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Problems set # 13 **Physics 169** May 12, 2015

1. Consider a ray of light traveling in vacuum from point P_1 to P_2 by way of the point Q on a plane miror as shown in Fig. 1. Show that Fermat's principle implies that, on the actual path followed, Q lies in the same vertical plane as P_1 and P_2 and obeys the law of reflection, that is $\theta_1 = \theta_2$. [Hints: Let the miror lie in the x-z plane, and let P_1 lie on the y axis at $(0, y_1, 0)$ and P_2 in the x-y plane at $(x_2, y_2, 0)$. Finally, let $Q = (x, 0, z)$. Calculate the time for the light to traverse the path P_1QP_2 and show that it is minimum when Q has $z = 0$ and satisfies the law of reflection.]

2. Nowadays dispersing prisms come in a great variety of sizes and shapes. Typically, a ray entering a dispersing prism will emerge having been deflected from its original direction by an angle δ, known as the angular deviation. Show that the minimum angle of deviation, $\delta_{\rm min}$, for a prism (with apex angle $\Phi$and index of refraction n) occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces, as shown in Fig. 2.

3 An interesting effect called total internal reflection can occur when light is directed from a medium having a given index of refraction toward one having a lower index of refraction. Consider a light beam traveling in medium 1 and meeting the boundary between medium 1 and medium 2, where n_1 is greater than n_2. Various possible directions of the beam are indicated by rays 1 through 5 in Fig. 3. The refracted rays are bent away from the normal because n_1 is greater than n_2. At some particular angle of incidence θ_c, called the critical angle, the refracted light ray moves parallel to the boundary so that $\theta_2 = \pi/2$. For angles of incidence greater than θ_c the beam is entirely reflected at the boundary. Consider a triangular glass prism with apex angle Φ and index of refraction n. What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

4 The index of refraction for violet light in silica flint glass is 1.66, and that for red light is 1.62; see Fig. 4. What is the angular dispersion of visible light passing through a prism of apex angle 60.0° if the angle of incidence is 50.0°?

5. A spherical mirror has a focal length of 10.0 cm. *(i)* Locate and describe the image for an object distance of 25.0 cm. *(ii)* Locate and describe the image for an object distance of 10.0 cm. *(iii)* Locate and describe the image for an object distance of 5.00 cm.

6. An automobile rearview mirror shows an image of a truck located 10.0 m from the mirror. The focal length of the mirror is -0.60 m. *(i)* Find the position of the image of the truck. *(ii)* Find the magnification of the image.

7. A small fish is swimming at a depth d below the surface of a pond. *(i)* What is the apparent depth of the fish as viewed from directly overhead? *(ii)* If your face is a distance d above the water

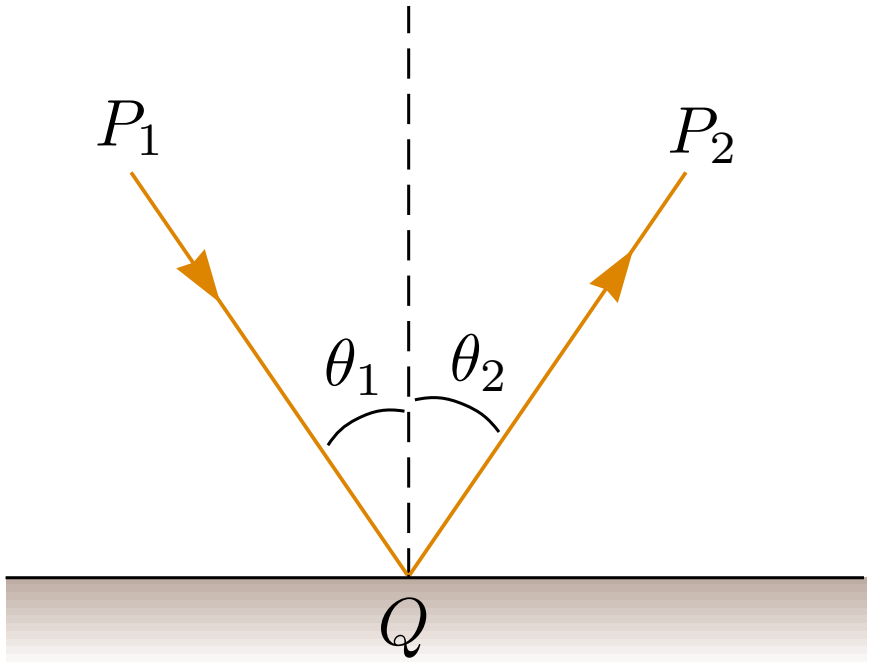

Figure 1: Reflection law. The incident ray, the reflected ray, and the normal all lie in the same plane; $\theta_1 = \theta_2$.

surface, at what apparent distance above the surface does the fish see your face?

8. A converging lens has a focal length of 10.0 cm. *(i)* An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(ii)* An object is placed 10.0 cm from the lens. Find the image distance and describe the image. *(iii)* An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

9. A diverging lens has a focal length of 10.0 cm. *(i)* An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(ii) (ii)* An object is placed 10.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(iii)* An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

10. Two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm. An object is placed 30.0 cm to the left of lens 1. Find the position and the magnification of the final image.

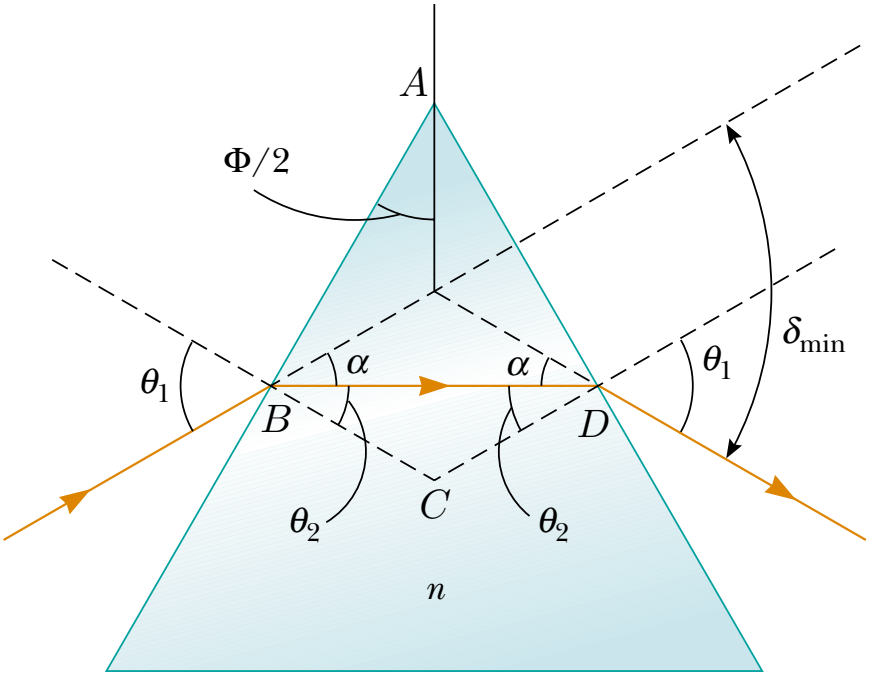

Figure 2: Geometry of a dispersing prism with a light ray passing through the prism at the minimum angle of deviation.

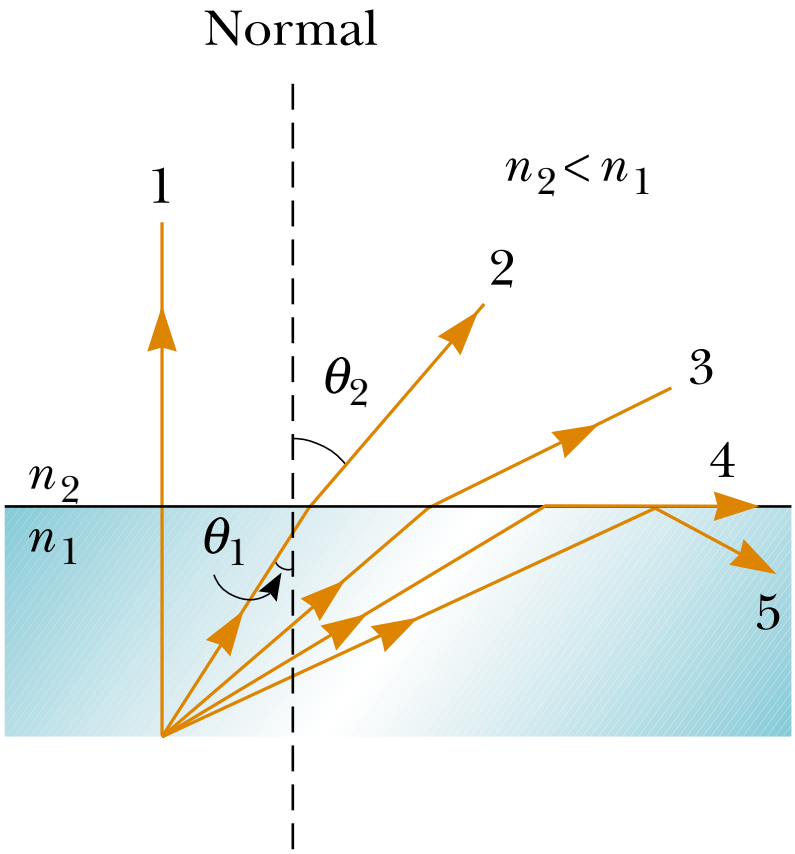

Figure 3: Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2, where $n_2 < n_1$. As the angle of incidence θ_1 increases, the angle of refraction θ_2 increases until $\theta_2 = \pi/2$ (ray 4). For even larger angles of incidence, total internal reflection occurs (ray 5).

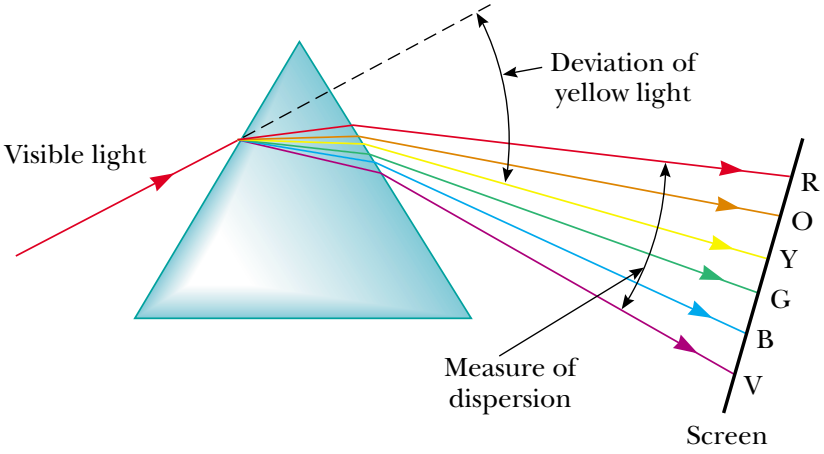

Figure 4: Newton's experiment showing that light is composed of colored components. A narrow beam of light is incident on a prism and produces a broadened and colored band which can be reconstituted back into a narrow white beam of light with a second prism.