Prof. Anchordoqui

Problems set # 3 **Physics 169** February 24, 2015

1. A point charge q is located at the center of a uniform ring having linear charge density λ and radius a, as shown in Fig. 1. Determine the total electric flux through a sphere centered at the point charge and having radius R, where $R < a$.

2. A point charge Q is located just above the center of the flat face of a hemisphere of radius R as shown in Fig. 2. What is the electric flux *(i)* through the curved surface and *(ii)* through the flat face?

3. The line ag in Fig. 3 is a diagonal of a cube. A point charge q is located on the extension of line ag, very close to vertex a of the cube. Determine the electric flux through each of the sides of the cube which meet at the point a.

4. A sphere of radius R surrounds a point charge Q , located at its center. *(i)* Show that the electric flux through a circular cap of half-angle (see Fig. 4) is $\Phi_E = \frac{Q}{2\epsilon_0}(1 - \cos\theta)$. What is the flux for *(ii)* $\theta = 90°$ and *(iii)* $\theta = 180°$.

5. An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$. *(i)* Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. *(ii)* Find an expression for the electric flux for $r > a$. *(iii)* Plot the flux versus r.

6. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Fig. 5. *(i)* Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface. *(ii)* What is the direction of the electric field at $r > c$? *(iii)* Find the electric field at $r \geq c$. *(iv)* Find the electric field in the region with radius r where $b < r < c$ *(v)* Construct a spherical gaussian surface of radius r, where $b < r < c$, and find the net charge enclosed by this surface. *(vi)* Construct a spherical gaussian surface of radius r, where $a < r < b$, and find the net charge enclosed by this surface. *(vii)* Find the electric field in the region $a < r < b$. *(viii)* Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r. Note that the charge inside this surface is less than $3Q$. *(ix)* Find the electric field in the region $r < a$. *(x)* Determine the charge on the inner surface of the conducting shell. *(xi)* Determine the charge on the outer surface of the conducting shell. *(xii)* Make a plot of the magnitude of the electric field versus r.

7.Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

8. A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Fig. 6. *(i)* Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. *(ii)* Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

9. An early (incorrect) model of the hydrogen atom, suggested by J. J. Thomson, proposed that a positive cloud of charge e was uniformly distributed throughout the volume of a sphere of radius R, with the electron an equal-magnitude negative point charge e at the center. *(i)* Using Gauss' law, show that the electron would be in equilibrium at the center and, if displaced from the center a distance $r < R$, would experience a restoring force of the form $F = -kr$, where k is a constant. *(ii)* Show that $k = \frac{e^2}{4\pi\epsilon_0 R^3}$. *(iii)* Find an expression for the frequency f of simple harmonic oscillations that an electron of mass m_e would undergo if displaced a small distance ($< R$) from the center and released. *(iv)* Calculate a numerical value for R that would result in a frequency of 2.47×10^{15} Hz, the frequency of the light radiated in the most intense line in the hydrogen spectrum.

10. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ. A line of uniform linear charge density λ is placed along the axis of the shell. Determine the electric field everywhere.

11. A particle of mass m and charge q moves at high speed along the x axis. It is initially near $x = -\infty$, and it ends up near $x = +\infty$. A second charge Q is fixed at the point $x = 0$, $y = -d$. As the moving charge passes the stationary charge, its x component of velocity does not change appreciably, but it acquires a small velocity in the y direction. Determine the angle through which the moving charge is deflected. [*Hint:* The integral you encounter in determining v_y can be evaluated by applying Gauss' law to a long cylinder of radius d, centered on the stationary charge.]

12. Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Fig. 7. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points *(i)* to the left of, *(ii)* in between, and *(iii)* to the right of the two sheets. *(iv)* Repeat the calculations when both sheets have positive uniform surface charge densities of value σ.

13. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius a is now removed from the sphere, as shown in Fig. 8. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \frac{\rho a}{3\epsilon_0}$. [*Hint*: The field within the cavity is the superposition of the field due to the original uncut sphere, plus the field due to a sphere the size of the cavity with a uniform negative charge density $-\rho$].

14. A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the center of the sphere. *(i)* Show that the magnitude of the electric field outside ($r > R$) the sphere is $E = \frac{AR^5}{5\epsilon_0 r^2}$. *(ii)* Show that the magnitude of the electric field inside ($r < R$) the sphere is $E = \frac{Ar^3}{5\epsilon_0}$. [*Hint*: The total charge Q on the sphere is equal to the integral of ρdV, where r extends from 0 to R; also, the charge q within a radius $r < R$ is less than Q. To evaluate the integrals, note that the volume element dV for a spherical shell of radius r and thickness dr is equal to $4r^2 dr$.]

15. A slab of insulating material (infinite in two of its three dimensions) has a uniform positive charge density ρ. An edge view of the slab is shown in Fig.9. *(i)* Show that the magnitude of the electric field a distance x from its center and inside the slab is $E = \rho x/\epsilon_0$. *(ii)* Suppose an electron of charge $-e$ and mass m_e can move freely within the slab. It is released from rest at a distance x from the center. Show that the electron exhibits simple harmonic motion with a frequency $s = \frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}$. *(iii)* A slab of insulating material has a nonuniform positive charge density $\rho = Cx^2$,

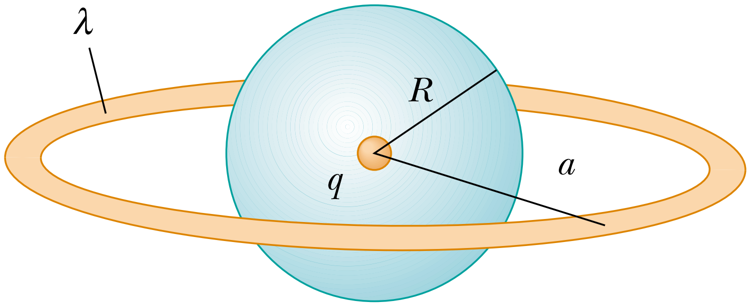

Figure 1: Problem 1.

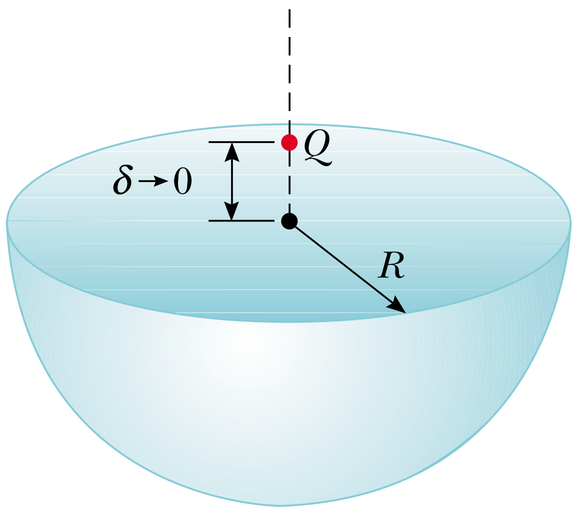

Figure 2: Problem 2.

where x is measured from the center of the slab as shown in Fig. 9, and C is a constant. The slab is infinite in the y and z directions. Derive expressions for the electric field in the exterior regions and the interior region of the slab ($-d/2 < x < d/2$).

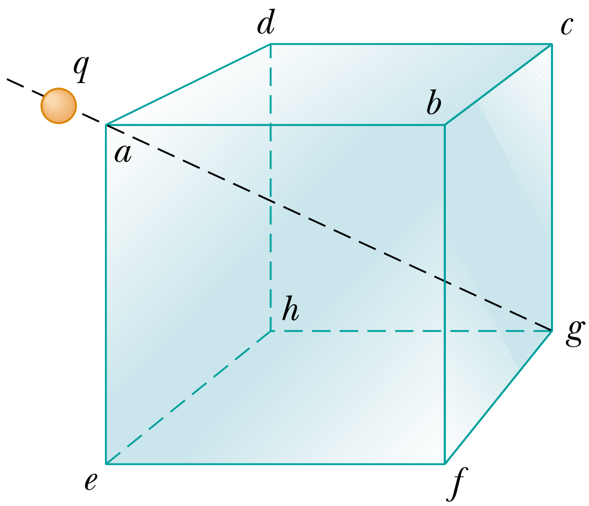

Figure 3: Problem 3.

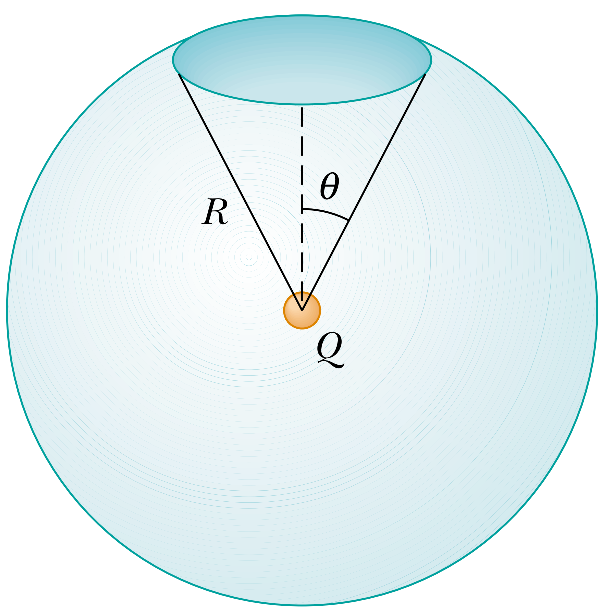

Figure 4: Problem 4.

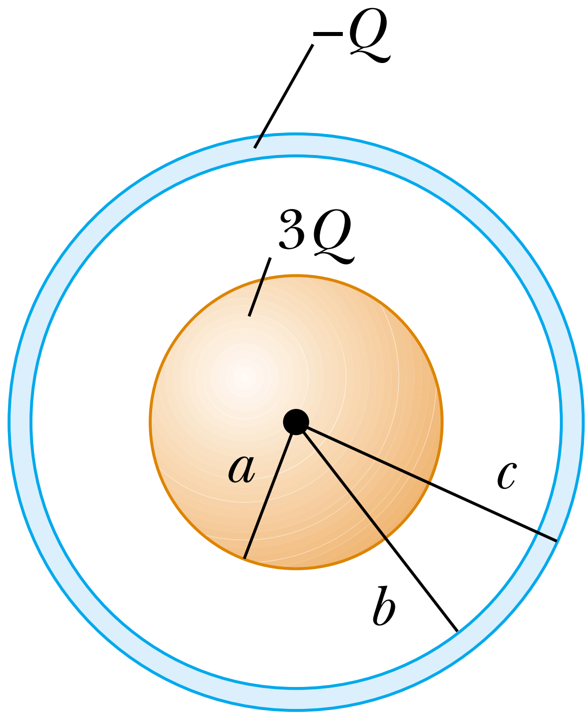

Figure 5: Problem 6.

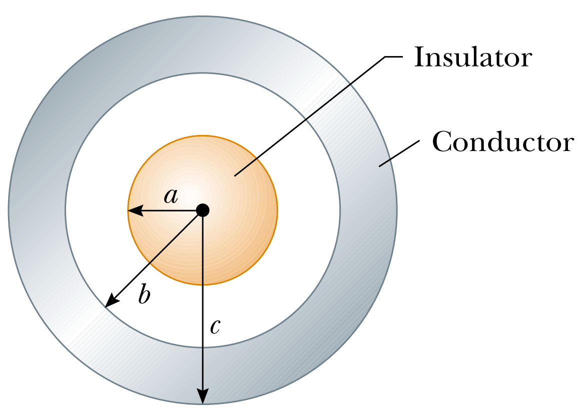

Figure 6: Problem 8.

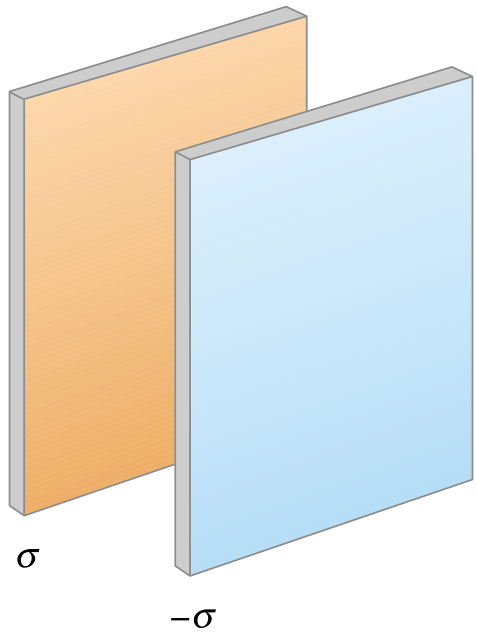

Figure 7: Problem 12.

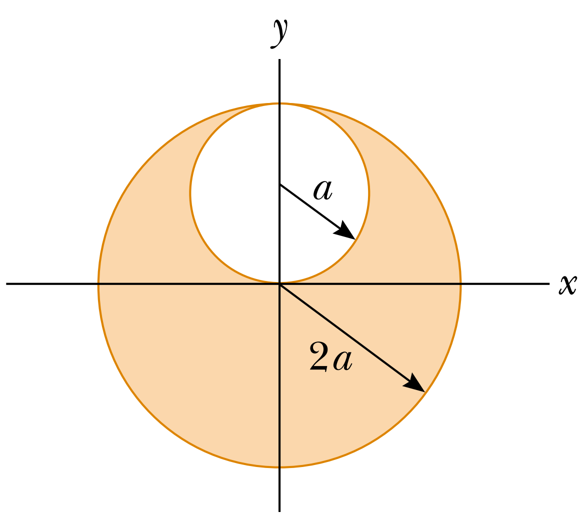

Figure 8: Problem 13.

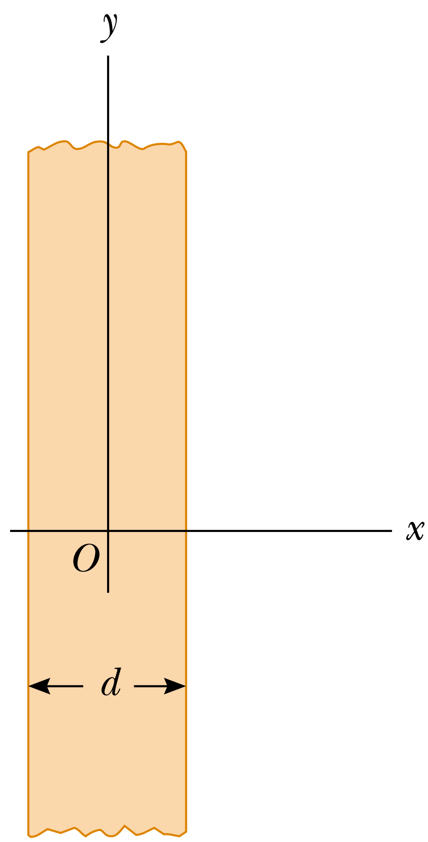

Figure 9: Problem 15.