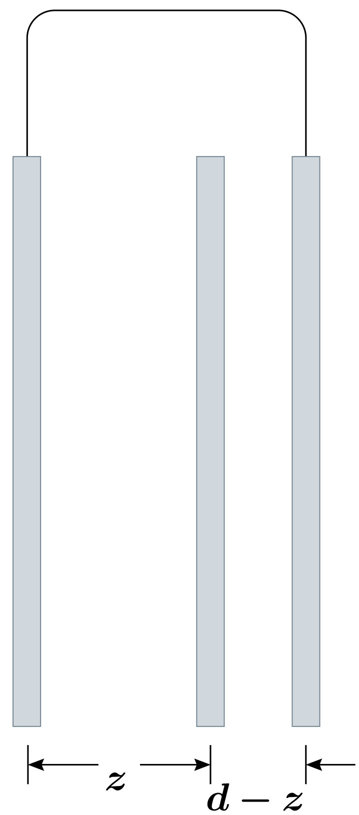

Figure 5: Problem 8.

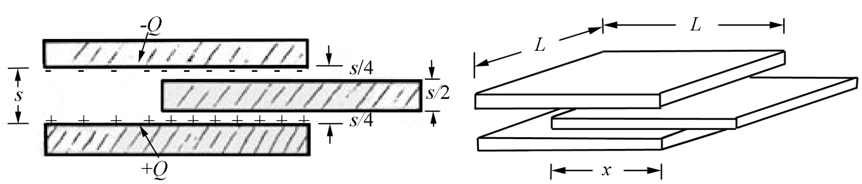

Figure 6: Problem 9.

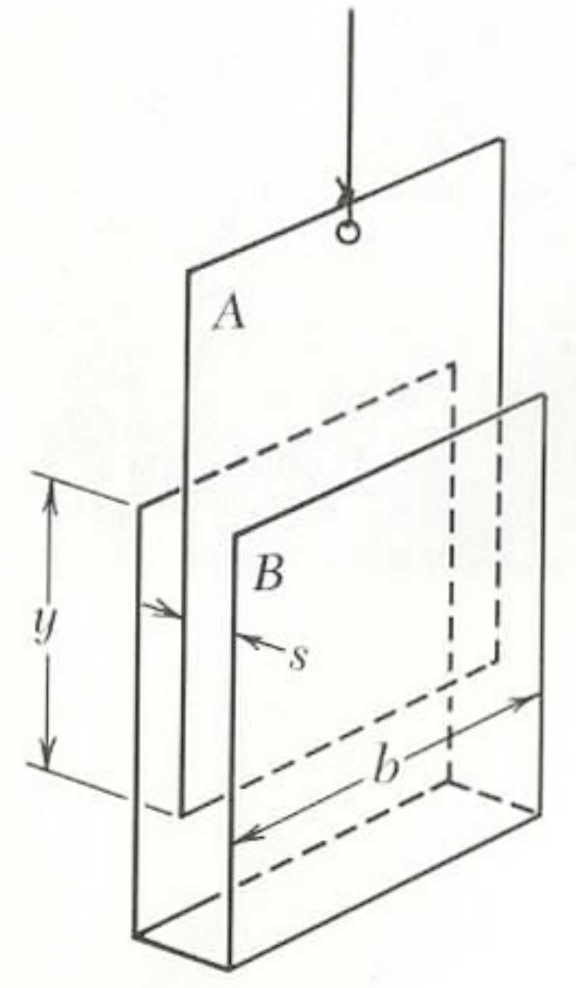

Figure 7: Problem 10.