Chapter 3

Electric Potential Energy; Electric Potential

3.1 The Important Stuff

3.1.1 Electric Potential Energy and Electric Potential

The last two chapters have dealt with *forces* and electric charge. The fundamental equation was Coulomb's law, by way of which we came to talk about the electric field as a quantity of greater utility.

In the first semester of the course, after discussing forces we discovered that *energy* (both potential and kinetic) were useful ideas, and we now discuss the role of energy in electricity.

A charge q moving through an electric field $\mathbf{E}$ experiences a force and so (in general) *work* is done on the charge. An example is shown in Fig. 3.1, where a charge q moves in a

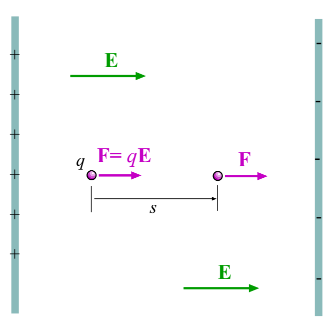

Figure 3.1: A positive charge q moves in a straight line between two large parallel plates, where the E field has a constant value; the force on the charge is also constant and equal to $q\mathbf{E}$. If q moves a distance s as shown, the work done on the charge is $W = Fs = qEs$.

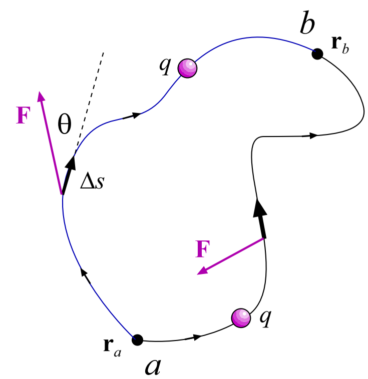

Figure 3.2: Charge q is moved from a to b by two paths. The work done the by electric force on q must be calculated by adding up $\mathbf{F}\Delta s\cos\theta$ for small steps $\Delta\mathbf{s}$ along the paths. Because of a special property of the electric force the value is the same for both paths.

uniform field $\mathbf{E}$. If it moves a distance s in the direction of the field, the work done on the charge is $W = Fs = qEs$.

Generally electric fields are not this simple; they are not uniform and so calculating the work in each case would be hard (it would require calculus!). But we are assured of a couple things:

• The work done on charge q is proportional to q.

• If the other charges producing the electric field stay in their places then the work done by the electric force as q moves from point a to point b *does not depend* on the path taken from a to b.

The second point is illustrated in Fig. 3.2. Here we pick two paths from a to b. The work done by the electric force might be a complicated thing to calculate but we get the same thing in both cases; the value depends only on the endpoints a and b.

Rather than calculate the work done by the electric forces it is easiest to think in term of a **electric potential energy** $\mathrm{PE}_{\mathrm{elec}}$ which can be evaluated at all points in space. The relation between the two is

$$W_{a\to b} = -(\mathrm{PE}_{\mathrm{elec}}(b) - \mathrm{PE}_{\mathrm{elec}}(a)) == -\Delta\mathrm{PE}_{\mathrm{elec}} \tag{3.1}$$

Of course, the electric potential energy $\mathrm{PE}_{\mathrm{elec}}$ is a scalar with units of joules.

Now we deal with the first of the two points: As charge q moves from a to b the work done (and hence the change in potential energy) is proportional to q. If we were to divide $\Delta\mathrm{PE}_{\mathrm{elec}}$ by q we would get a number which does not depend on q, just all of the *other* charges in the world and points a and b. If we call this quantity ΔV, then

$$\Delta V = \frac{\Delta\mathrm{PE}_{\mathrm{elec}}}{q} \tag{3.2}$$

The quantity V is called the **electric potential**, which should *not* be confused with *electric potential energy*. The two are related, but the electric potential V give the *potential energy per unit charge*, just as the electric field $\mathbf{E}$ gave the electric force *per units charge*.

The definition given in Eq. 3.2 only gives the *difference* in values of V, just as it is only the *differences* in the potential energy $\mathrm{PE}_{\mathrm{elec}}$ that have any real meaning.

V is a scalar and from Eq. 3.2 it must have units of J/C. Because V is such an important quantity, we give this combination of units a special name, the volt:

$$1\,\mathrm{volt} = 1\,\mathrm{V} = 1\,\tfrac{\mathrm{J}}{\mathrm{C}} \tag{3.3}$$

It will often happen that we will discuss the change in potential energy of an elementary charge (like an electron or a proton) when it moves through a potential difference of 1 volt. When this happens the change in *potantial energy* has magnitude

$$\Delta\mathrm{PE}_{\mathrm{elec}} = |q\Delta V| = (e)(1\,\mathrm{V}) = (1.602\times10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602\times10^{-19}\,\mathrm{J}$$

This gives us a convenient unit of energy for this process; since the energy here is "one electron times one volt" we define a unit of energy called the **electron volt**:

$$1\,\mathrm{electron\ volt} = 1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J} \tag{3.4}$$

For example when an electron (charge $-e$) goes through a potential gain of $+5.0\,\mathrm{V}$ it has a change in potential energy of $-5.0\,\mathrm{eV}$.

3.1.2 Calculating the Electric Potential

In certain simple situations we can calculate the electric potential

If the point P is a distance r from a point charge q, then the potential at P is given by

$$V = k\frac{q}{r} \tag{3.5}$$

Keep in mind that V is a *scalar* (a single number; no direction), and even though Eq. 3.5 looks like the formula for the *electric field* near a point charge (it should; they are related), we are calculating a it different quantity here. Note, there is only a single power of r in the denominator.

To get the electric potential for a point P which is in the vicinity of a *group* of point charges $q_1, q_2, \ldots$, just add up the electric potentials due to each charge:

$$V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + \cdots \tag{3.6}$$

where r_1 is the distance from P to q_1, r_2 is the distance from P to q_2, and so on.

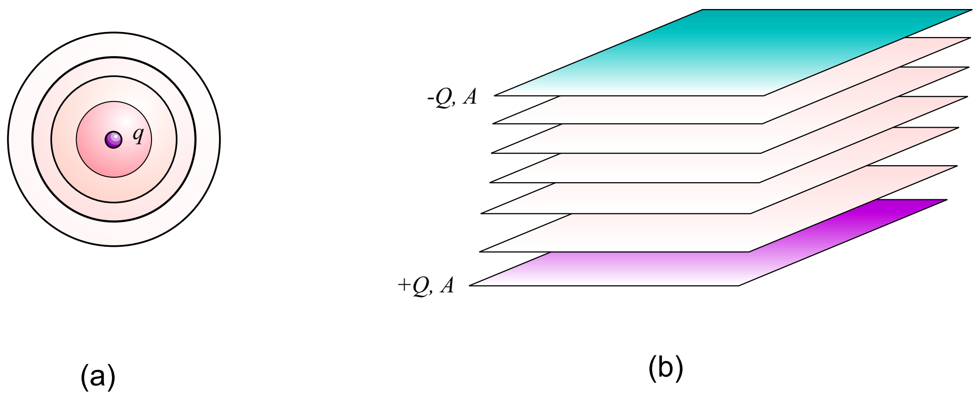

Figure 3.3: Equipotential surfaces for two simple charge configurations. (a) Around a point charge the equipotentials are spherical surfaces. (b) Between two parallel charged plates the equiotentials are parallel planes.

Between two large oppositely-charged parallel plates the electric field is uniform; if the coordinate axis perpendicular to the plates is z and $\mathbf{E}$ points in the positive z direction then the work done by the electric field as a charge q has a change in position given by Δz is

$$W = (qE)\Delta z \qquad \Longrightarrow \qquad \Delta \mathrm{PE}_{\mathrm{elec}} = -W = -qE\Delta z = \Delta(-qEz)$$

This means that we can take the electric potential between the plates as

$$V_{\mathrm{plates}} = -Ez \tag{3.7}$$

Here, the z axis points from the positive plate to the negative plate. If we are given the potential difference of the plates themselves and the distance between the plates ($\Delta z = d$) then we can find the magnitude of $\mathbf{E}$ from: $E = \Delta V/d$.

3.1.3 Equipotentials; Relation Between E and V

For any configuration of charges we can draw (or imagine) surfaces on which the potential V has the same value. Such a surface is called an **equipotential**. Simple examples are shown in Fig. 3.3; for a point charge the equipotentials are spherical surfaces surrounding the charge. For the parallel charged plates (where the potential is proportional to z between the plates) the equipotentials are parallel planes.

A less trivial example is given in Fig. 3.4, where we see the profiles of the equipotential surfaces for the electric dipole.

In general the equipotential surfaces are *perpendicular to the electric field lines.*

The relation between the electric field $\mathbf{E}$ and the potential V can be expressed using the equipotentials and field lines, as illustrated in Fig. 3.5. If we consider a small displacement

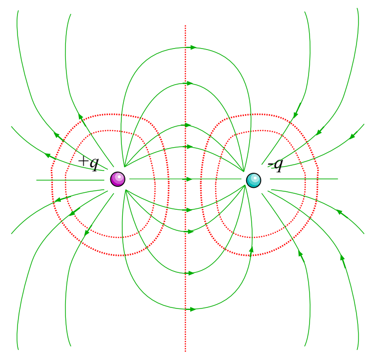

Figure 3.4: Equipotential surfaces for the electric dipole.

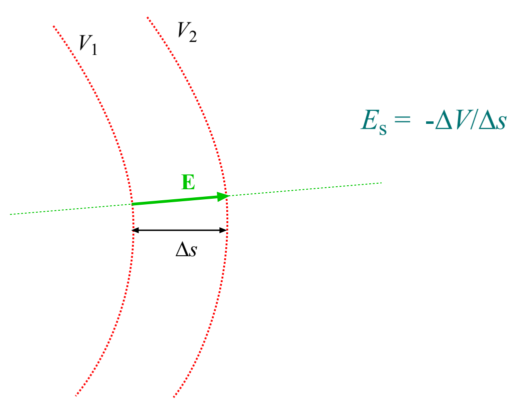

Figure 3.5: Relation between $\mathbf{E}$ and V. If we go from equipotential V_1 to equipotential V_2 along a field line (perpendicular to the equipotentials, along a coordinate s) then the component of the $\mathbf{E}$ field in this direction is $E_s = -\Delta V/\Delta s$. We are assuming that Δs is very small.

Δs along a field line, and the displacement takes you from potential V_1 to potential V_2, then the component of the $\mathbf{E}$ field in the direction of s is given by

$$E_s = -\frac{\Delta V}{\Delta s} \tag{3.8}$$

There's a minus sign here because the electric field always points from higher to lower electric potential; when ΔV is negative, E_s is positive.

When the E field is expressed as in Eq. 3.8 it is clear that the units of the E field can also be given as $\frac{\mathrm{V}}{\mathrm{m}}$ (volts per meter). This *is* exactly the same as the units we had been using, $\frac{\mathrm{N}}{\mathrm{C}}$ (newtons per coulomb).

3.1.4 Capacitance

We've seen one example of a capacitor already (the parallel plates); in general a **capacitor** is a pair of conductors on which we intend to place opposite charges $\pm q$. (To be brief we will say that the charge on a capacitor is "q".)

When the two plates of the capacitor are charged there will be a potential difference ΔV between them (with the positively charged plate at the higher potential). Again, to be brief we will just say that the potential difference of the capacitor is "V".

As one might expect, the greater the charge placed on the plates of the capacitor, the greater the potential difference. In fact, it turns out that they are always *proportional* and the constant of proportionality is called the **capacitance** C of the capacitor:

$$q = CV \tag{3.9}$$

From Eq. 3.9 (which gives $C = \frac{q}{V}$), the units of capacitance have to be C/V. We define this combination of units as the **farad**:

$$1\,\text{farad} = 1\,\mathrm{F} = 1\,\tfrac{\mathrm{C}}{\mathrm{V}} \tag{3.10}$$

A farad is actually quite a large amount of capacitance; more commonly one sees capacitors with capacitances on the order of mF or μF.

The constant ϵ_0 is often expressed in terms of this unit:

$$\epsilon_0 = 8.85 \times 10^{-12}\,\tfrac{\mathrm{C}^2}{\mathrm{N\cdot m^2}} = 8.85 \times 10^{-12}\,\tfrac{\mathrm{F}}{\mathrm{m}}$$

We can work out the capacitance of two parallel plates (which are separated by a "small" distance) using this definition and our earlier results. If the plates of the capacitor have area A and are given a charge q then the charge density on the plates (assumed uniform) is $\sigma = \frac{q}{A}$. Then the magnitude of the E field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

If the spacing between the plates is d then putting $\Delta z = d$ in Eq 3.7 gives the magnitude of the potential difference, $V = \Delta V = Ed$, or $E = V/d$. Substituting, this gives

$$\frac{V}{d} = \frac{q}{\epsilon_0 A} \qquad \Longrightarrow \qquad q = \left(\frac{\epsilon_0 A}{d}\right) V$$

Comparing this with Eq. 3.9 gives

$$C = \frac{\epsilon_0 A}{d} \tag{3.11}$$

3.1.5 Dielectrics

The formula given above for the capacitance of a parallel-plate capacitor assumes that there is nothing (except for air, which has a small effect) between the plates. If the volume between the plates is filled with an insulating material (a **dielectric**), then the plates can still store charges but the capacitance needs a correction factor from the value given in Eq. 3.11 (which we now call C_0 for clarity). In general the capacitance will be larger than the air-filled value by a factor of κ:

$$C = \kappa C_0 = \frac{\kappa \epsilon_0 A}{d} \tag{3.12}$$

The (unitless) number κ is characteristic of the substance we put between the plates. Some examples are:

$$\kappa_{\rm Teflon} = 2.1 \qquad \kappa_{\rm Mica} = 5.4 \qquad \kappa_{\rm Water} = 80.4$$

With a dielectric between the plates the electric field is still related to V by $E = V/d$ (because the electric field is still uniform) but it is related to the charges on the plates by:

$$E = \frac{\sigma}{\kappa \epsilon_0} = \frac{q}{\kappa \epsilon_0 A} \tag{3.13}$$

so if we keep the *charge* on the plates the same (as would happen if the capacitor were isolated) then the electric field *decreases* from the value it has without the dielectric in place.

3.1.6 Capacitors and Energy

In putting charge onto the plates of a capacitor one must do *work* in transferring charge from one plate to the other. Thus a capacitor stores energy; if the potential difference of a capacitor is V, the energy stored in the capacitor is given by

$$E = \tfrac{1}{2} C V^2 \tag{3.14}$$

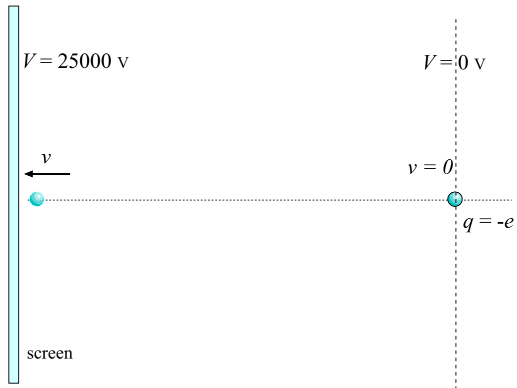

Figure 3.6: Electron in Example 1 moves through a potential difference and gains speed.

If we use Eq. 3.9, we can write this in terms of the charge q on the capacitor,

$$E = \frac{q^2}{2C} \tag{3.15}$$

3.2 Worked Examples

3.2.1 Electric Potential Energy and Electric Potential

1. In a television tube, electrons strike the screen after being accelerated through a potential difference of $25000\,\mathrm{V}$. The speeds of the electrons are quite large, and for accurate calculations of the speeds, the effects of special relativity must be taken into account. Ignoring such effects, find the electron speed just before the electron strikes the screen. [CJ6 19-5]

The problem is diagrammed in Fig. 3.6. Initially the electron is at a potential $V = 0$ and its speed is zero. Later it is at the screen where the potential is $+25000\,\mathrm{V}$ and its speed is v. In moving toward the screen the kinetic energy of the electron increases and its potential energy decreases such that the *total* energy change of the electron is zero:

$$\Delta\mathrm{PE} + \Delta\mathrm{KE} = 0$$

The change in potential energy of the electron is

$$\Delta\mathrm{PE} = q\Delta V = (-e)\Delta V$$

The change in kinetic energy of the electron is

$$\Delta\mathrm{KE} = \tfrac{1}{2}m_{\mathrm{e}}v^2 - 0 = \tfrac{1}{2}m_{\mathrm{e}}v^2$$

Put these together and get

$$(-e)\Delta V + \tfrac{1}{2}mv^2 = 0 \qquad \Longrightarrow \qquad v^2 = \frac{2e\Delta V}{m_{\mathrm{e}}}$$

Plug in the numbers:

$$v^2 = \frac{2(1.60\times10^{-19}\,\mathrm{C})(25000\,\mathrm{V})}{(9.11\times10^{-31}\,\mathrm{kg})} = 8.8\times10^{15}\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

And then:

$$v = 9.4\times10^{7}\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

(This is about one-third the speed of light so the caution about the need for relativity was appropriate!)

3.2.2 Calculating the Electric Potential

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2. Two point charges, $3.40\,\mu\mathrm{C}$ **and** $-6.10\,\mu\mathrm{C}$ **are separated by** $1.20\,\mathrm{m}$**. What is the electric potential midway between them?** [CJ6 19-12]

At the point midway between the charges the distance to *each* charge is 0.60 m. Use Eq. 3.6 to get the electric potential due to this set of point charges:

$$\begin{aligned} V &= k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + \cdots \\ &= (8.99\times10^{9}\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\left(\frac{(3.40\times10^{-6}\,\mathrm{C})}{(0.60\,\mathrm{m})} + \frac{(-6.10\times10^{-6}\,\mathrm{C})}{(0.60\,\mathrm{m})}\right) = -4.05\times10^{4}\,\mathrm{V}\end{aligned}$$

The potential at the given point is $-4.05\times10^{4}\,\mathrm{V}$.

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3. Oppositely charged plates are separated by $5.33\,\mathrm{mm}$**. A potential difference of** $600\,\mathrm{V}$ **exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned** $2.90\,\mathrm{mm}$ **from the positive plate?** [SF7 16-7]

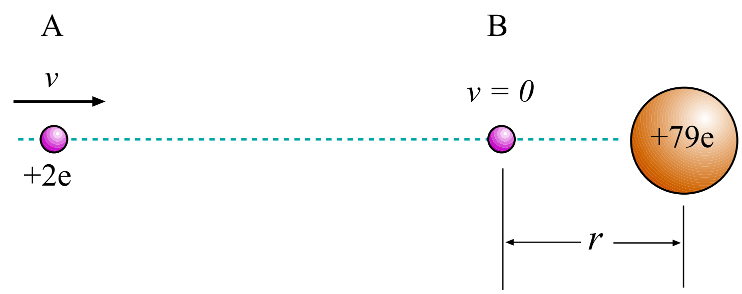

Figure 3.7: Alpha particle (with charge $+2e$) is fired at gold nucleus (with charge $+79e$) in Example 4. At position A it is *very* far away from the gold nucleus and has speed v. At position B, at a distance r from the nucleus, it has come to rest and is turning around.

(a) The electric field is uniform between the plates and we can its magnitude from Eq. 3.7 which gave

$$E = \frac{\Delta V}{d} = \frac{(600\,\mathrm{V})}{(5.33 \times 10^{-3}\,\mathrm{m})} = 1.13 \times 10^{5}\,\tfrac{\mathrm{V}}{\mathrm{m}}$$

(b) The force on the electron can be found from $\mathbf{F} = q\mathbf{E}$. We have the magnitude of the E field from (a), so the magnitude of the force on an electron is

$$F = |qE| = (1.60 \times 10^{-19}\,\mathrm{C})(1.13 \times 10^{5}\,\tfrac{\mathrm{N}}{\mathrm{C}}) = 1.80 \times 10^{-14}\,\mathrm{N}$$

(c) To push the electron back to the negative plate we must oppose the force found in part (b). Since it is 2.90 mm from the *positive* plate, we need to push the electron a distance

$$s = 5.33\,\mathrm{mm} - 2.90\,\mathrm{mm} = 2.43\,\mathrm{mm}$$

The distance Thus we push with a force of 1.80×10^{-14} N for a distance of 2.43 mm. Then the work done is

$$W = Fs = (1.80 \times 10^{-14}\,\mathrm{N})(2.90 \times 10^{-3}\,\mathrm{m}) = 4.38 \times 10^{-17}\,\mathrm{J}$$

So the work done is 4.38×10^{-14} J.

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4. In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (having charges of $+2e$ and masses of $6.64 \times 10^{-27}\,\mathrm{kg}$) were fired toward a gold nucleus with charge $+79e$. An alpha particle, initially very far from the gold nucleus, is fired at $2.00 \times 10^{7}\,\frac{\mathrm{m}}{\mathrm{s}}$, as shown in Fig. 3.7. How close does the alpha particle get to the gold nucleus before turning around? Assume the gold nucleus remains stationary. [SF7 16-19]

Because the electric force is a *conservative* force, the total energy of the alpha particle is conserved as it flies toward the gold nucleus. The total energy is the same at position A and position B in Fig. 3.7 and we can use this to figure out the distance at which the alpha particle momentarily comes to rest.

Consider the energy at position A. The alpha particle has kinetic energy $\frac{1}{2}mv^2$. Now, at *all* distances from the gold nucleus the electric potential is

$$V = k\frac{Q}{r} = k\frac{(79e)}{r}$$

which becomes very small as r becomes very large; we assume position A is so far away from the nucleus that we can ignore the electric potential of the nucleus and so the total energy at A is just the kinetic energy:

$$E_A = \tfrac{1}{2}mv^2$$

where $v = 2.00 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}}$.

At position B the speed of the alpha particle is zero, so it has no kinetic energy. But now we do have to think about the electric potential energy. At a distance r the electric potential is $k(79e)/r$ and the potential *energy* of the alpha particle is $+2e$ (its charge) times this amount, so

$$\mathrm{PE}_B = k\frac{(2e)(79e)}{r} = k\frac{(158e^2)}{r} = E_B$$

From energy conservation we can equate these two expressions for the energy. We get:

$$\tfrac{1}{2}mv^2 = k\frac{(158e^2)}{r} \qquad \Longrightarrow \qquad r = \frac{2k(158)e^2}{mv^2}$$

Plug in the numbers and get:

$$\begin{aligned} r &= \frac{2(8.99 \times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})(158)(1.60 \times 10^{-19}\ \mathrm{C})^2}{(6.64 \times 10^{-27}\ \mathrm{kg})(2.00 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}})^2} \\ &= 2.74 \times 10^{-14}\ \mathrm{m} \end{aligned}$$

3.2.3 Capacitance

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5. What voltage is required to store $7.2 \times 10^{-5}\ \mathrm{C}$ **of charge on the plates of a** $6.0\text{-}\mu\mathrm{F}$ **capacitor?** [CJ6 19-36]

Use equation Eq. 3.9, which relates charge, voltage and capacitance:

$$q = CV \qquad \Longrightarrow \qquad V = \frac{q}{C}$$

Plug in the numbers:

$$V = \frac{(7.2 \times 10^{-5}\,\mathrm{C})}{(6.0 \times 10^{-6}\,\mathrm{F})} = 12.0\,\mathrm{V}$$

We must put a potential difference of 12.0 V across the plates of the capacitor.

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6. An air–filled capacitor consists of two parallel plates, each with an area of $7.60\,\mathrm{cm}^2$ and separated by a distance of $1.80\,\mathrm{mm}$. If a 20.0-V potential difference is applied to these plates, calculate (a) the electric field between the plates, (b) the capacitance, and (c) the charge on each plate. [SF7 16-25]

(a) In between the plates the field is uniform so if we have the potential difference of the plates and their separation, Eq. 3.8 (or Eq. 3.7) gives the magnitude of the electric field in between the plates:

$$E = \frac{\Delta V}{\Delta z} = \frac{V}{d} = \frac{(20.0\,\mathrm{V})}{(1.80 \times 10^{-3}\,\mathrm{m})} = 1.11 \times 10^{4}\,\frac{\mathrm{V}}{\mathrm{m}}$$

(b) To get the capacitance of the parallel plates, use Eq. 3.11, (be careful to convert the units of the area properly...),

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})(7.60 \times 10^{-4}\,\mathrm{m}^2)}{(1.80 \times 10^{-3}\,\mathrm{m})} = 3.74 \times 10^{-12}\,\mathrm{F}$$

The answer can also be expressed as

$$C = 3.74\,\mathrm{pF}$$

(c) Having the capacitance and potential difference we can get the charge on each plate from Eq. 3.9,

$$\begin{aligned} q &= CV = (3.74 \times 10^{-12}\,\mathrm{F})(20.0\,\mathrm{V}) = 7.47 \times 10^{-11}\,\mathrm{C} \\ &= 74.7 \times 10^{-12}\,\mathrm{C} = 74.7\,\mathrm{pC} \end{aligned}$$

which means that one plate has a charge of $+74.7\,\mathrm{pC}$ and the other has a charge of $-74.7\,\mathrm{pC}$.

3.2.4 Dielectrics

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7. A parallel plate capacitor has a capacitance of $7.0\,\mu\mathrm{F}$ when filled with a dielectric. The area of each plate is $1.5\,\mathrm{m}^2$ and the separation between the plates is $1.0 \times 10^{-5}\,\mathrm{m}$. What is the dielectric constant of the dielectric? [CJ6 19-35]

Eq. 3.12 gives the capacitance of a parallel plate capacitor when it is filled with dielectric of constant κ. Solving for κ gives

$$\kappa = \frac{dC}{\epsilon_0 A}$$

Plug in the numbers:

$$\kappa = \frac{(1.0 \times 10^{-5}\,\mathrm{m})(7.0 \times 10^{-6}\,\mathrm{F})}{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})(1.5\,\mathrm{m}^2)} = 5.3$$

3.2.5 Capacitors and Energy

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8. A parallel-plate capacitor has $2.00\,\mathrm{cm}^2$ plates that are separated by $5.00\,\mathrm{mm}$ with air between them. If a 12.0-V battery is connected to this capacitor, how much energy does it store? [SF7 16-43]

Using Eq. 3.11 we find the capacitance of this capacitor (note that $2.00\,\mathrm{cm}^2$ converts to $2.00 \times 10^{-4}\,\mathrm{m}^2$):

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\,\tfrac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}) \frac{(2.00 \times 10^{-4}\,\mathrm{m}^2)}{(5.00 \times 10^{-3}\,\mathrm{m})} = 3.54 \times 10^{-13}\,\mathrm{F}$$

Using Eq 3.14 we find the energy stored in this capacitor:

$$\text{Energy} = \tfrac{1}{2} C V^2 = \tfrac{1}{2}(3.54 \times 10^{-13}\,\mathrm{F})(12.0\,\mathrm{V})^2 = 2.55 \times 10^{-11}\,\mathrm{J}$$