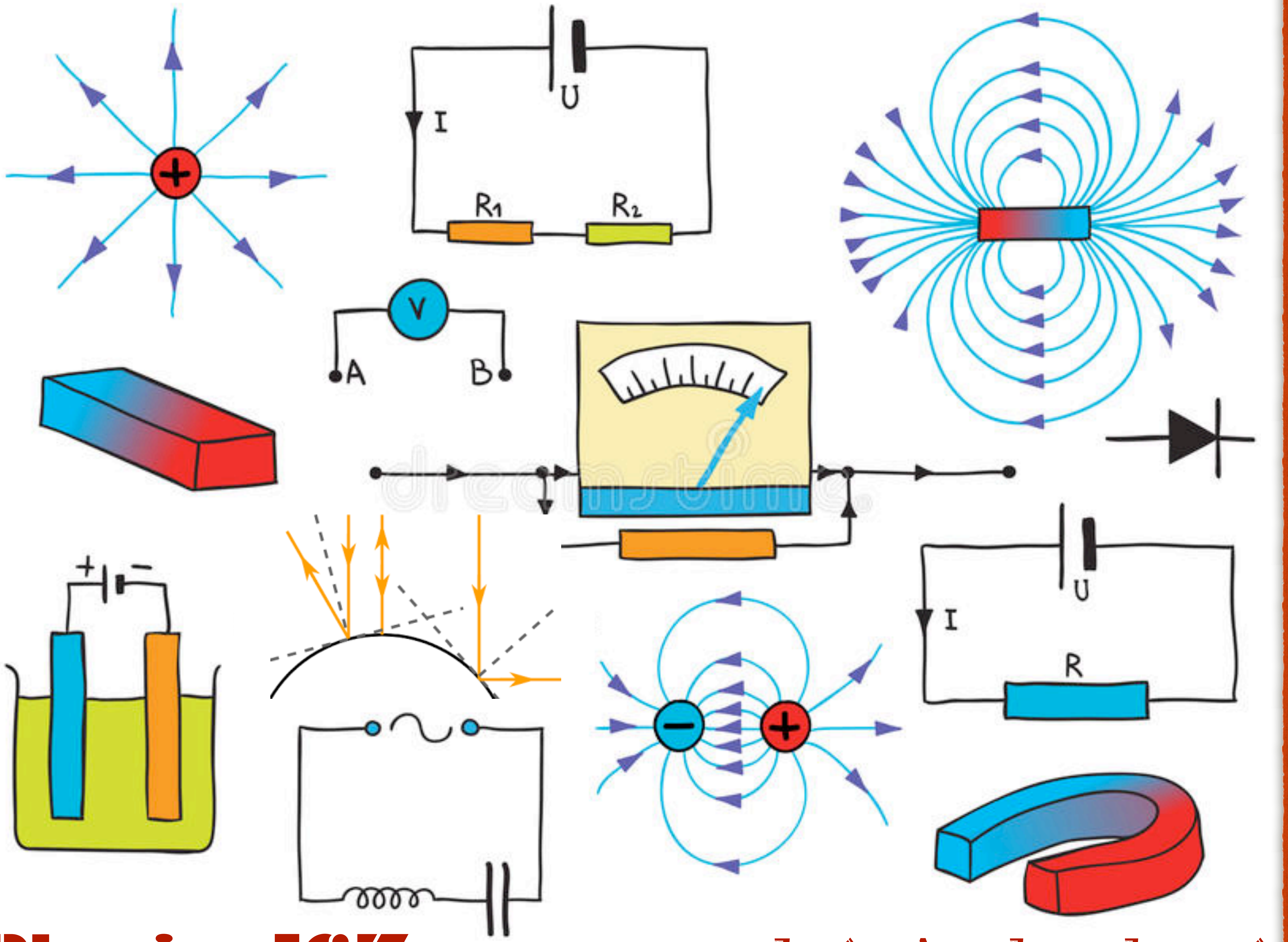
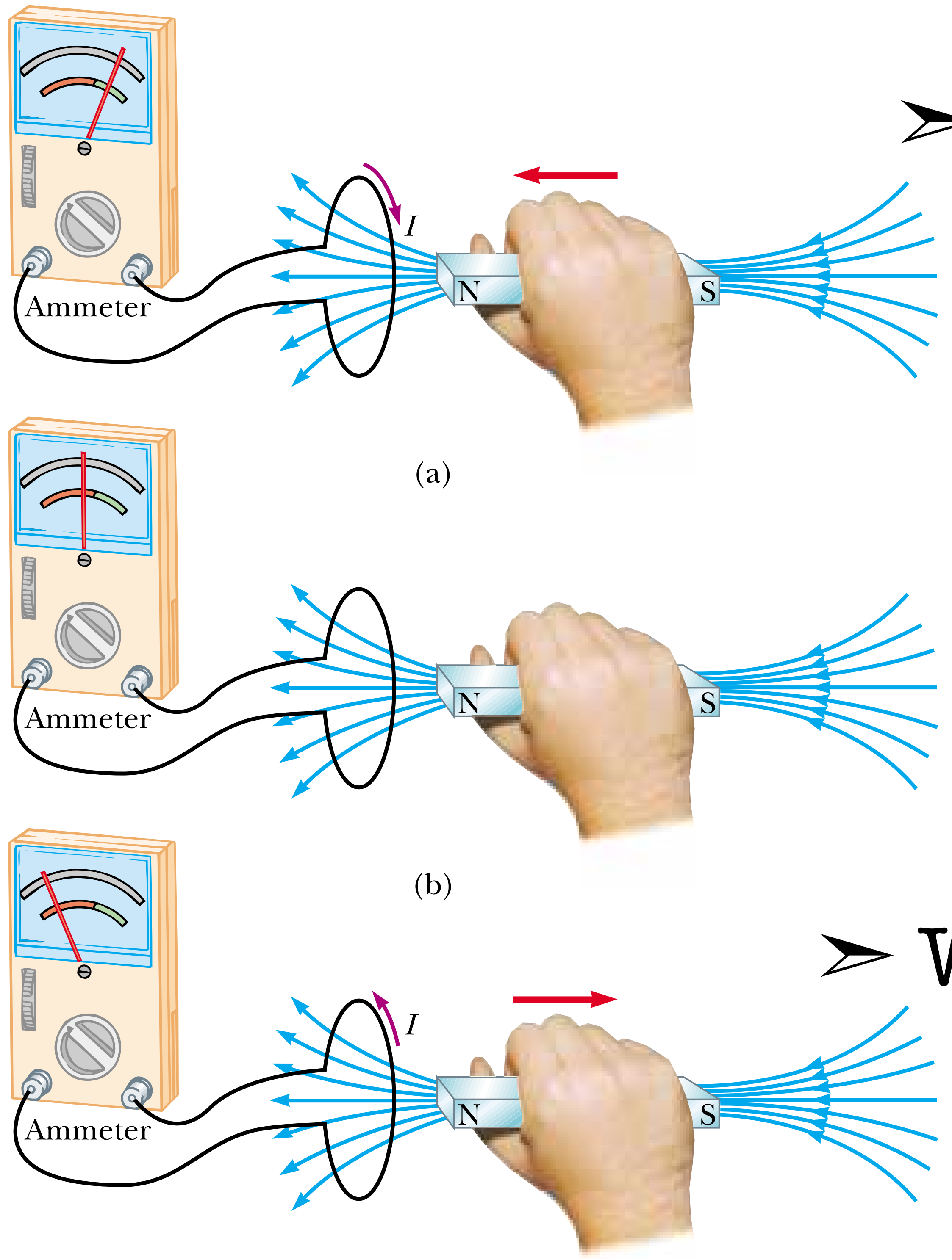


Physics 167

Luis Anchordoqui

Question ☛

- ➢ Steady electric current can give steady magnetic field
- ➢ Because of symmetry between electricity and magnetism ☛ we can ask:
- ➢ Steady magnetic field can give steady electric current
- ➢ OR Changing magnetic field can give steady electric current



- ➢ When a magnet is moved toward a loop of wire ☛ sensitive ammeter deflects indicating that current is induced in the loop
- ➢ When magnet is held stationary ☛ there is no induced current in the loop even when the magnet is inside the loop
- ➢ When magnet is moved away from loop ☛ ammeter deflects in opposite direction indicating that induced current is in opposite direction

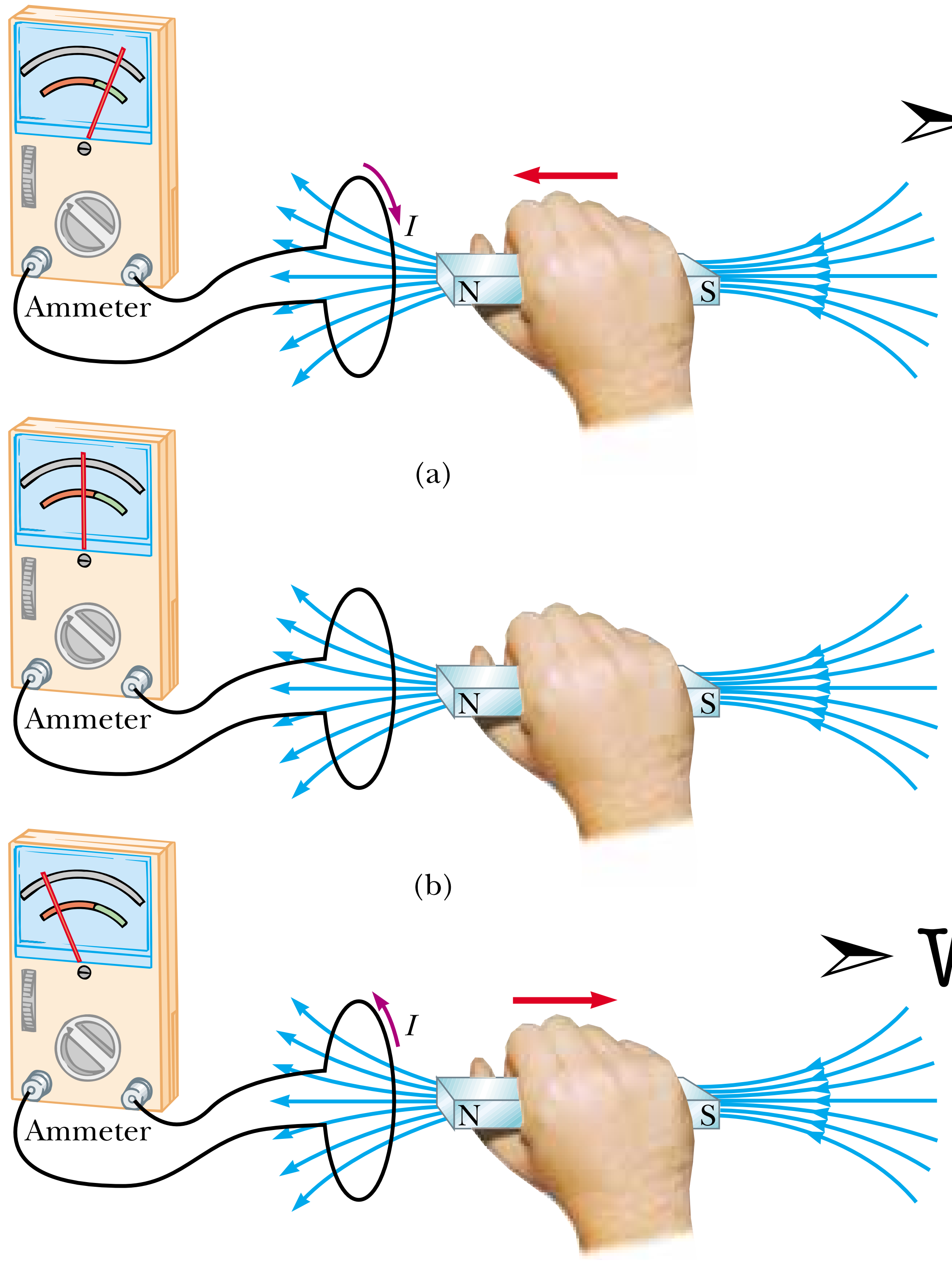
ANSWER ☛

➢ Steady electric current can give steady magnetic field

➢ Because of symmetry between electricity and magnetism ☛ we can ask:

➢ Steady magnetic field can give steady electric current ✗

➢ OR Changing magnetic field can give steady electric current ✓



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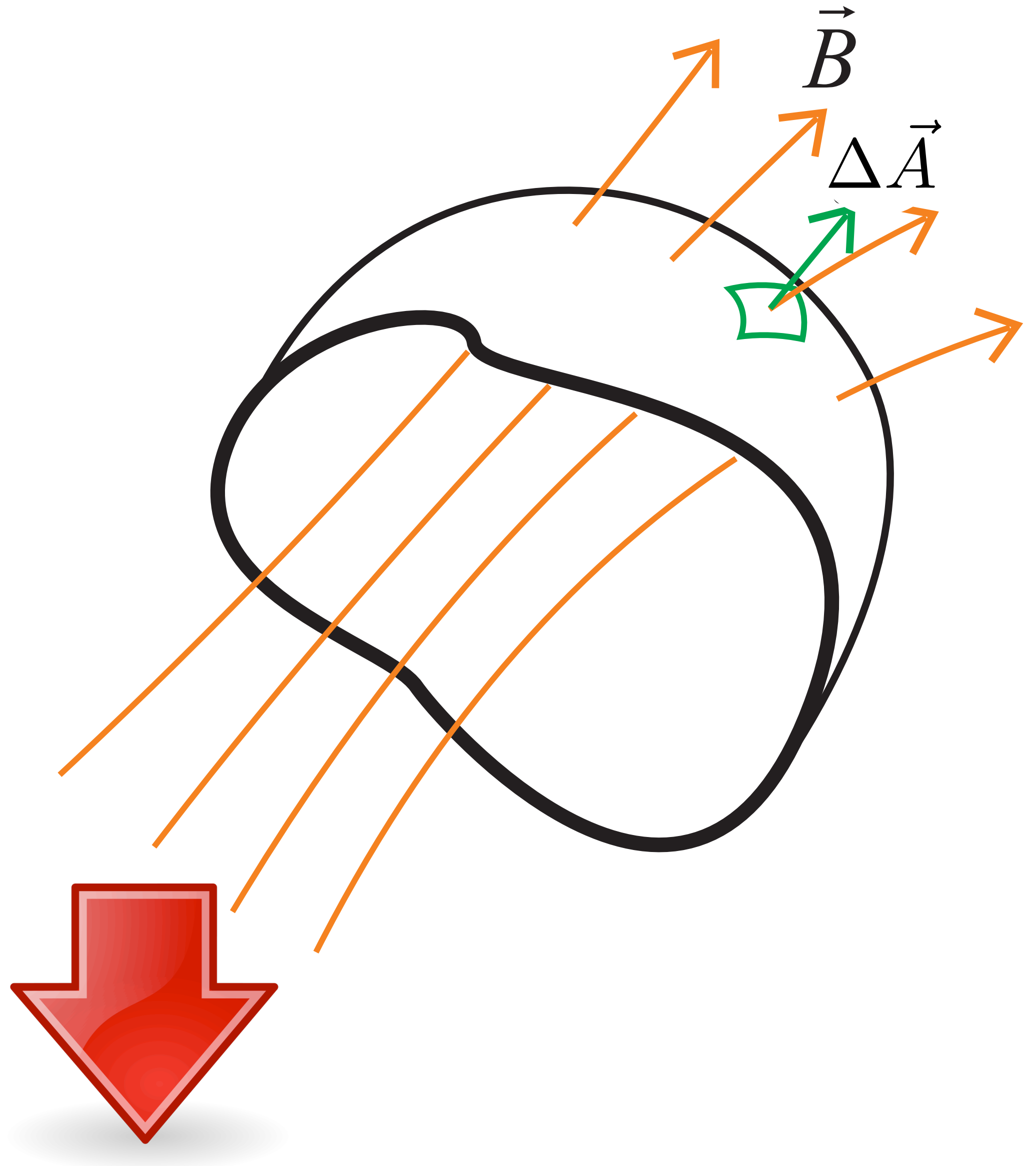
Magnetic Flux



① Magnetic flux through surface S

$$\Phi_M = \sum \vec{B} \cdot \Delta\vec{A}$$

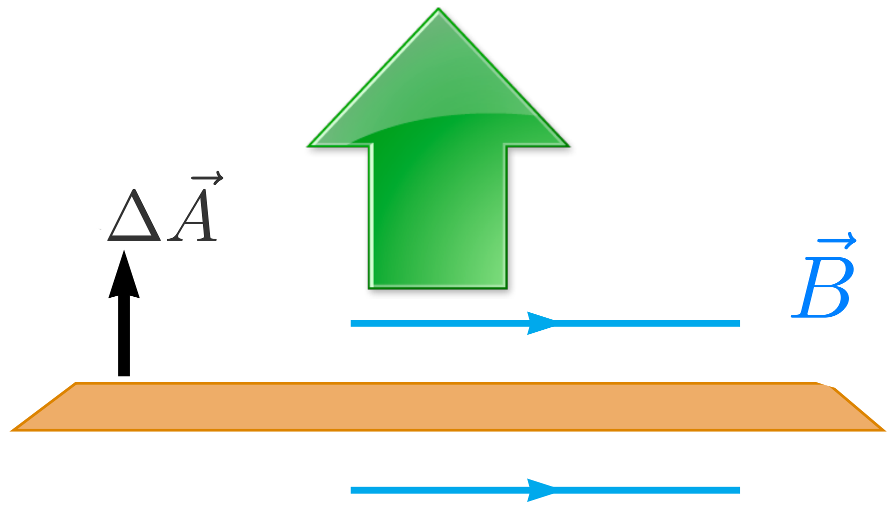
Unit of Φ_M ☛ Weber (Wb)

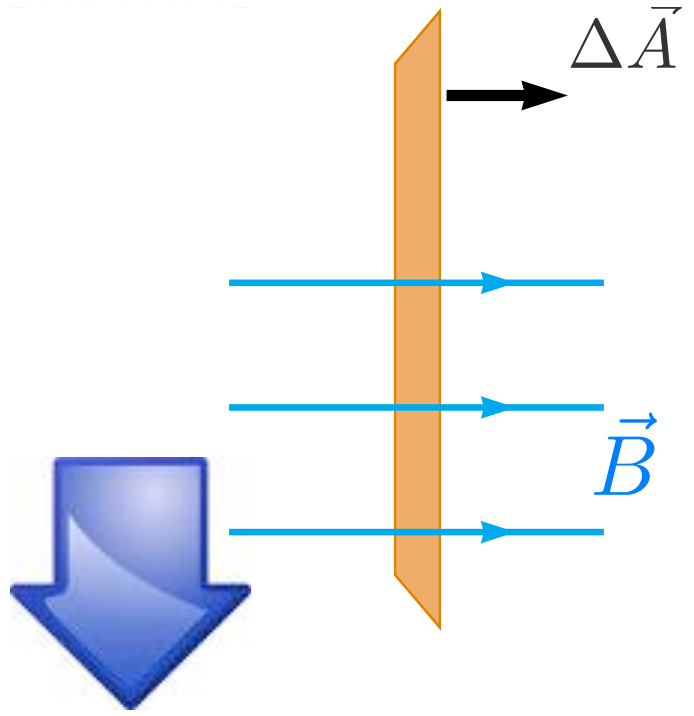
$$1\,\text{Wb} = 1\,\text{Tm}^2$$

② Graphical

Φ_M ☛ number of magnetic field lines passing through surface

Flux through plane is zero when magnetic field is parallel to plane surface





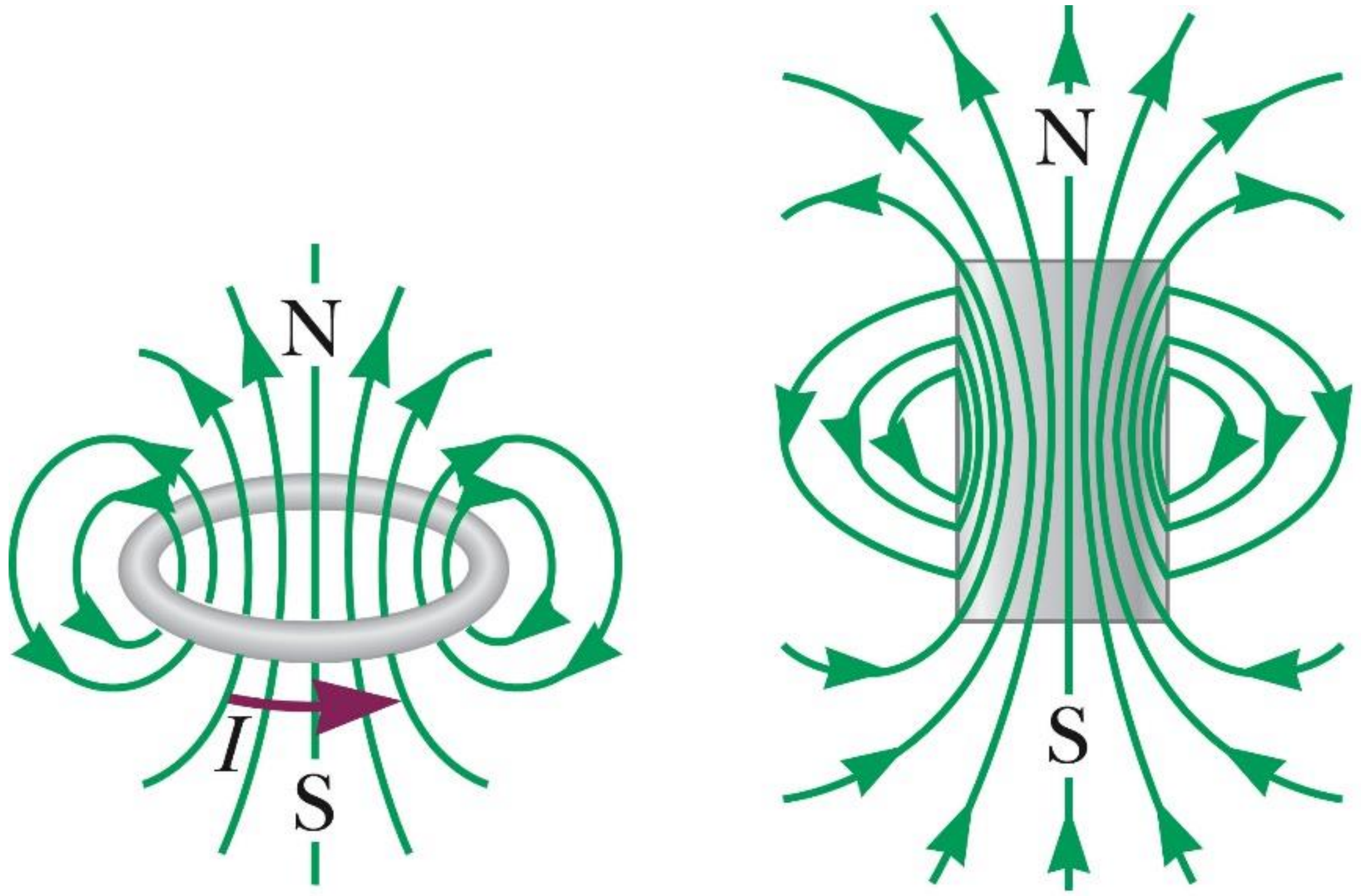
Flux through plane is maximum when magnetic field is perpendicular to plane

Gauss's Law for Magnetic Field

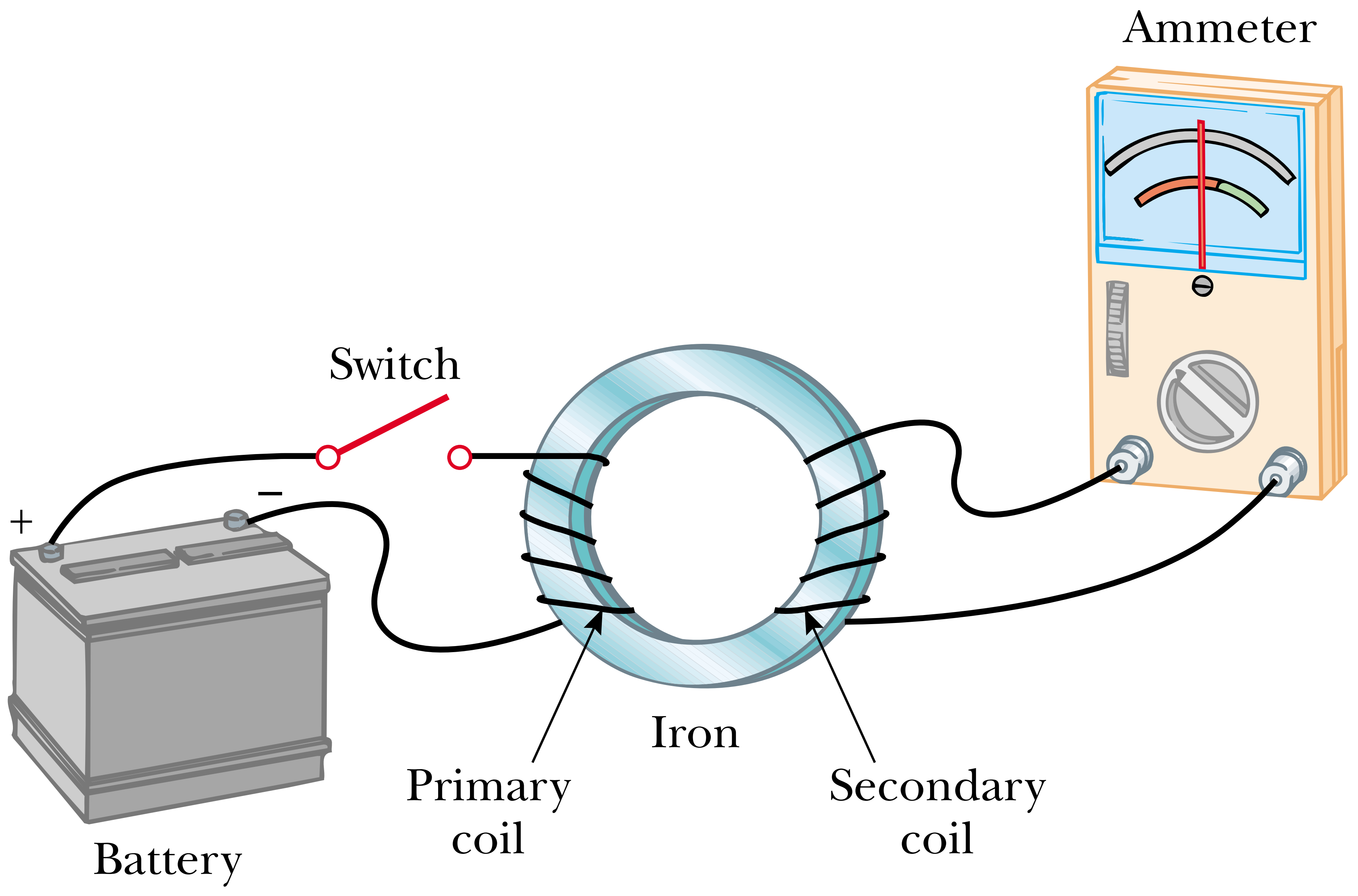
- ➢ The net magnetic flux Φ_B through any closed surface is equal to zero

$$\sum_{\substack{\text{closed}\\\text{surface}}} \vec{B} \cdot \Delta\vec{A} = 0$$

- ➢ There are no magnetic charges
- ➢ Magnetic field lines always close in themselves
- ➢ No matter how the (closed) Gaussian surface is chosen, the net magnetic flux through it always vanishes



Faraday's law



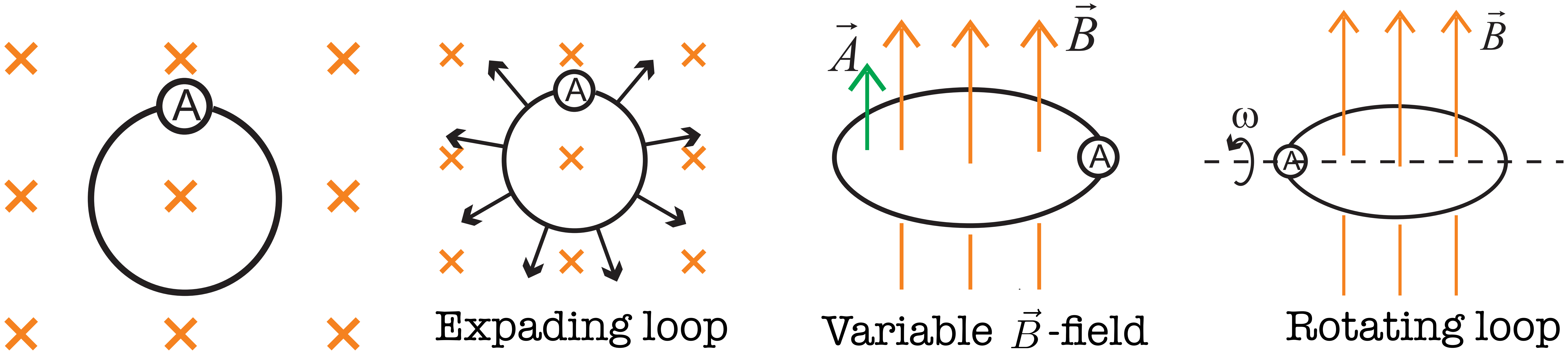
Faraday's experiment ➧

When switch in primary circuit is closed ammeter in secondary circuit deflects momentarily

emf induced in secondary circuit is caused by changing magnetic field through secondary coil

Faraday's law of induction ➧ Induced emf ☛ $|\mathcal{E}| = N \left| \frac{\Delta \Phi_M}{\Delta t} \right|$

number of coils in circuit



Loop at rest

$\vec{B} = \text{Constant}$

$\vec{A} = \text{Constant}$

$\mathcal{E} = 0$

Expading loop

$\vec{B} = \text{Constant}$

$\hat{A} = \text{Constant}$

$\Delta A/\Delta t \neq 0$

$\therefore\ |\mathcal{E}| > 0$

Variable $\vec{B}$-field

$\hat{B} = \text{Constant}$

$\Delta B/\Delta t \neq 0$

$\vec{A} = \text{Constant}$

$\therefore\ |\mathcal{E}| > 0$

Rotating loop

$\vec{B} = \text{Constant}$

$\vec{A} = \text{Constant}$

$\Delta \hat{A}/\Delta t \neq 0$

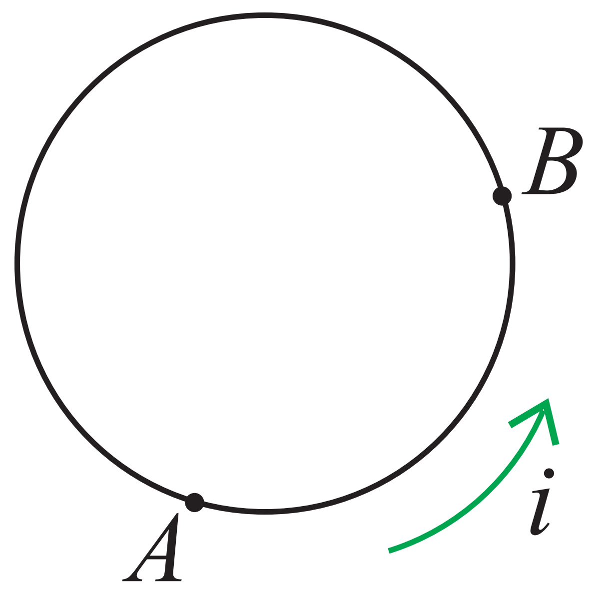
$\therefore\ |\mathcal{E}| > 0$

Note

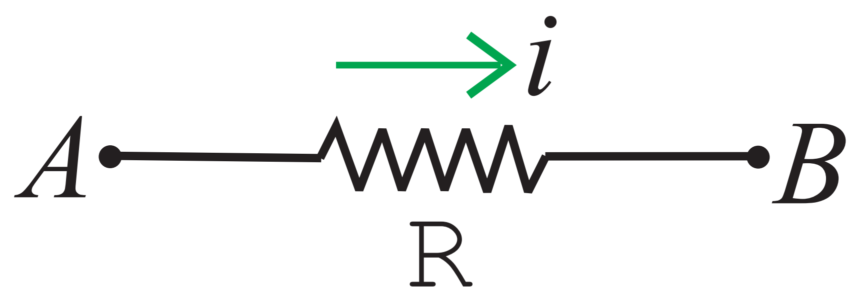
- Induced emf drives a current throughout circuit similar to function of a battery
- Difference here is that induced emf is distributed throughout circuit consequence

  we cannot define a potential difference between any two points in circuit

➢ Suppose there is an induced current in loop ☛ can we define ΔV_{AB} ?



Recall



$$\Delta V_{AB} = V_A - V_B = iR > 0$$

$$\Rightarrow V_A > V_B$$

➢ Going anti-clockwise (same as i)

➢ If we start from A going to B then we get $V_A > V_B$

➢ If we start from B going to A then we get $V_B > V_A$

$\therefore$ We cannot define ΔV_{AB} !!

This situation is like when we study **interior of a battery**

A battery A loop	provides energy needed to drive charge carriers around circuit by	chemical reactions changing magnetic flux

sources of emf **non-electric means**

Lenz's law

① Flux of magnetic field due to induced current **opposes** change in flux that causes induced current

② Induced current is in such a direction as to **oppose** changes that produces it

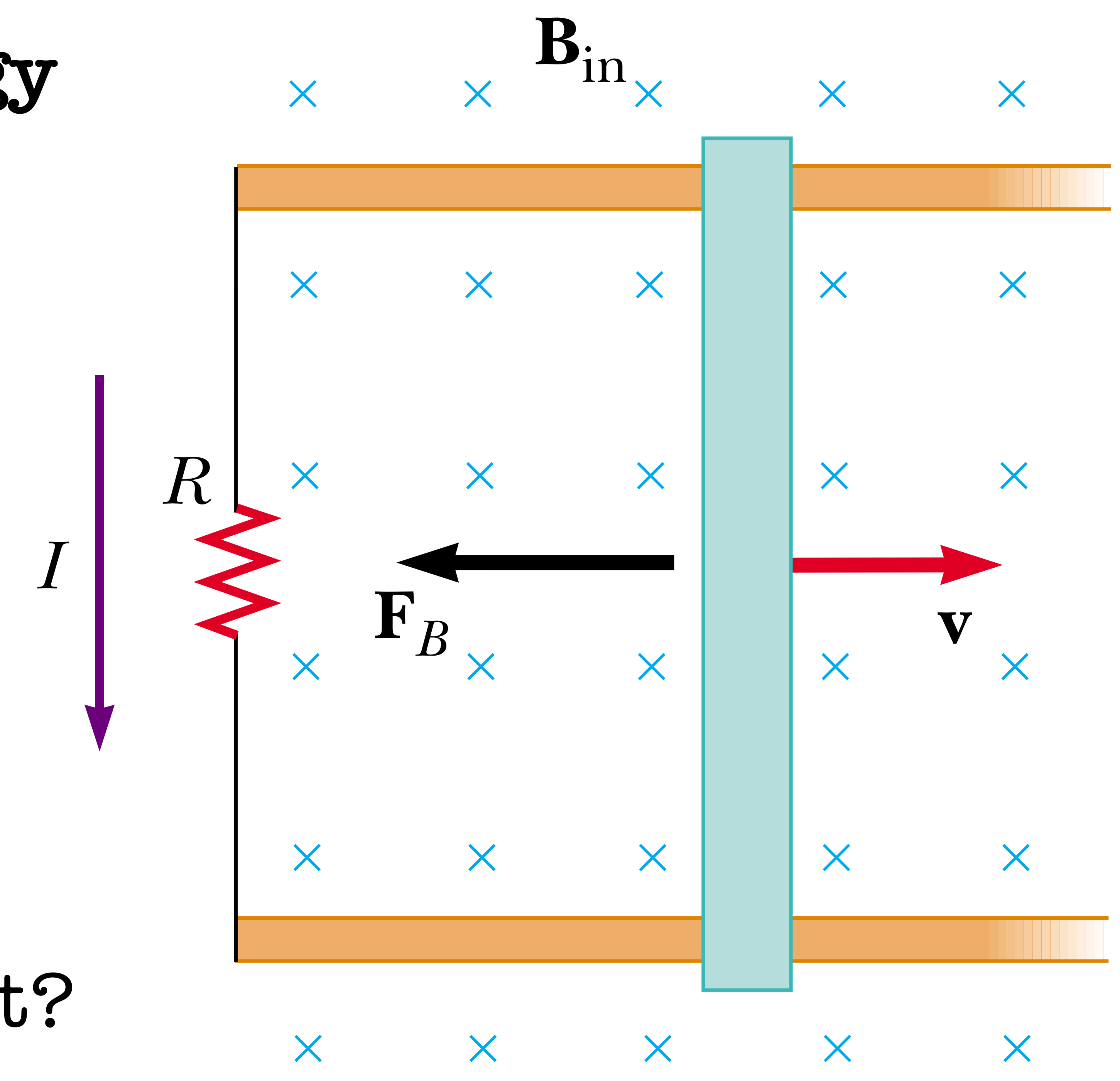
③ Incorporating Lenz's law into Faraday's Law ☛ $\mathcal{E} = -N\dfrac{\Delta\Phi_M}{\Delta t}$

If $\dfrac{\Delta\Phi_M}{\Delta t} > 0 \Rightarrow \Phi_M \uparrow$ $\mathcal{E}$ appears ⇒ Induced current appears

so that

⇒ $\vec{B}$ -field due to induced current ⇒ cahnge in Φ_M ⇒ $\Phi_M \downarrow$

④ Lenz's Law is consequence from **principle of conservation of energy**



➢ Suppose bar is given slight push to right

➢ This motion sets up a counterclockwise current in the loop

BUT ☛

➢ What happens if we assume that current is clockwise

such that direction of magnetic force exerted on bar is to the right?

➢ This force would accelerate the rod and increase its velocity

➢ This (in turn) would cause area enclosed by loop to increase more rapidly

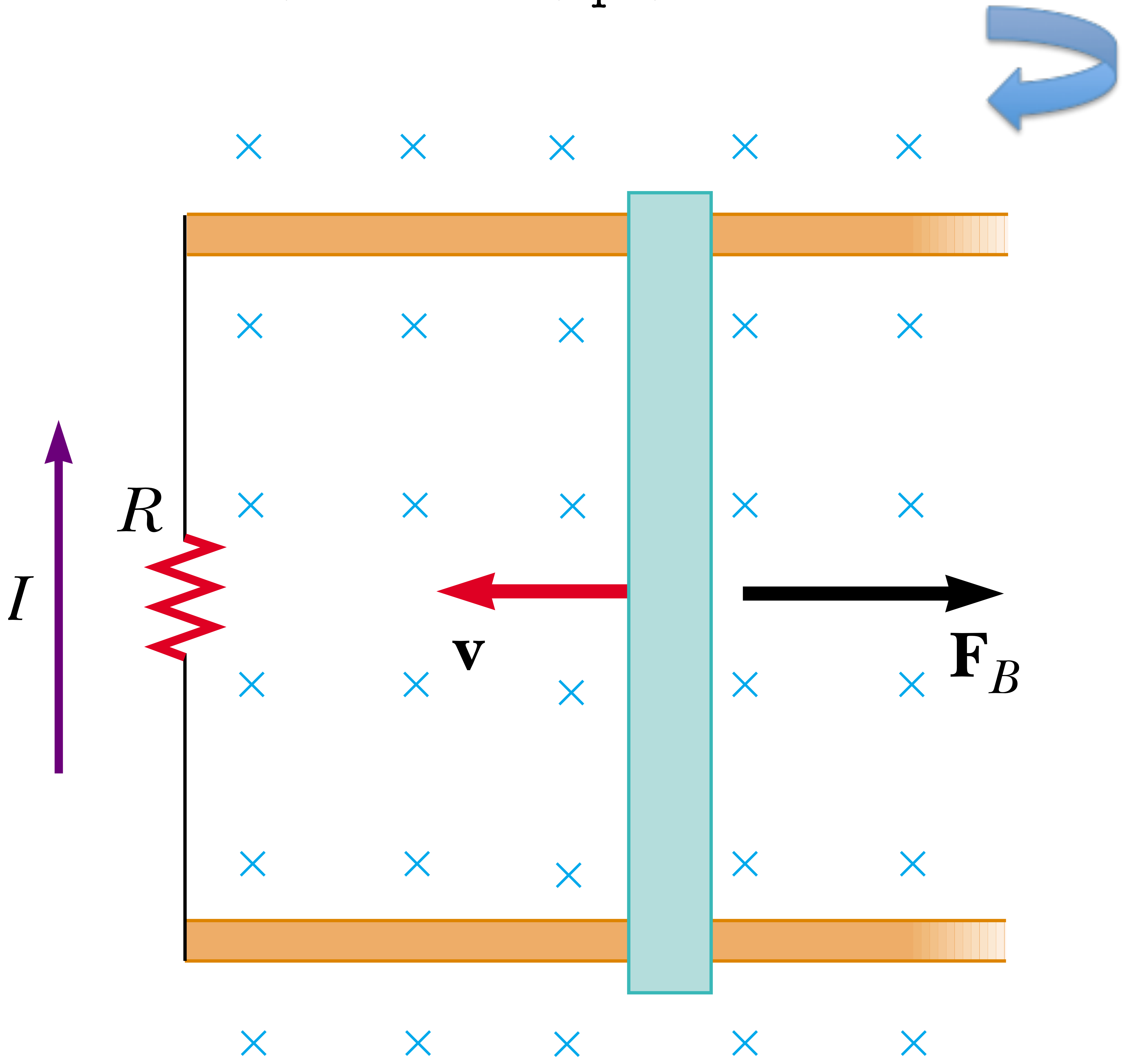
this would result in increase in induced current which would cause increase in force which would produce increase in current … and so on…

➢ System would acquire energy with no input of energy

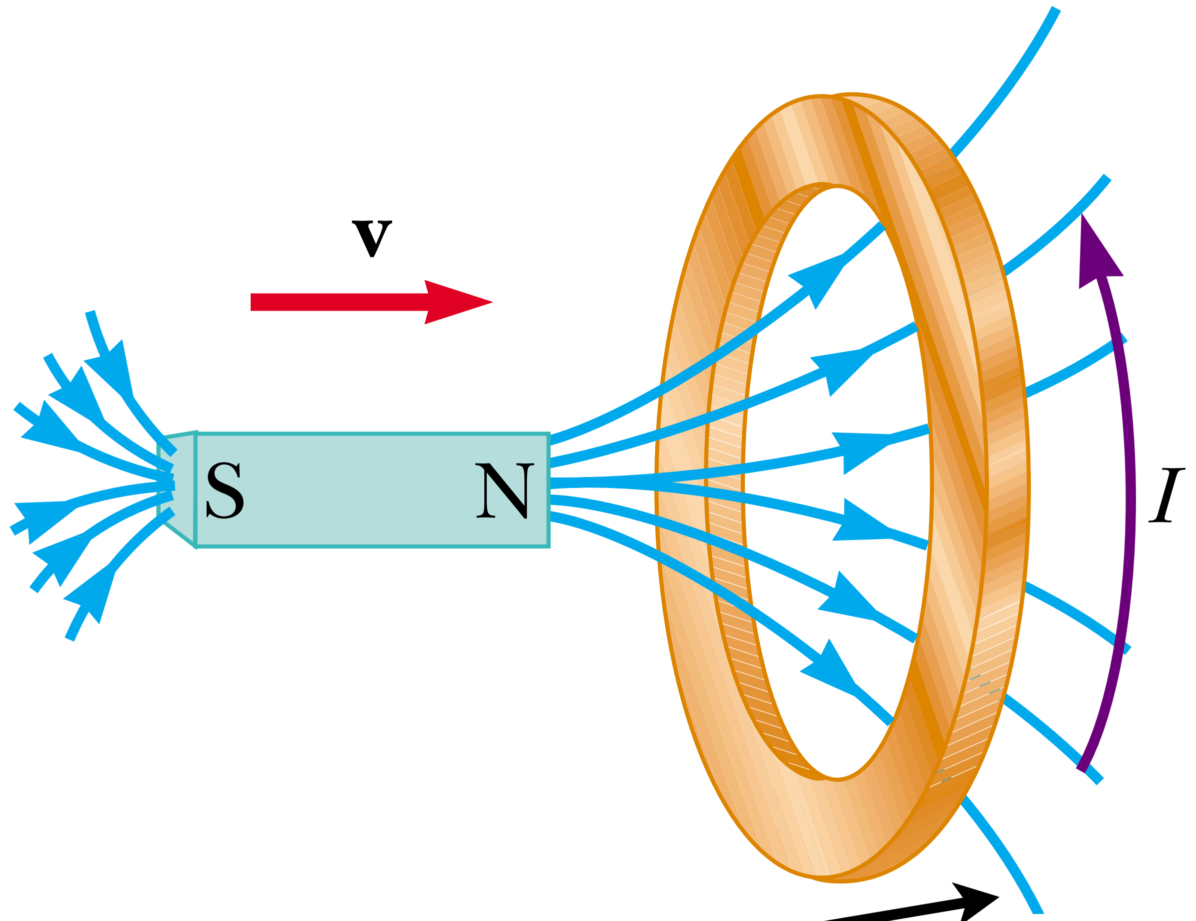
➢ This is clearly inconsistent with all experience and violates law of energy conservation

➢ We are forced to conclude that current must be counterclockwise

Likewise ☛ if bar is push to the left



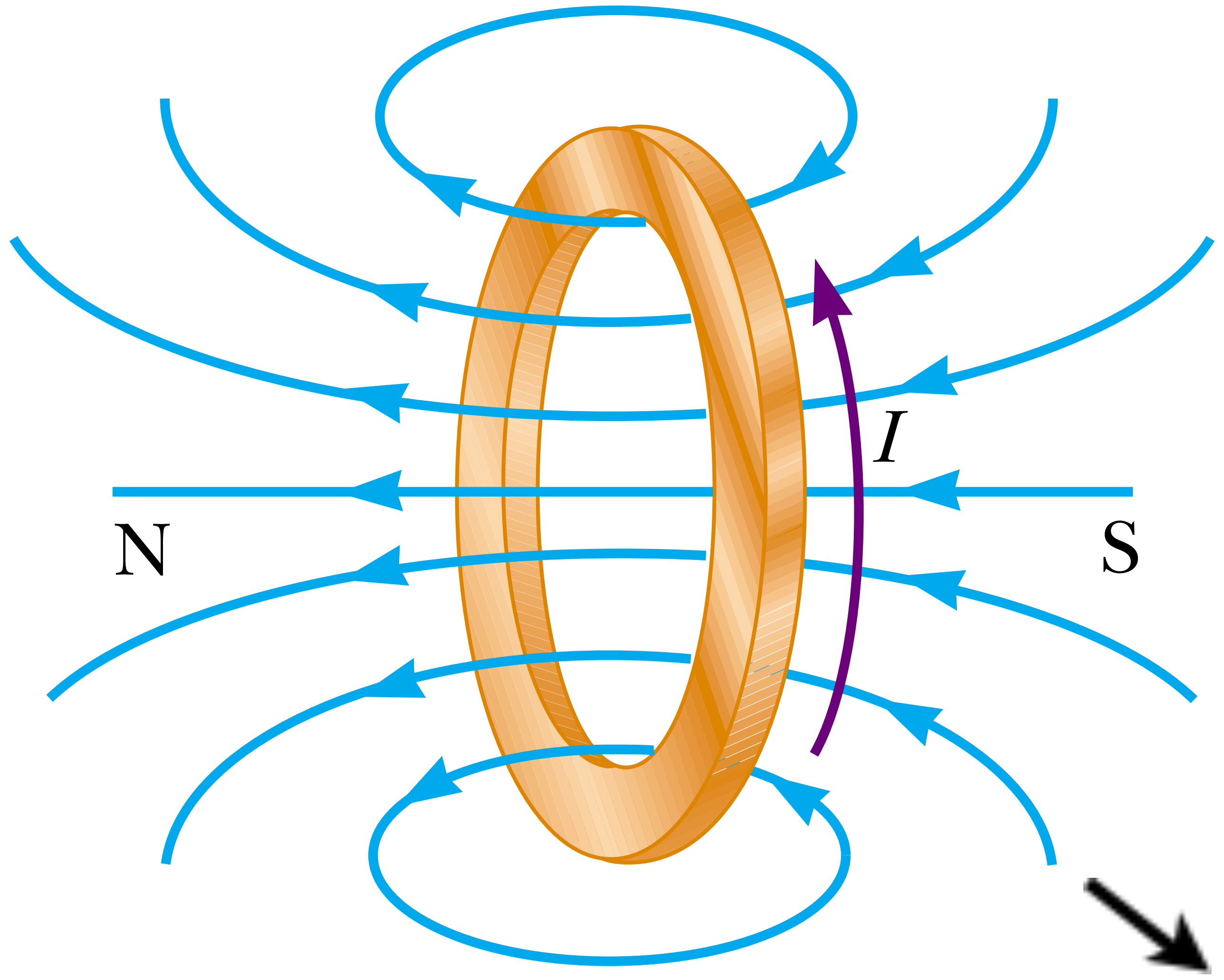
- When magnet is moved toward stationary conducting loop current is induced in the direction shown



- Magnetic field lines shown are those due to bar magnet

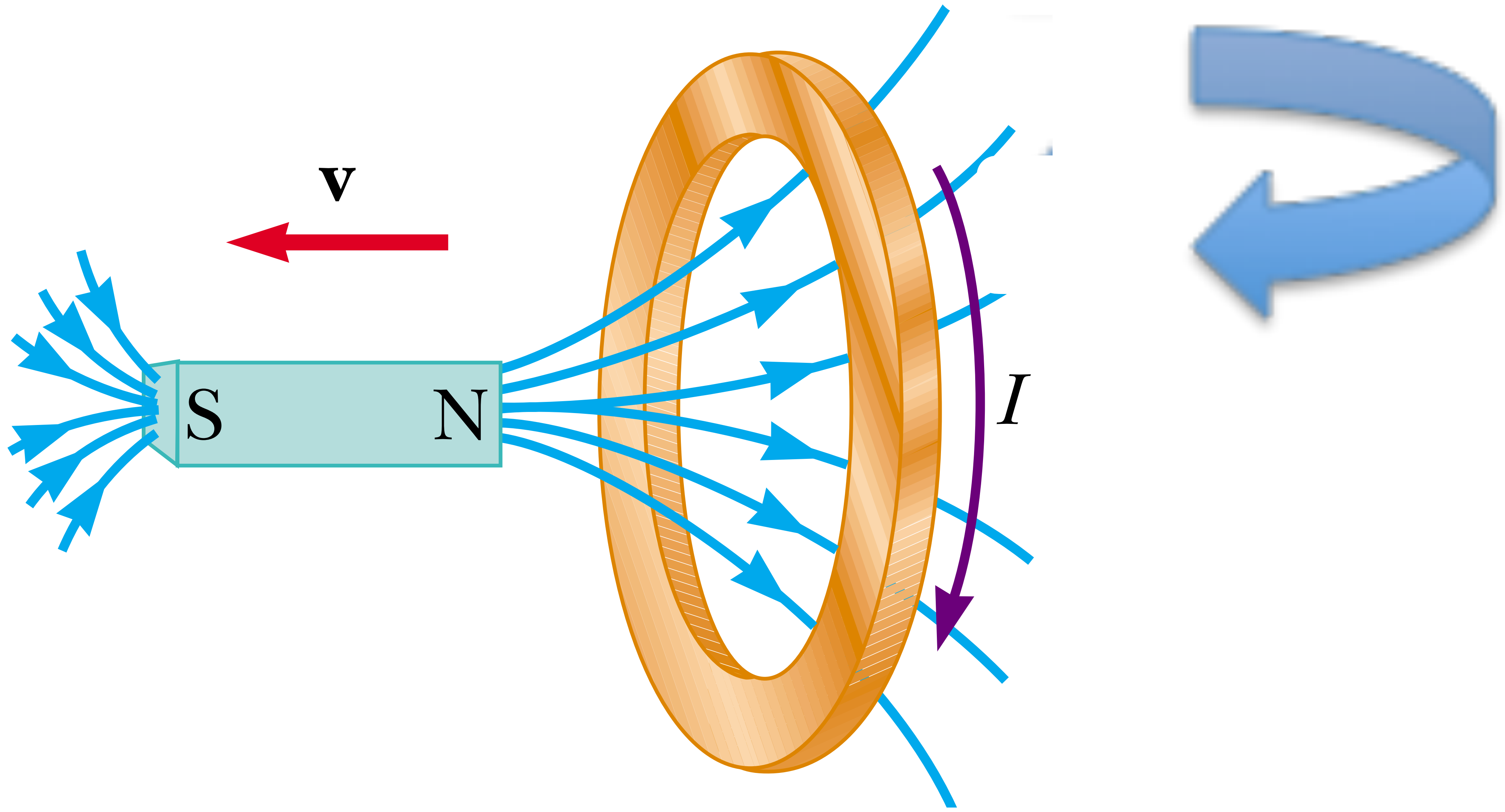
- This induced current produces its own magnetic field directed to the left

that counteracts the increasing external flux



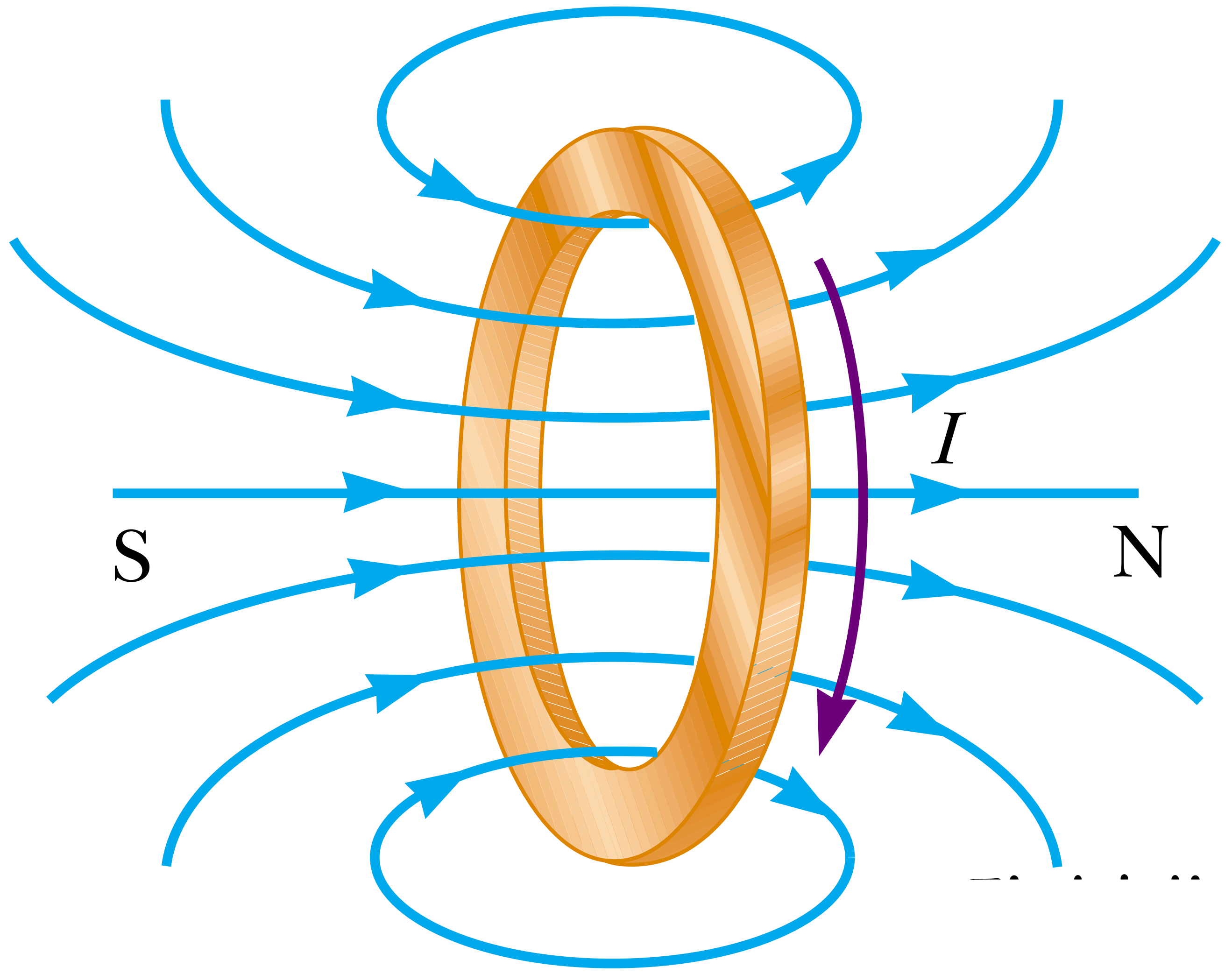
- Magnetic field lines shown are those due to induced current in ring

➢ When magnet is moved away from stationary conducting loop current is induced in direction shown



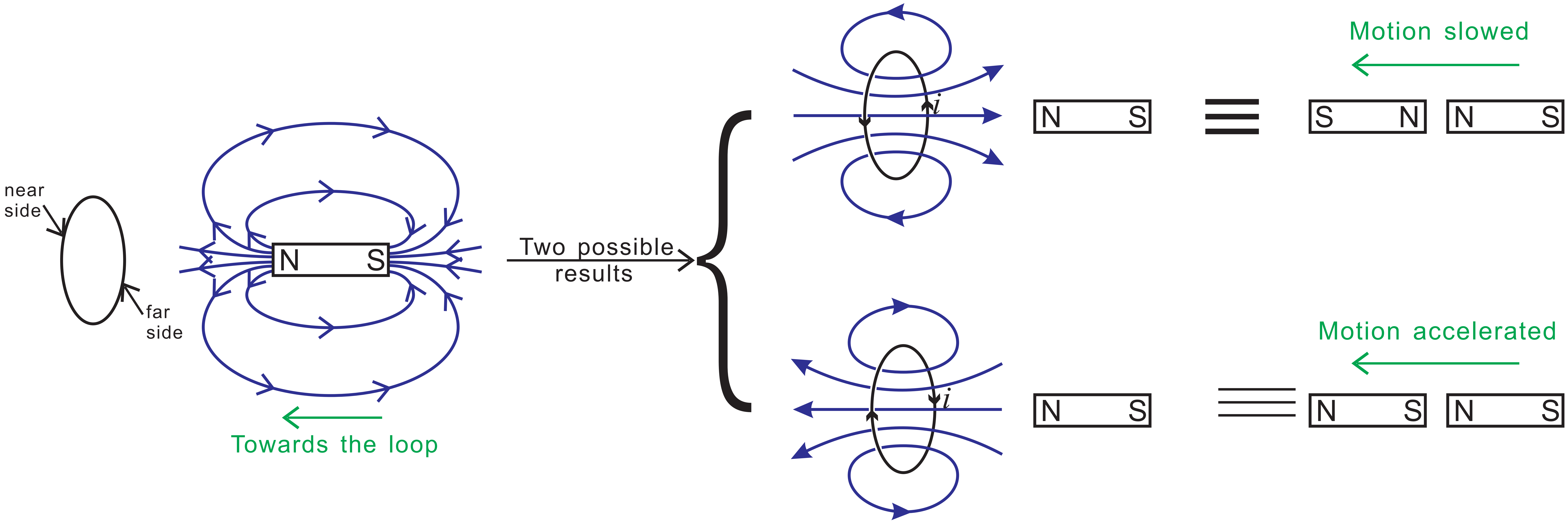
➢ Magnetic field lines shown are those due to bar magnet

➢ This induced current produces magnetic field directed to the right and so counteracts decreasing external flux

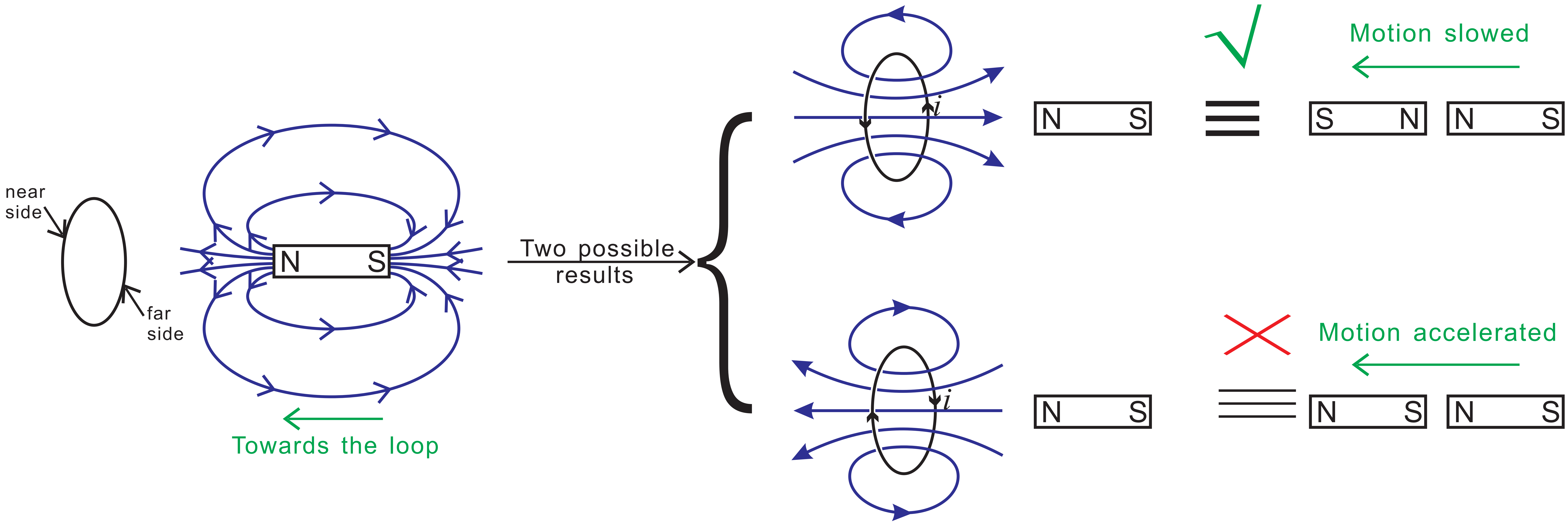


➢ Field lines shown are those due to induced current in ring

Question


near side
far side
N
S
Towards the loop
Two possible results
i
N
S
S
N
N
S
Motion slowed
Motion accelerated
N
S
N
S
N
S

Answer


near side
far side
N S
Towards the loop
Two possible results
i
N S
S N
N S
Motion slowed
i
N S
N S
N S
Motion accelerated

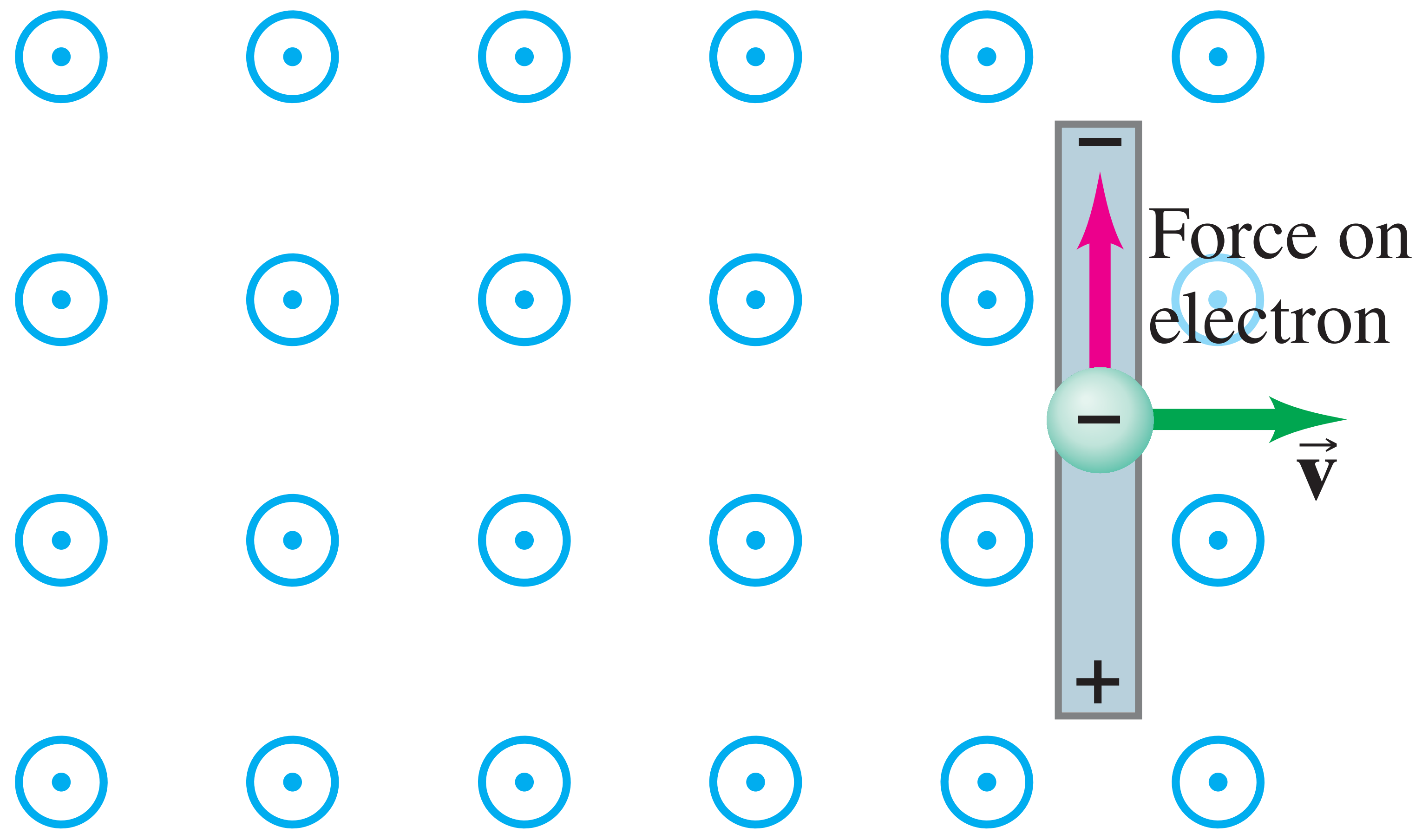
Motional EMF

➢ Straight conductor of length ℓ is moving through uniform $\vec{B}$-field directed out of the page

➢ Assume conductor is moving with constant $\vec{v} \perp \vec{B}$ under influence of some external agent

➢ Electrons in conductor experience force ➡ $\vec{F}_B = q\vec{v} \times \vec{B}$

directed along the length perpendicular to both $\vec{v}$ and $\vec{B}$



➢ Under influence of this force electrons move to upper end of conductor and accumulate there leaving net positive charge at lower end

➢ Because of this charge separation electric field $\vec{E}$ is produced inside conductor

➢ Charges accumulate at both ends until downward magnetic force qvB on charges remaining in conductor is balanced by the upward electric force qE

- At this point ☛ electrons move only with random thermal motion
- Equilibrium requires that ➡ $\vec{F}_E + \vec{F}_B = 0$

$$\Rightarrow q\vec{E} + q\vec{v} \times \vec{B} = 0$$

$$\Rightarrow \vec{E} = -\vec{v} \times \vec{B}$$

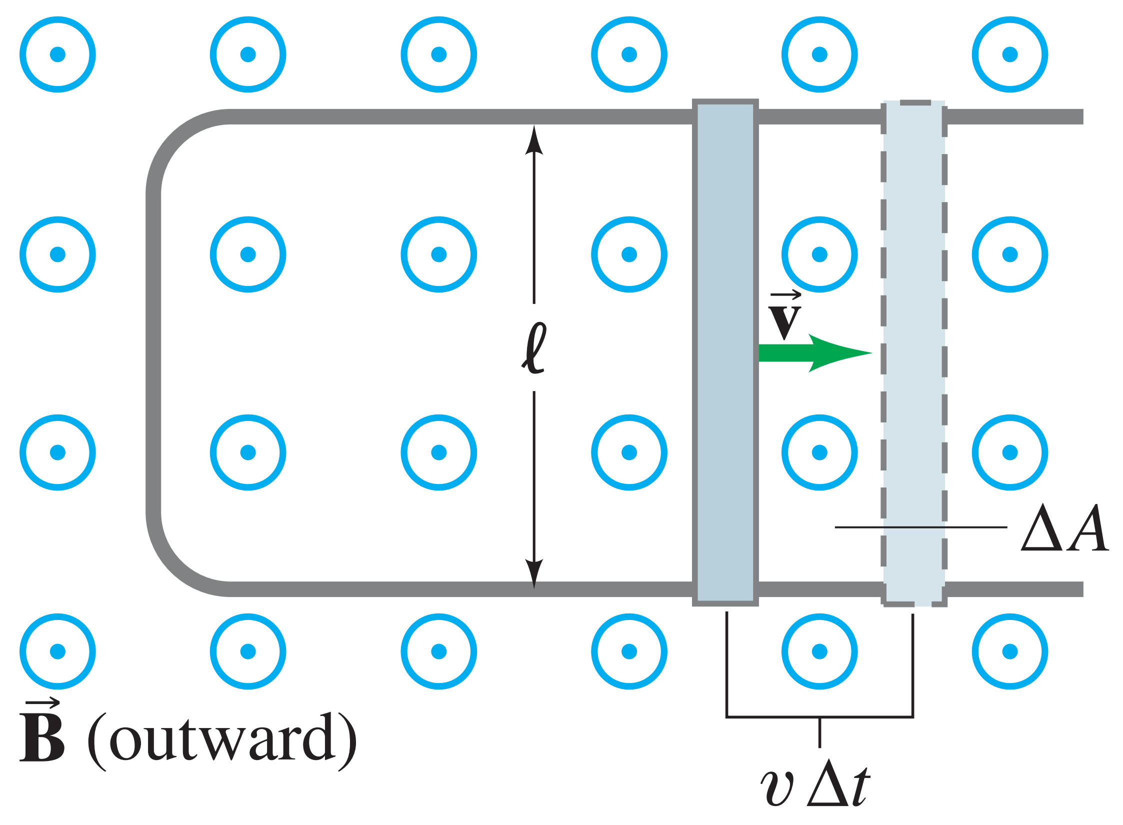
- Voltage across ends of conductor ☛ $\Delta V = -E\ell$

∴ **Voltage** ☛ $\mathcal{E} = \Delta V = vB\ell$

Potential difference is maintained between ends of conductor as long as the conductor continues to move through the uniform magnetic field

EMF Induced in Moving Conductor

➢ Assume that uniform magnetic field B is perpendicular to area bounded by U-shaped conductor and movable road resting on it



➢ If rod is made to move at speed v to right ☛ it travels $\Delta x = v\Delta t$ in time Δt

➢ Area of loop increases ☛ $A = \ell\Delta x = \ell v\Delta t$ in time Δt

➢ By Faraday's law ☛ there is induced emf $\mathcal{E}$ whose magnitude is

$$\mathcal{E} = \frac{\Delta\Phi_M}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{B\ell v\Delta t}{\Delta t} = B\ell v$$

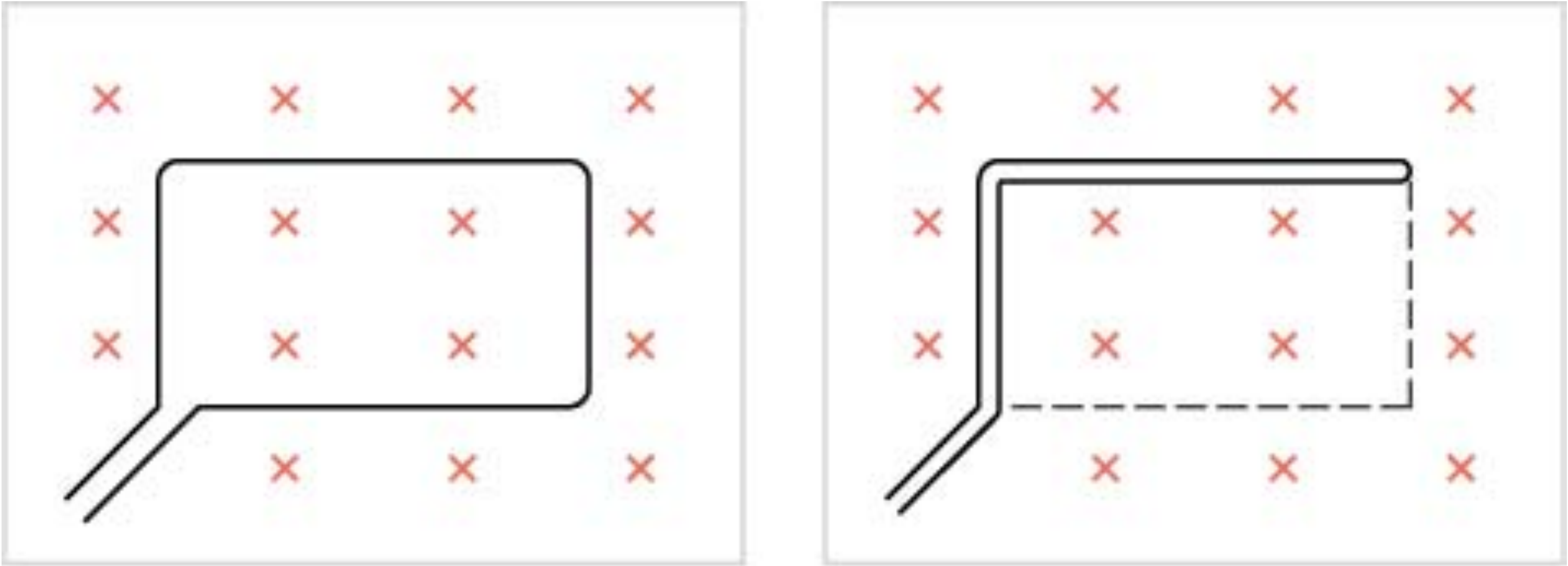
➢ Induced current is clockwise ⇒(to counter the increasing flux)

Example

➢ A rectangular loop of wire with sides of 0.20 and 0.35 m lies in a plane perpendicular to a constant magnetic field of 0.65 T

➢ In a time of 0.18 s, one-half of the loop is folded back on the other half as shown

➢ What is the average EMF induced in the loop?



$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t} = \frac{NB\Delta A}{\Delta t}$$

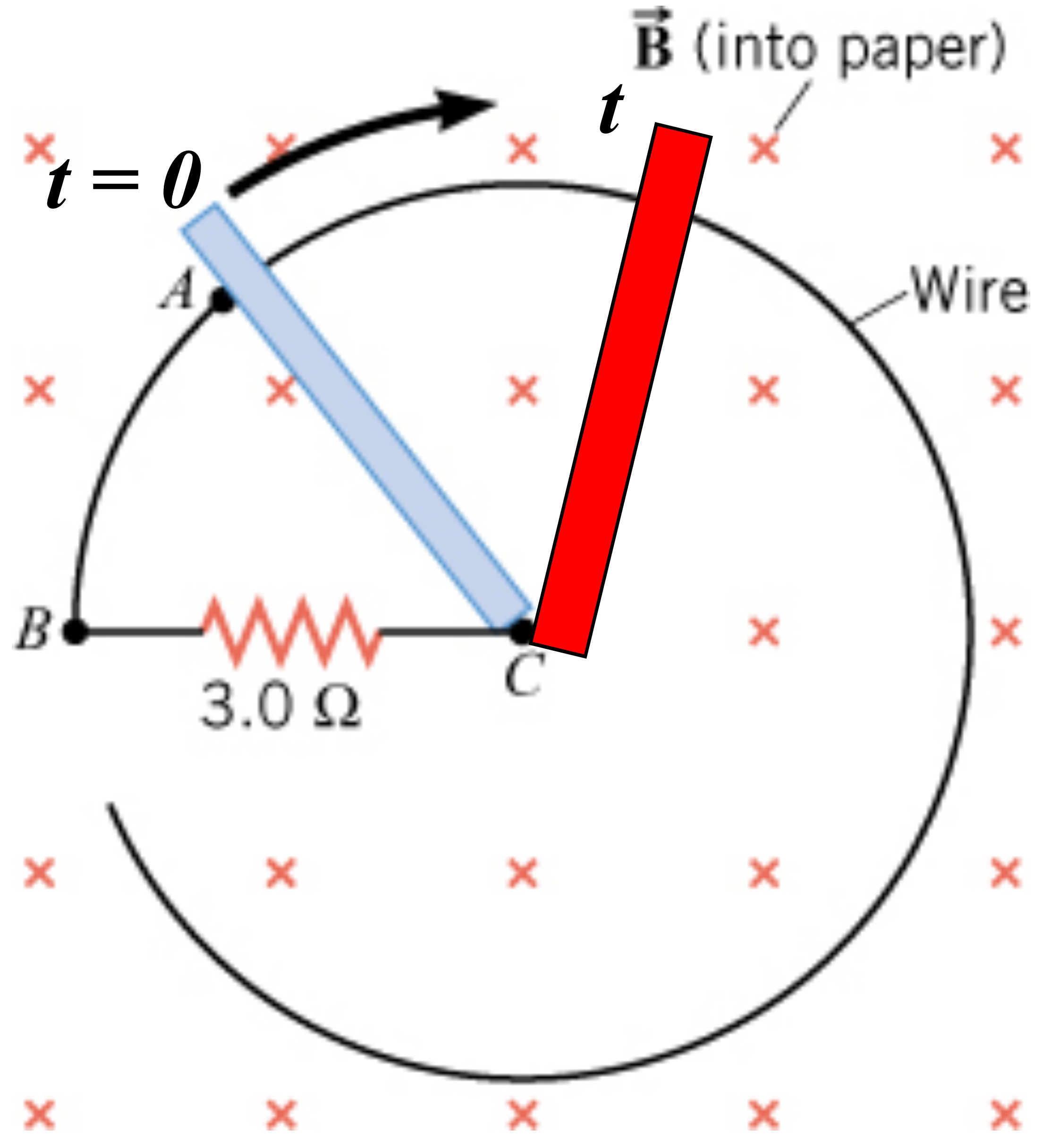
➢ Initially $A = (0.2\ m)\ (0.35\ m) = 0.07\ m^2$

➢ In the final state the area is very small, I.e. $A \approx 0$

$$\mathcal{E} = -\frac{1\cdot(0.65T)(0-0.07m^2)}{0.18s} = 0.25V$$

Example

- The drawing shows a copper wire bent into a circular shape with a radius of 0.5 m
- Radial section BC is fixed in place, while copper bar AC sweeps around at an angular speed of 15 rad/s
- The bar makes electrical contact with the wire at all times
- The wire and the bar have negligible resistance
- A uniform magnetic field exists everywhere, is perpendicular to plane of circle and has a magnitude of 3.8×10^{-3} T
- What is the magnitude of the current induced in the loop ABC?



$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t} = \frac{NB\Delta A}{\Delta t}$$

BUT ☛ $\Delta A = \frac{\pi r^2}{2\pi}\cdot\Delta\theta$

$$\frac{\Delta A}{\Delta t} = \frac{\pi r^2}{2\pi}\cdot\frac{\Delta\theta}{\Delta t} = \frac{\pi r^2}{2\pi\cdot rad}\frac{15rad}{1.0s} = \frac{15r^2}{2s}$$

$$\frac{\Delta A}{\Delta t} = \frac{15(0.5m)^2}{2s} = 1.88\frac{m^2}{s}$$

$$\mathcal{E} = 1\cdot(3.8\times10^{-3}T)\frac{\Delta A}{\Delta t} = (3.8\times10^{-3}T)\left(1.88\frac{m^2}{s}\right)$$

$$\mathcal{E} = 7.1mV$$

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R} = \frac{7.1\times10^{-3}V}{3\Omega} = 2.4mA$$

Electric Generator

Faraday's Law

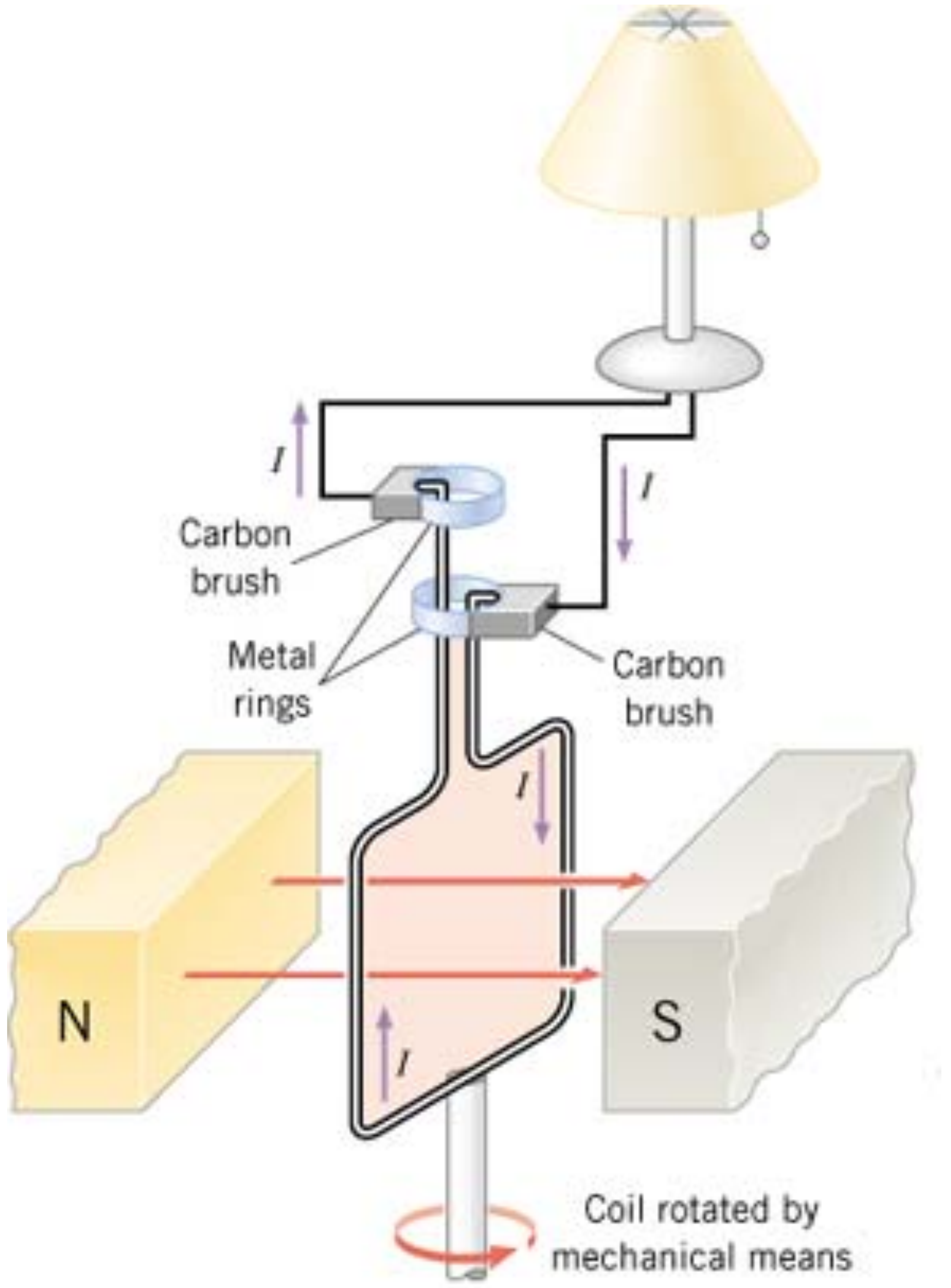
➢ A changing magnetic flux through a loop of wire generates a potential difference (EMF) and current

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

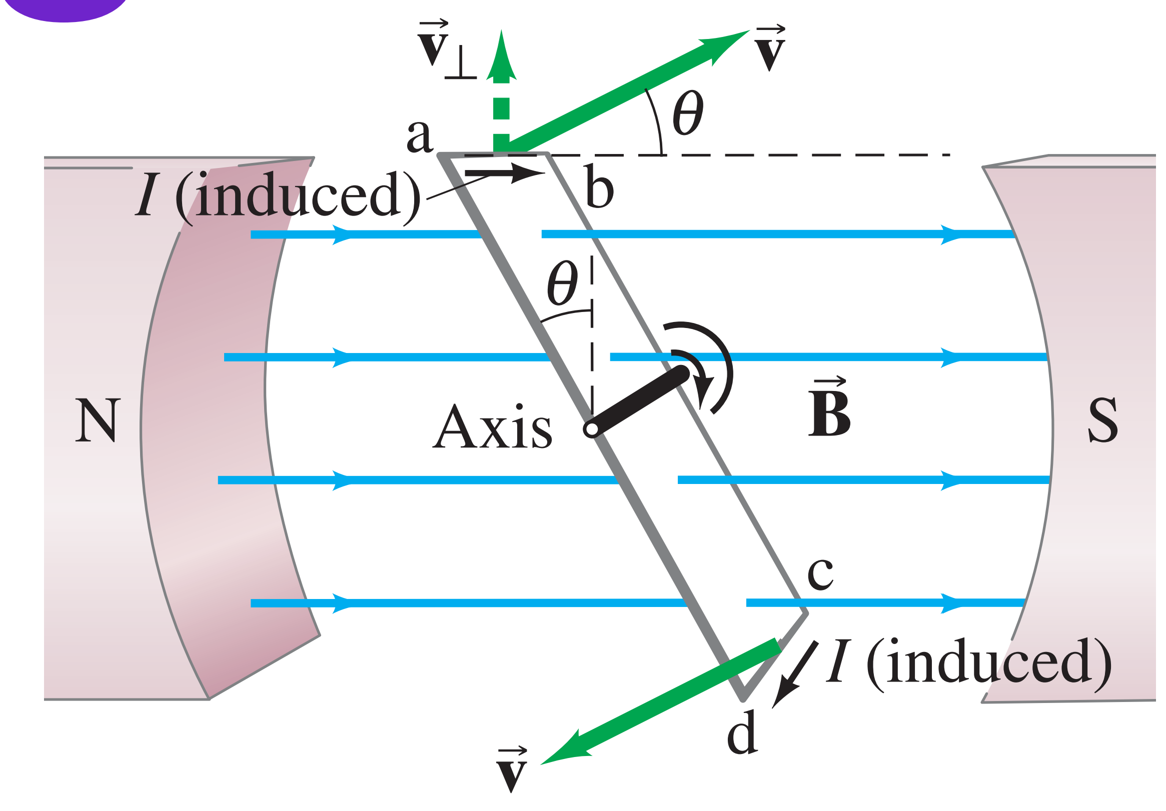
➢ Imagine a rotating loop in a costant magnetic field

Is the flux changing ?

➢ **Rotate with an angular speed of ω**



Deriving Generator Equation



ℓ ☛ length of ab

➢ Loop is being made to rotate clockwise in a uniform magnetic field B

➢ Velocity of two lengths ab and cd at this instant are shown

➢ Although sections of wire bc and da are moving ☛ force on e's in these sections is toward side of wire not along wire's length $\Rightarrow$ emf generated is due only to force on charges in sections ab and cd

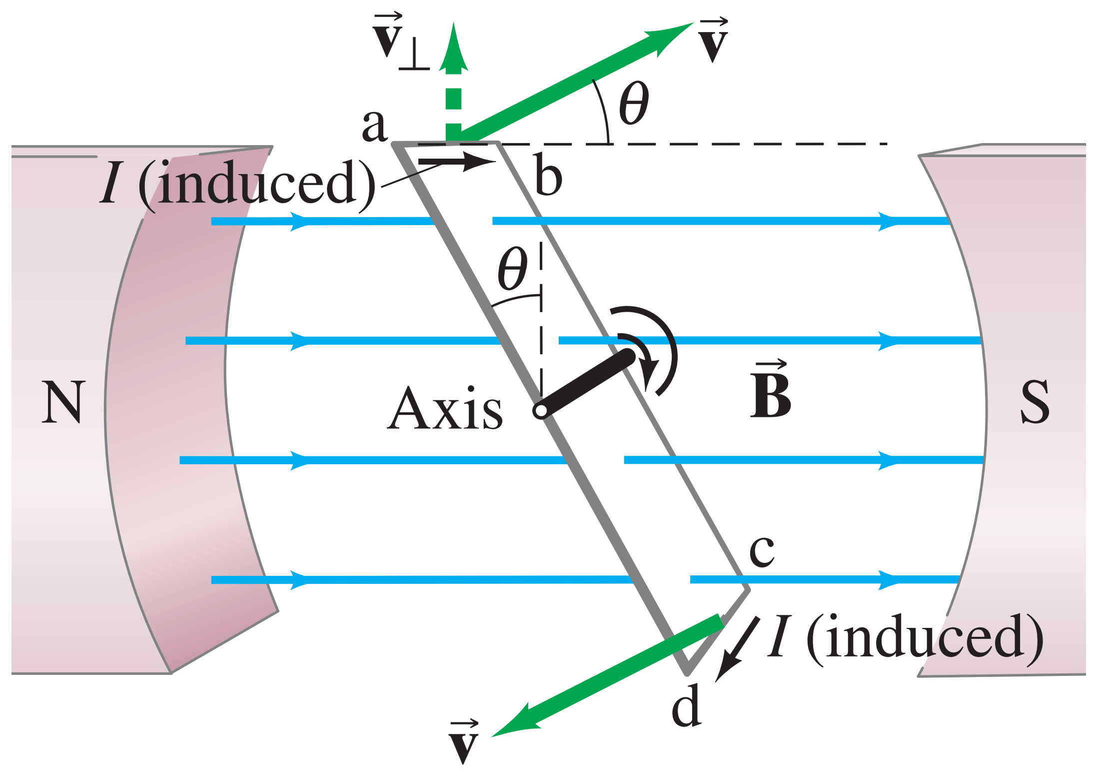
➢ RHR ☛ direction of the induced current in ab is from a toward b

➢ In the lower section ☛ it is from c to d $\Rightarrow$ flow is continuous in the loop

➢ Magnitude of the emf generated in ab is ☛ $\mathcal{E} = B\ell v_\perp$
emf induced in cd has same magnitude and is in same direction ☛ emfs add

$$v_\perp = v \sin\theta \Rightarrow \mathcal{E} = 2NB\ell v \sin\theta$$

⬇ number of loops in coil



If coil is rotating with constant angular velocity $\omega \Rightarrow \theta = \omega t$

$$v = \omega r = \omega(h/2)$$

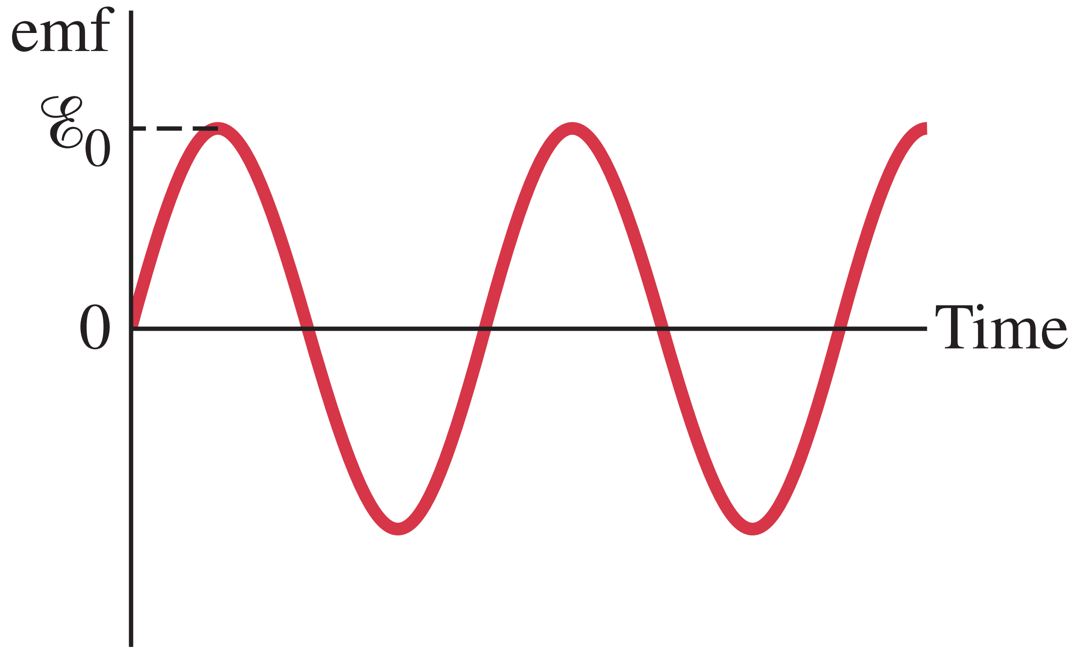
r ☛ distance from the rotation axis

h ☛ length of bc or ad

$$\mathcal{E} = 2NB\omega\ell(h/2)\sin(\omega t) \Rightarrow \mathcal{E} = NB\omega A\sin(\omega t)$$

$A = \ell h$ ☛ area of loop

$$V_{\text{rms}} = \frac{NB\omega A}{\sqrt{2}}$$



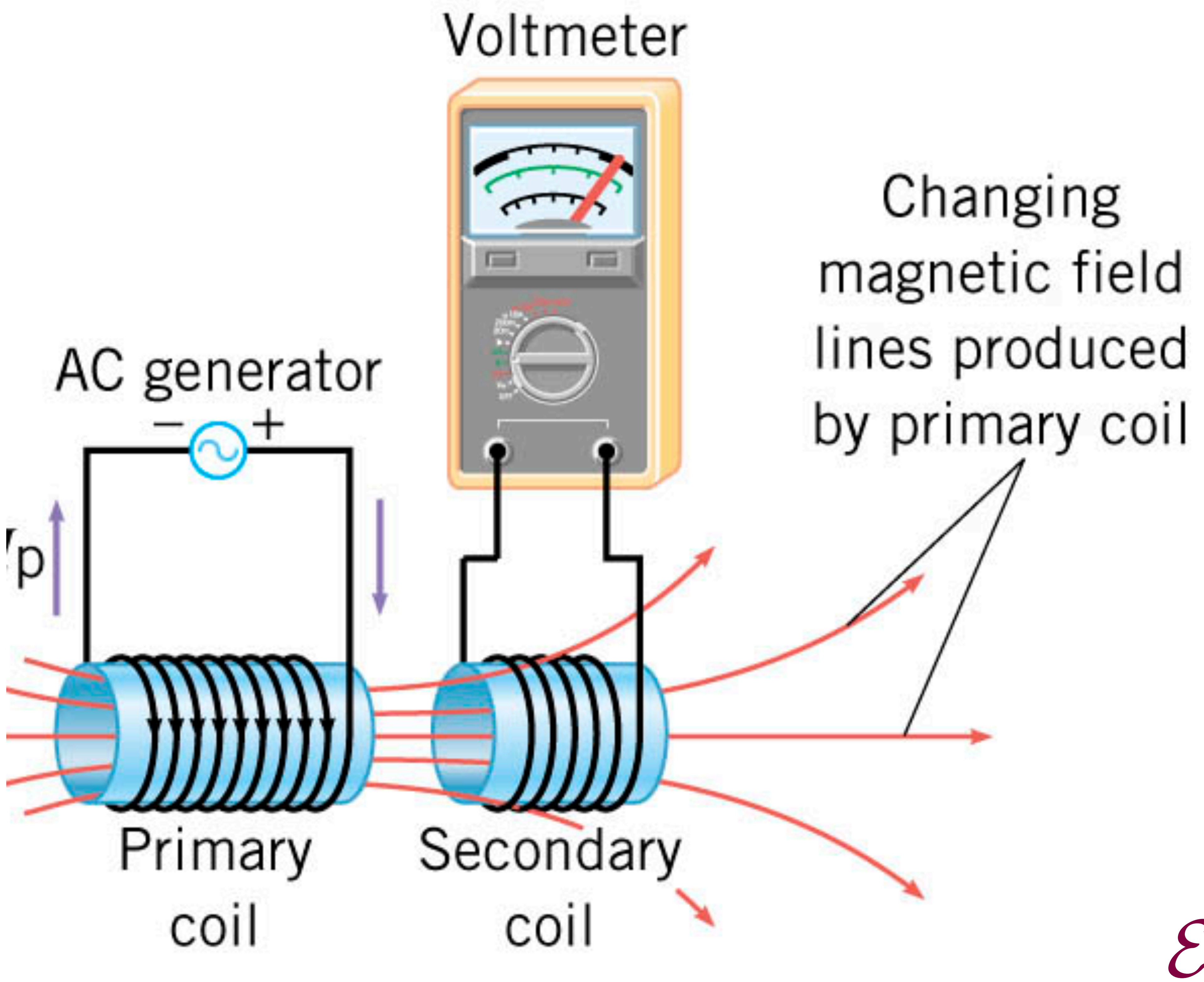
$$\mathcal{E} = \mathcal{E}_0 \sin(\omega t)$$

$$\mathcal{E}_0 = NB\omega A$$

Mutual Inductance

➢ Let's place two coils side by side

➢ Let's connect one to an AC generator (primary coil) and the other to voltmeter (secondary coil)



➢ Primary coil creates a magnetic field, and some of those field lines pass thru secondary coil

➢ This produces a change in magnetic flux in secondary coil, leading to an induced emf!

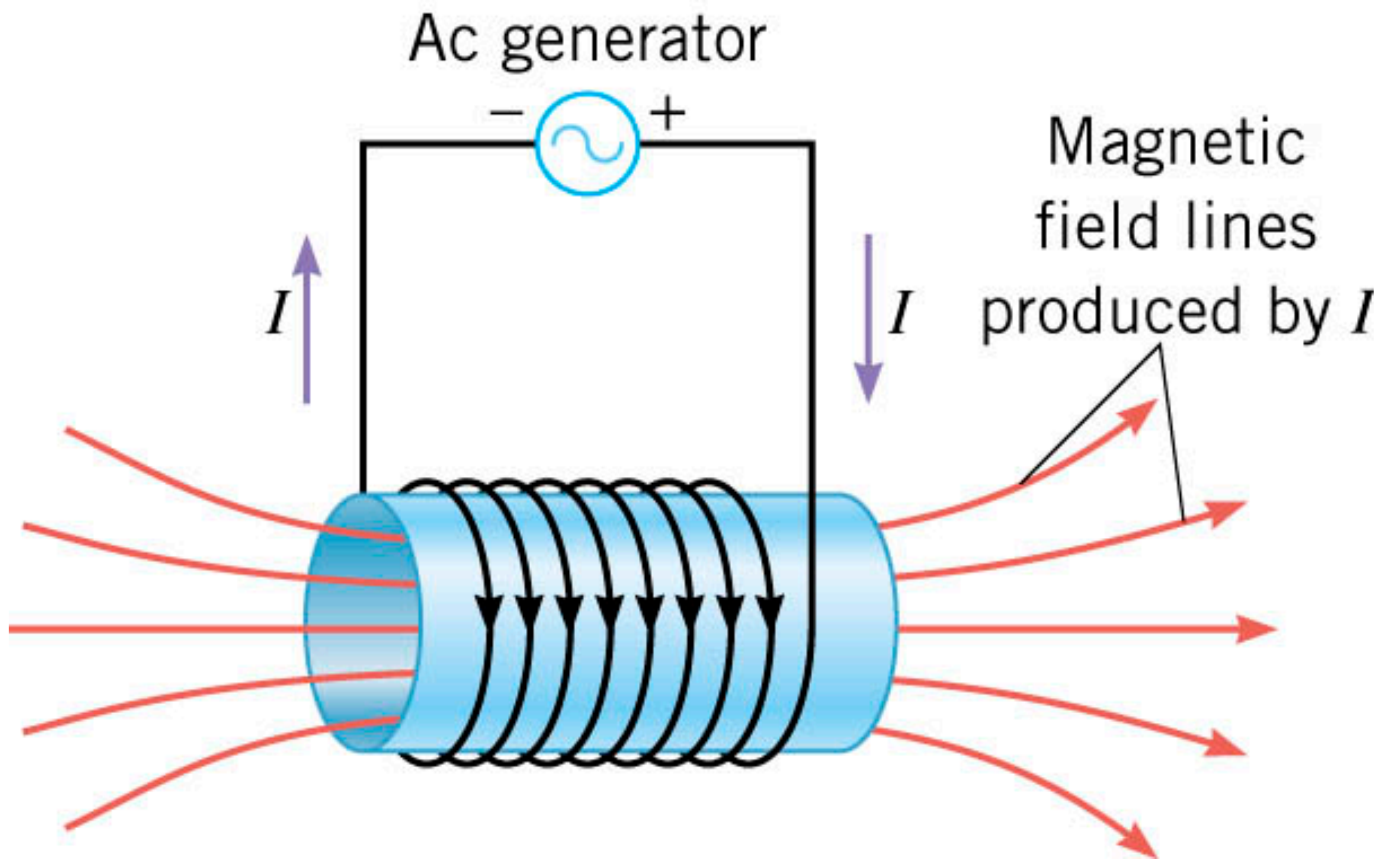
This is called Mutual Inductance

$$\mathcal{E}_{\text{sec}} = -M\,\frac{\Delta I_{\text{prim}}}{\Delta t}$$

M → **Proportionality constant**

Self Inductance

- ➢ Consider just one coil connected to an AC generador
- ➢ AC current produces a changing magnetic field which produces a change in magnetic flux within coil



This leads to an **induced emf** in coil!

This process is called **Self Induction**

$$\mathcal{E} = -L\,\frac{\Delta I}{\Delta t}$$

Proportionality constant

Solenoid Inductance

➢ We equate Faraday's law to inductance

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

and salve for L

$$L = N\frac{\Delta\Phi_B}{\Delta I}.$$

➢ We know $\mathbf{\Phi}_B = BA$ and magnetic field for solenoid ☛ $B = \mu_0 NI/\ell$, so magnetic flux inside solenoid is

$$\Phi_B = \frac{\mu_0 NIA}{\ell}$$

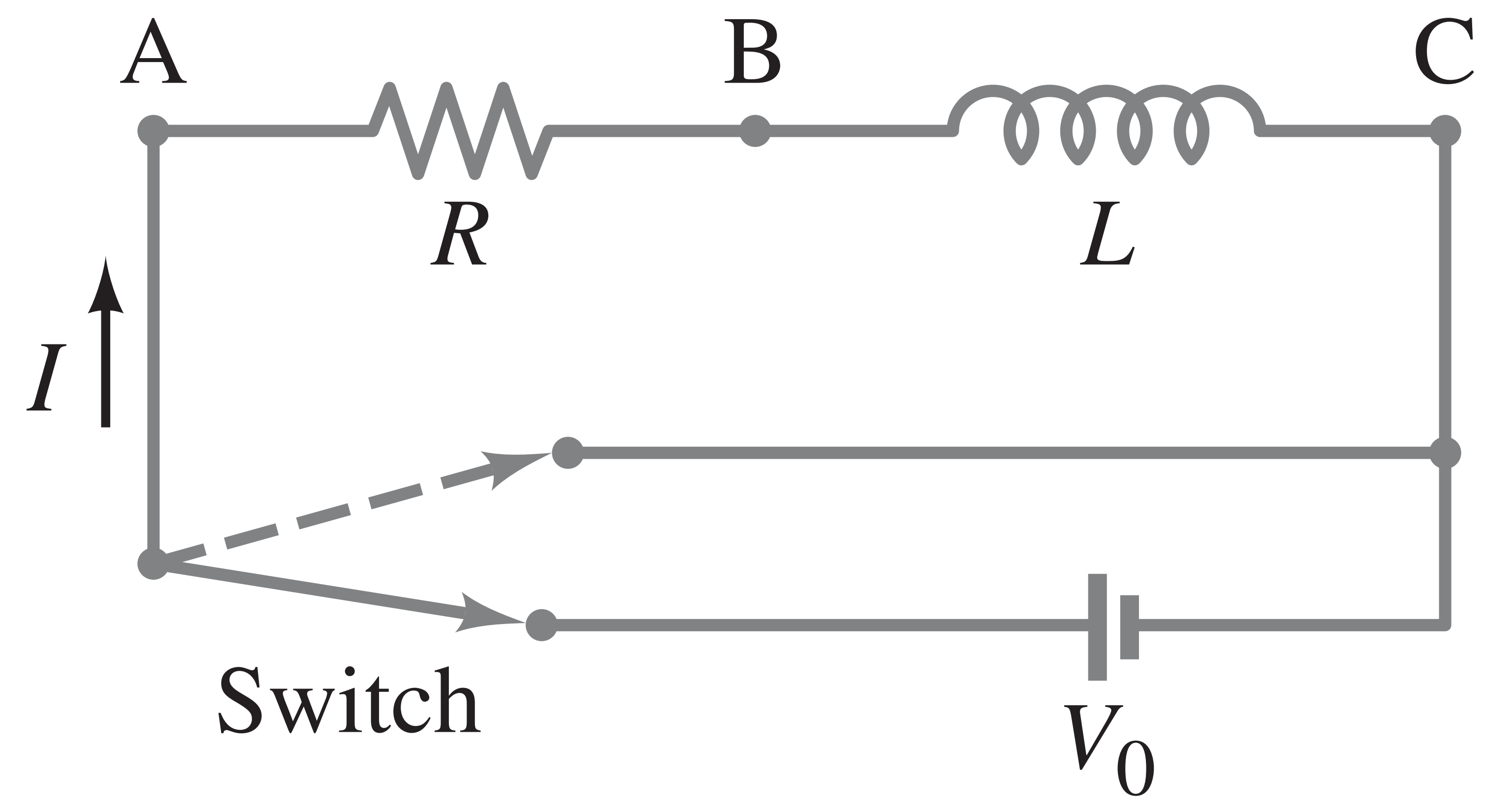
➢ Any change in current, ΔI, causes a change in flux

$$\Delta\Phi_B = \frac{\mu_0 N\,\Delta I\,A}{\ell}$$

➢ We put this into our equation above for L

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{\mu_0 N^2 A}{\ell}$$

Energy Stored in a Magnetic Field



- We saw that energy stored in capacitor ☛ $\frac{1}{2}CV^2$.
- By using similar argument it can be shown that energy stored in inductance is

$$U = \text{energy} = \tfrac{1}{2}LI^2$$

for solenoid

$$L = \mu_0 N^2 A/\ell \qquad\qquad B = \mu_0 NI/\ell$$

$$I = B\ell/\mu_0 N$$

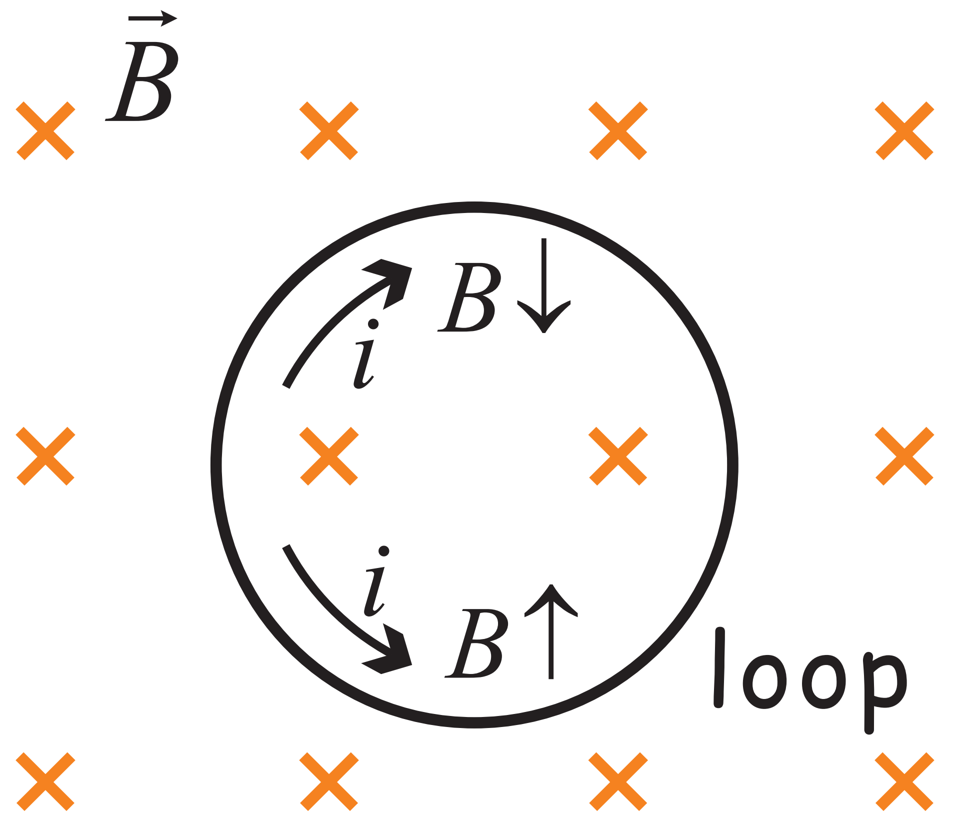
$$U = \text{energy} = \tfrac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{\ell}\right)\left(\frac{B\ell}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}A\ell$$

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$Energy\ density = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

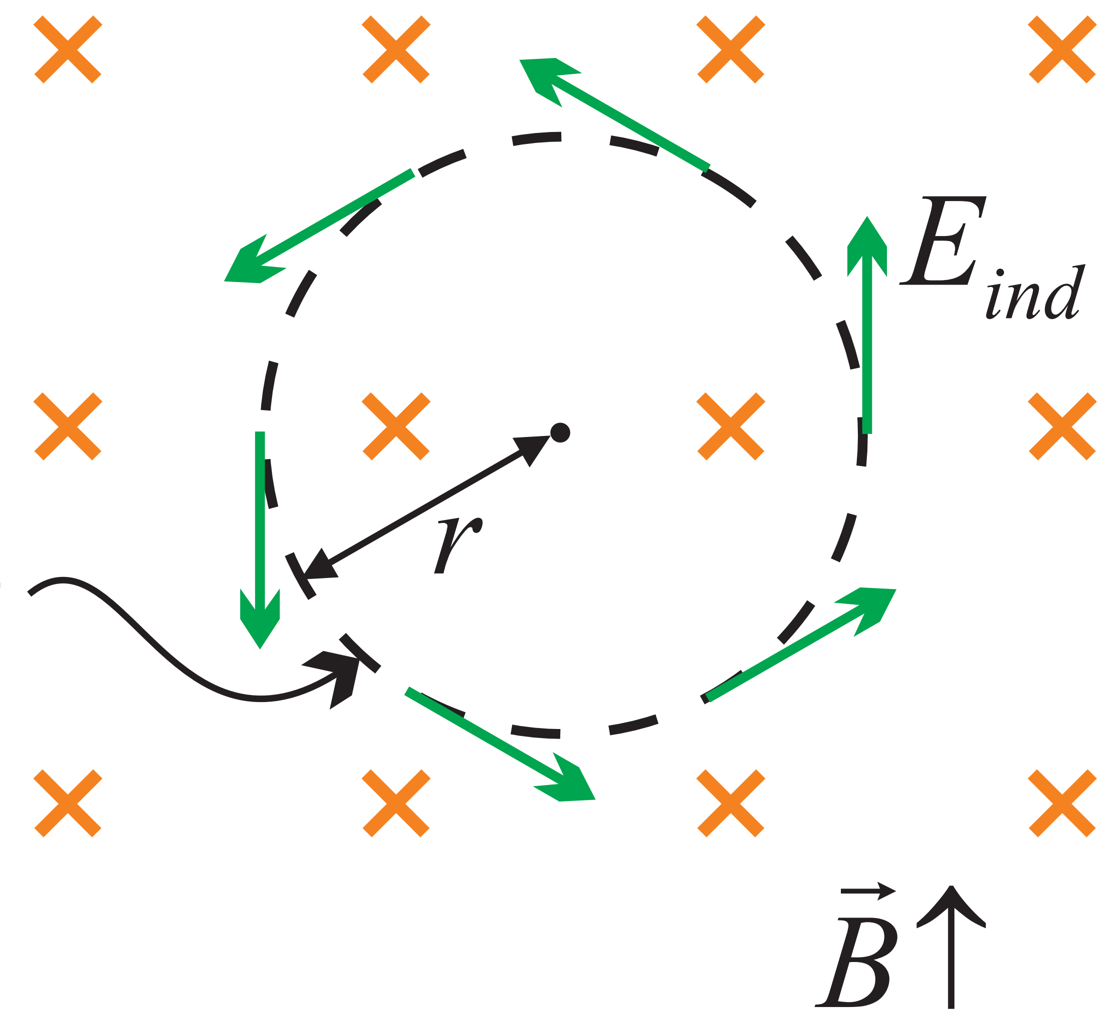
Induced Electric Field

- ➢ We have seen that a changing magnetic flux induces an emf and a current in a conducting loop



$\therefore \vec{B}\uparrow \Rightarrow$ anti-clockwise current
$\vec{B}\downarrow \Rightarrow$ clockwise current

- ➢ In the same way we can relate induced current in conducting loop to an electric field by claiming that electric field is created in conductor as a result of the changing magnetic flux



- ➢ $\mathcal{E} = \sum_{\substack{\text{closed}\\\text{path}}} E_{\|}\,\Delta\ell \neq 0$ ☛ Non-conservative force field

Summary

Regular $\vec{E}$-field created by charges

$\vec{E}$ -field lines start from

$+q$ and end on $-q$ charge

can define electric potential

so that we can discuss potential difference

between two points

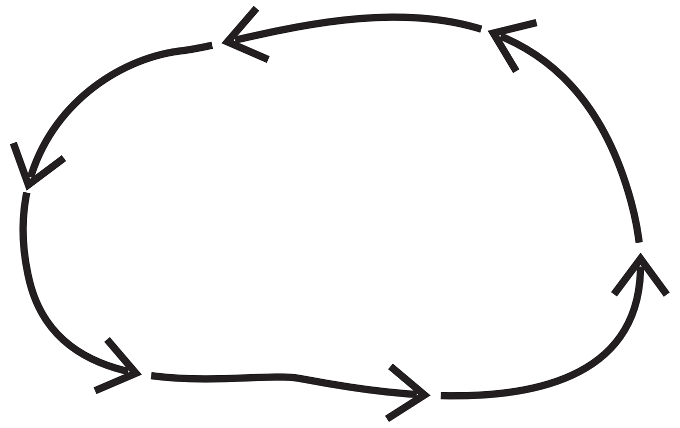
$\Downarrow$

Conservative force field

Induced $\vec{E}$ -field

created by changing B-field

$\vec{E}$ -field lines form closed loops



Electric potential cannot be defined

(or, potential has no meaning)

$\Downarrow$

Non-conservative force field

Classification of electric and magnetic effects **depend on frame of reference of observer** !!!

e.g. For motional emf ☛ observer in reference frame of moving loop

will **NOT** see an induced $\vec{E}$ -field but just a **regular** $\vec{E}$ -field

To be continued in Lesson 11

same bat-time, same bat-channel


aNy