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Problems set # 10 **Physics 169** April 28, 2015

1. A 12 V battery is connected into a series circuit containing a 10 Ω resistor and a 2 H inductor. In what time interval will the current reach *(i)* 50% and *(ii)* 90% of its final value?

2. In the circuit shown in Fig. 1, let $L = 7.00$ H, $R = 9.00$ Ω, and $\varepsilon = 120$ V. What is the self-induced emf 0.200 s after the switch is closed?

3. The switch in Fig. 2 is open for $t < 0$ and then closed at time $t = 0$. Find the current in the inductor and the current in the switch as functions of time thereafter.

4. Assume that the magnitude of the magnetic field outside a sphere of radius R is $B = B_0(R/r)^2$, where B_0 is a constant. Determine the total energy stored in the magnetic field outside the sphere and evaluate your result for $B_0 = 5.00 \times 10^{-5}$ T and $R = 6.00 \times 10^6$ m, values appropriate for the Earth's magnetic field.

5. A large coil of radius R_1 and having N_1 turns is coaxial with a small coil of radius R_2 and having N_2 turns. The centers of the coils are separated by a distance x that is much larger than R_1 and R_2. What is the mutual inductance of the coils? Suggestion: John von Neumann proved that the same answer must result from considering the flux through the first coil of the magnetic field produced by the second coil, or from considering the flux through the second coil of the magnetic field produced by the first coil. In this problem it is easy to calculate the flux through the small coil, but it is difficult to calculate the flux through the large coil, because to do so you would have to know the magnetic field away from the axis.

6. Two inductors having self-inductances L_1 and L_2 are connected in parallel as shown in Fig. 3(a). The mutual inductance between the two inductors is M. Determine the equivalent self-inductance $L_{\rm eq}$ for the system shown in Fig. 3(b).

7. A long solenoid, which has $n = 400$ turns per meter, carries a current given by $I = 30\ A(1 - e^{-1.6t})$. Inside the solenoid and coaxial with is a coil that has a radius of 6 cm and consists of a total of N = 250 turns of fine wire. *(i)* What emf is induced in the coil by the changing current? *(ii)* As indicated in Fig. 4, the current in the solenoid flows in the clockwise direction, what is the direction of the current induced in the coil, clockwise or anticlockwise? Explain how you arrived at your answer *(iii)* Consider now the situation in which an inductor is discharging, i.e., $I = 30\ Ae^{-1.6t}$, the direction of the current is still clockwise. What is the direction of the current induced in the coil now (clockwise or anticlockwise)? Explain how you arrived at your answer.

8. In the circuit of Fig. 5, the battery emf is 50.0 V, the resistance is 250 Ω, and the capacitance is 0.500 μF. The switch S is closed for a long time and no voltage is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. What is the value of the inductance?

9. An inductor consists of two very thin conducting cylindrical shells, one of radius a and one of radius b, both of length h, see Fig. 6. Assume that the inner shell carries current I out of the page, and that the outer shell carries current I into the page, distributed uniformly around the circumference in both cases. The z-axis is out of the page along the common axis of the cylinders and the r-axis is the radial cylindrical axis perpendicular to the z-axis. *(i)* Use Ampere's law to

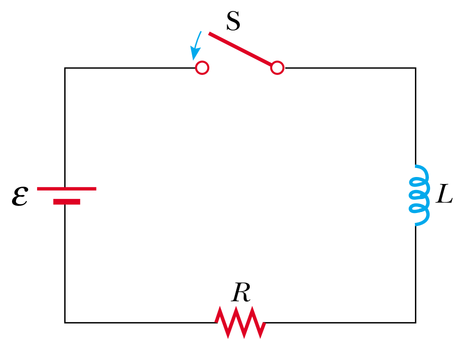

Figure 1: Problem 2.

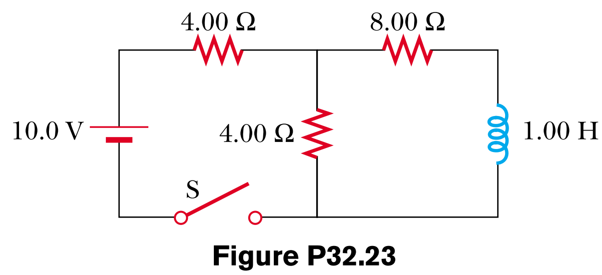

Figure 2: Problem 3.

find the magnetic field between the cylindrical shells. Indicate the direction of the magnetic field on the sketch. What is the magnetic energy density as a function of r for $a < r < b$? *(ii)* Calculate the inductance of this long inductor recalling that $\mathcal{U} = \frac{1}{2}LI^2$ and using your results for the magnetic energy density in (i). *(ii)* Calculate the inductance of this long inductor by using the formula $\Phi = LI = \int_{\substack{\text{open}\\ \text{surface}}} \vec{B} \cdot d\vec{A}$ and your results for the magnetic field in (i). To do this you must choose an appropriate open surface over which to evaluate the magnetic flux. Does your result calculated in this way agree with your result in (ii)?

10. The energy of an RLC circuit decreases by 1.00% during each oscillation when $R = 2.00\ \Omega$. If this resistance is removed, the resulting LC circuit oscillates at a frequency of 1.00 kHz. Find the values of the inductance and the capacitance.

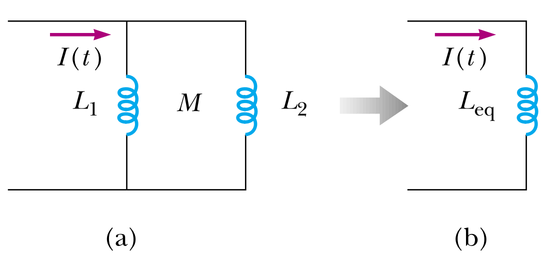

Figure 3: Problem 6.

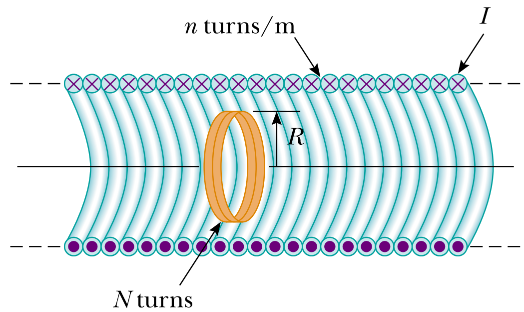

Figure 4: Problem 7.

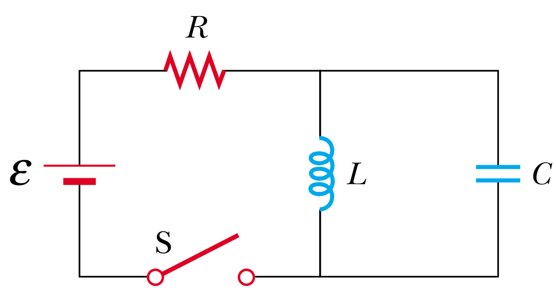

Figure 5: Problem 8.

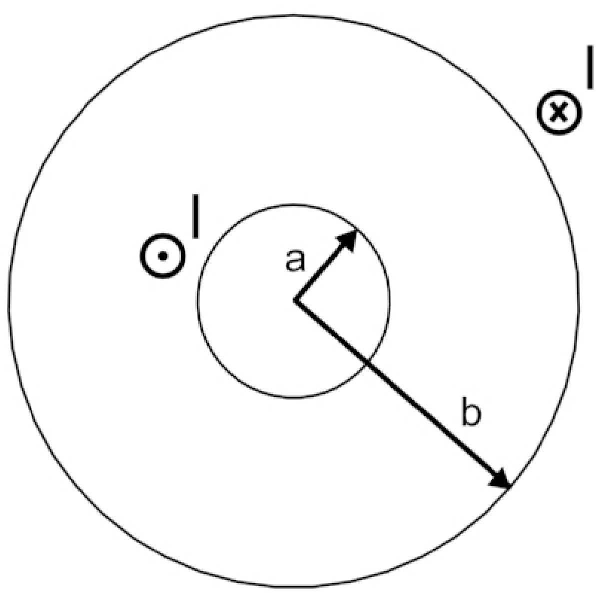

Figure 6: Problem 9.