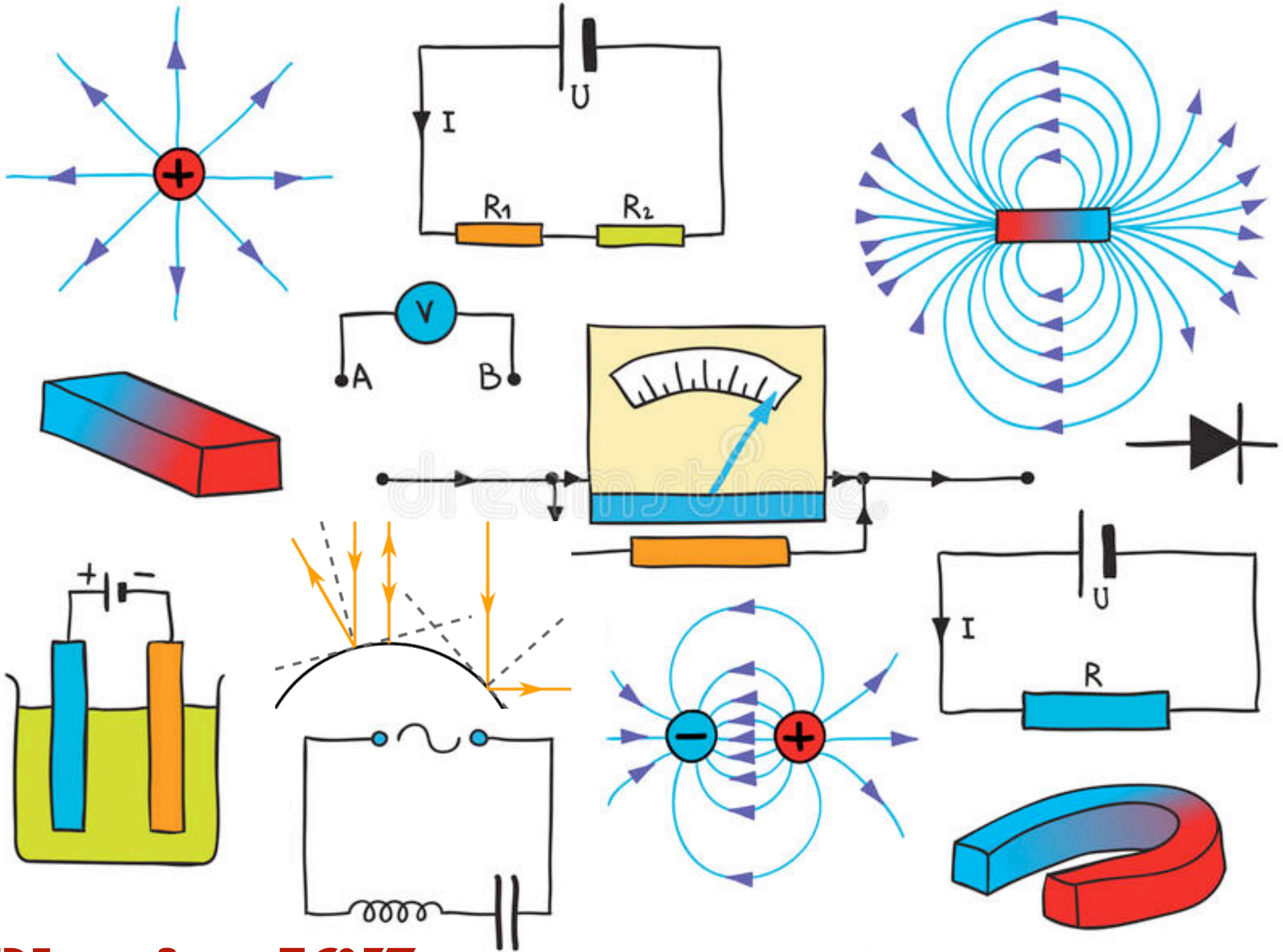


Physics 167

Luis Anchordoqui

Magnetic Fields

- ➢ Magnetism has been observed since roughly 500 B.C.
- ➢ Certain rocks on the Greek peninsula of Magnesia were noticed to attract and repel one another
- ➢ Hence the word Magnetism
- ➢ So just like charged objects, magnetized objects can exert forces on each other - repulsive or attractive
- ➢ A magnet has two poles ☛ NORTH and SOUTH
- ➢ Similar to electric charges but magnetic poles always come in pairs

Like poles repel and opposite poles attract

- ➢ No single pole (called magnetic monopole) has been observed in nature

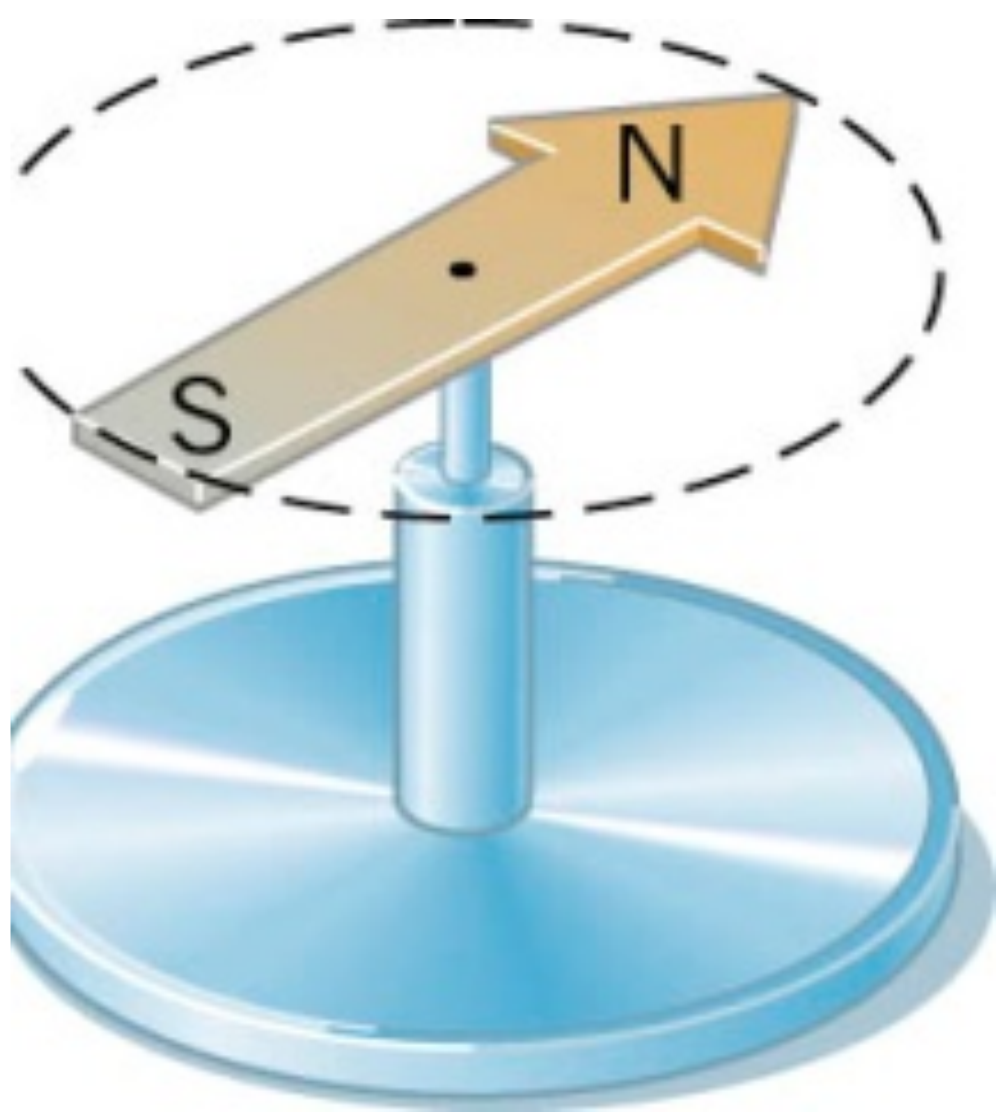
We get two magnets, each with two poles!

➢ **Electric charges** produce **electric fields** $\vec{E}$ and **magnets** produce **magnetic fields** $\vec{B}$

➢ We used a small positive charge (test charge) to determine what electric field lines look like around a point charge

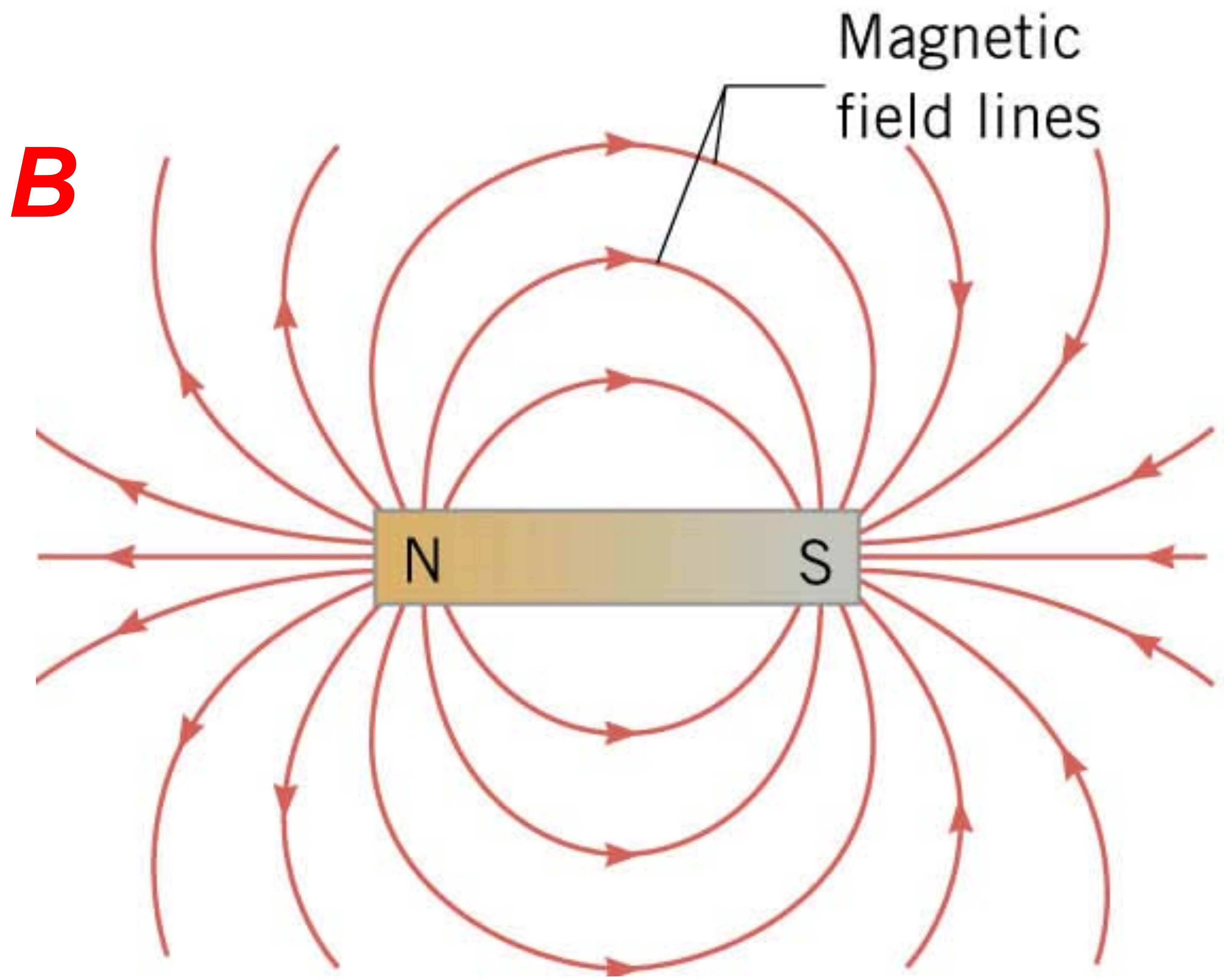
➢ Can we do a similar thing to determine what the **magnetic field lines** look like around a magnet??

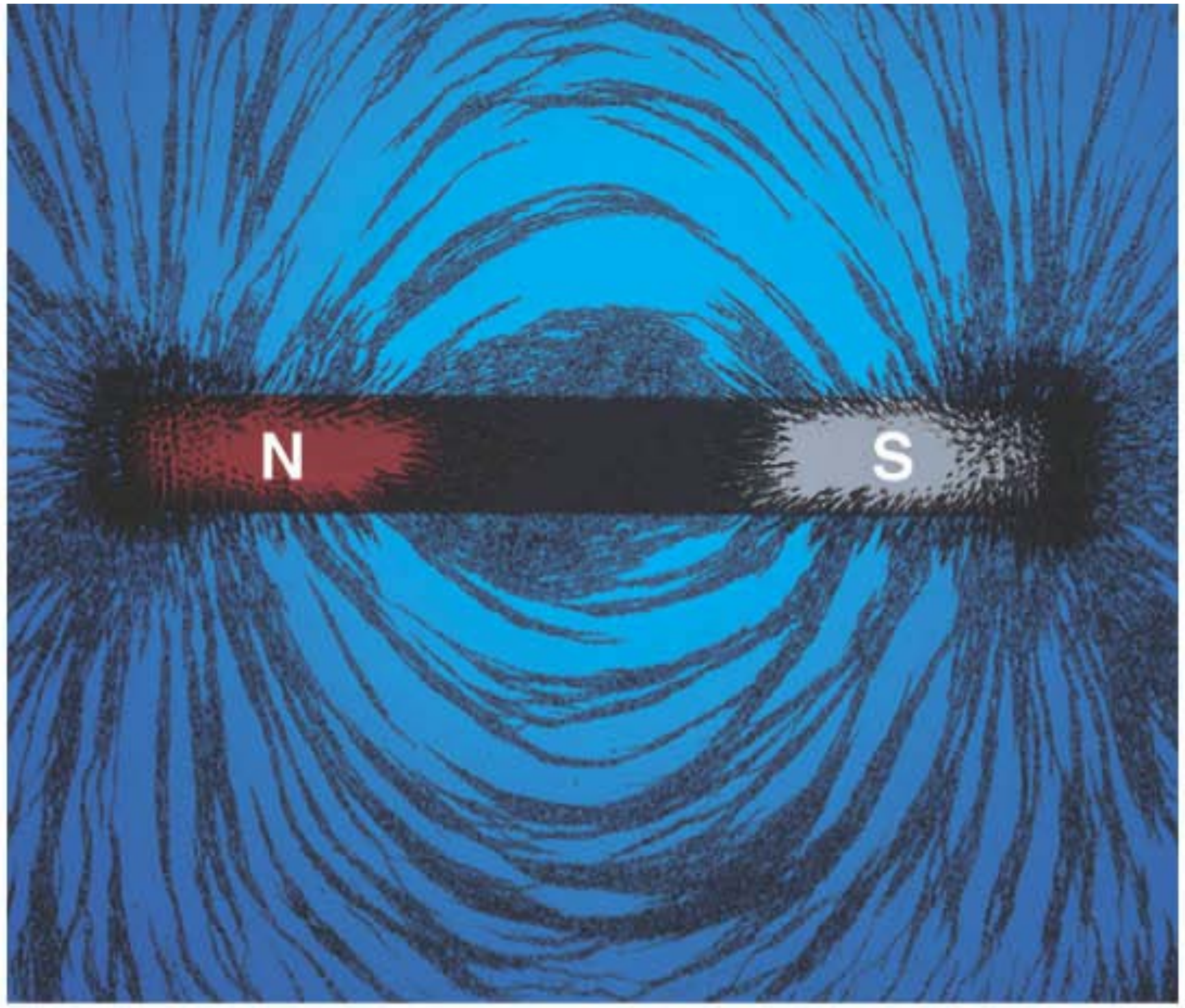
YES!! We can use a small magnet called a compass!!



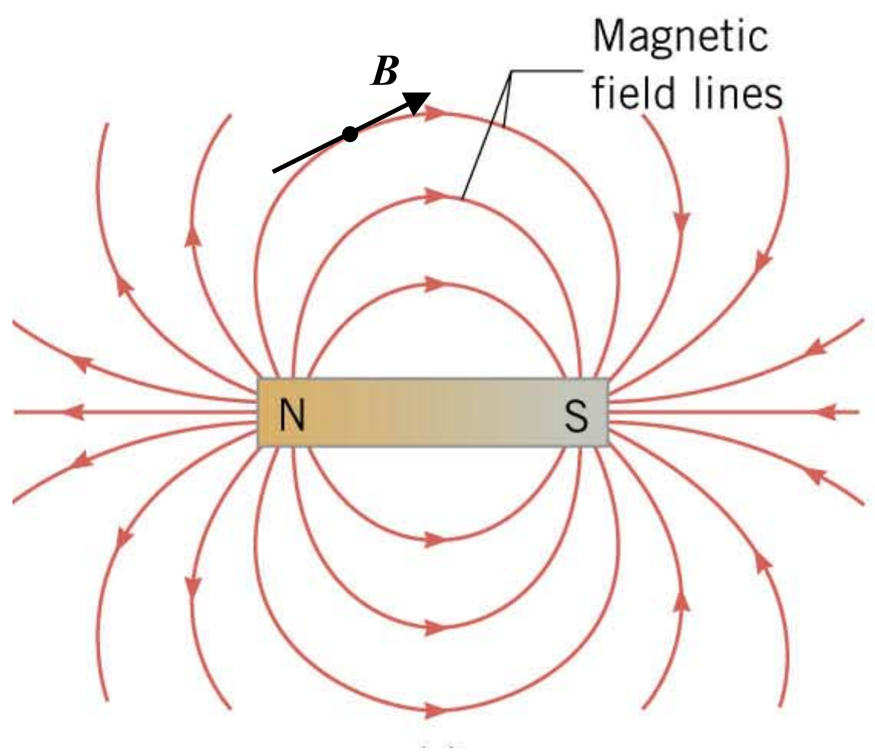
➢ Compass needle is free to pivot, and its tip (North pole) will point toward South pole of another magnet

➢ Fields lines around a bar magnet look like this





Properties of Magnetic Field Lines



1. They point away from **North** poles and point toward **South** poles

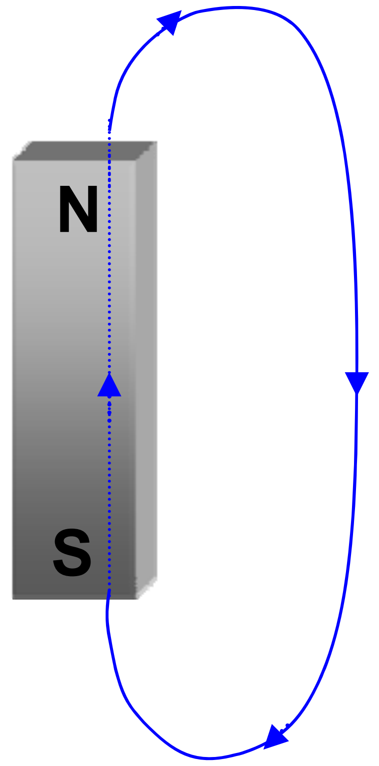
   The compass needle will line up in the direction of the field!

2. Magnetic field at any point in space is tangent to the field line at that point
3. The higher the density of field lines, the stronger the field

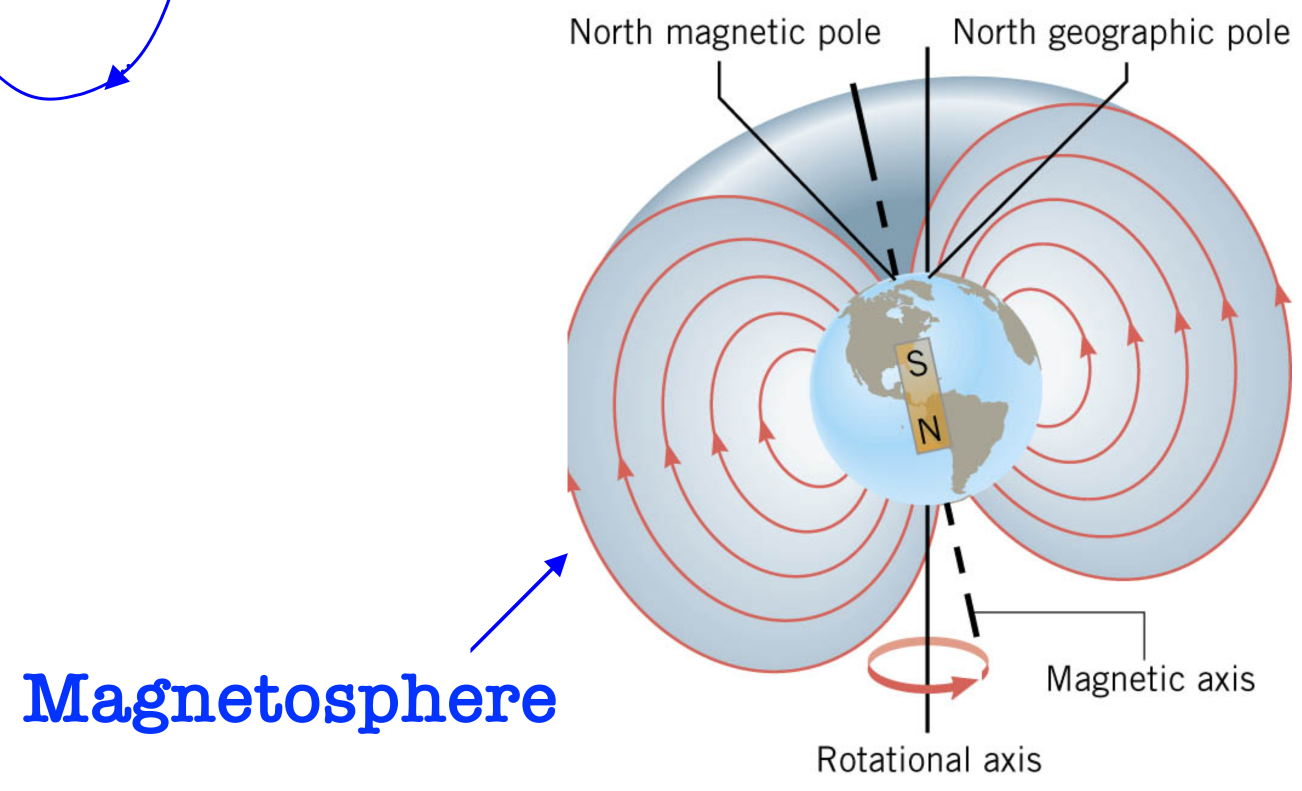
   Thus, the strongest field is near the poles !!

4. The field lines must form closed loops, i.e. they don't start or stop in mid space

   There are no magnetic monopoles!



➢ Since a compass needle points North on the surface of earth, **earth must have a magnetic field,** and its **South pole**, called **Magnetic North**, must be in northern hemisphere



➢ Magnetic north does not coincide with geographic north, and it tends to move around over time

➢ Earth's magnetic field is not well understood

➢ May be due to distribution of currents flowing in liquid nickel core

Magnetic Force

➢ **Charges** feel forces in **electric fields**

➢ **Magnets** feel forces in **magnetic fields**

Until 1820, everyone thought electricity and magnetism had absolutely nothing to do with each other

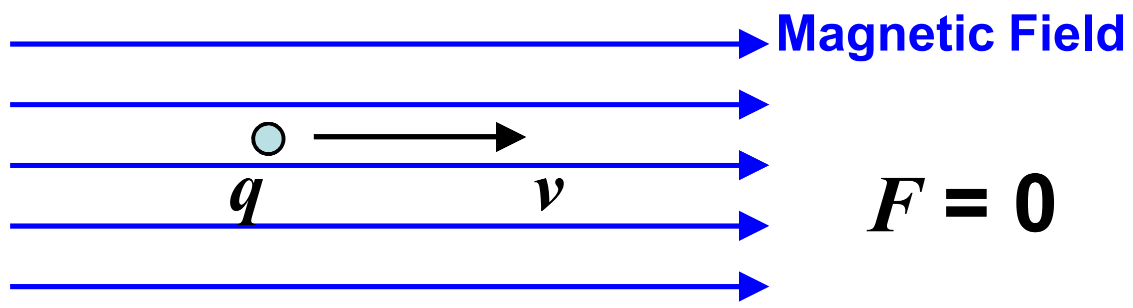
But, it turns out that eléctrical charges WILL also feel a force in magnetic fields, under certain conditions

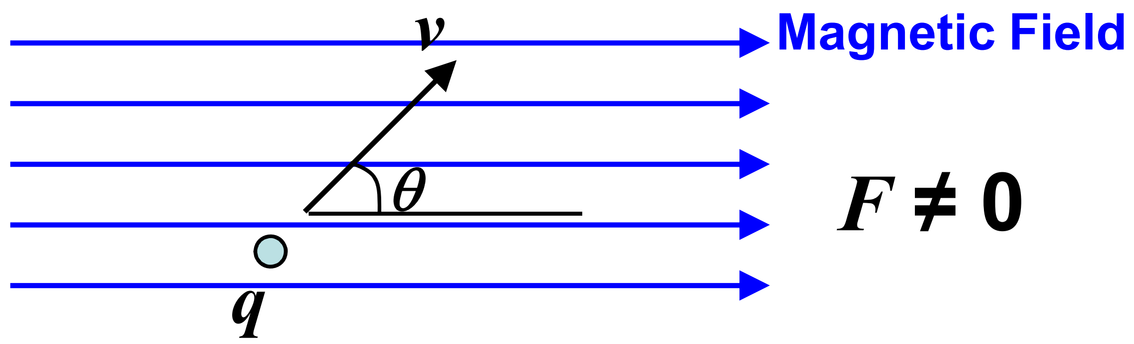
1. The charge must be moving i.e. has a nonzero velocity

There is no magnetic force on a stationary charge!

2. The charge's velocity must have a component that is perpendicular to magnetic field

➢ If a charge is moving in a magnetic field, but it moves along same direction as the field (parallel to it) there is no force





➢ So, if there is a force on a moving charged particle in a magnetic field, how do we calculate that force?

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$$

$$F = qvB\sin\theta$$

B is the magnetic field

θ is the angle between B and v

➢ Units of B

$$B = \left[\frac{\text{Force}}{\text{Charge} \times \text{Velocity}}\right] = \left[\frac{\text{N} \cdot \text{s}}{\text{C} \cdot \text{m}}\right] = [\text{Tesla}] = [\text{T}]$$

➢ 1 T is a pretty big field

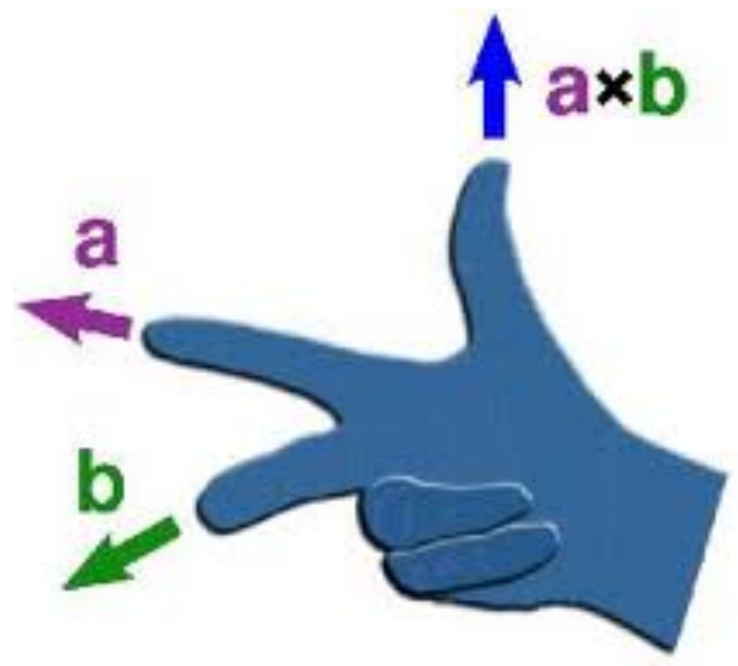
➢We also use another unit of magnetic field ☛ gauss

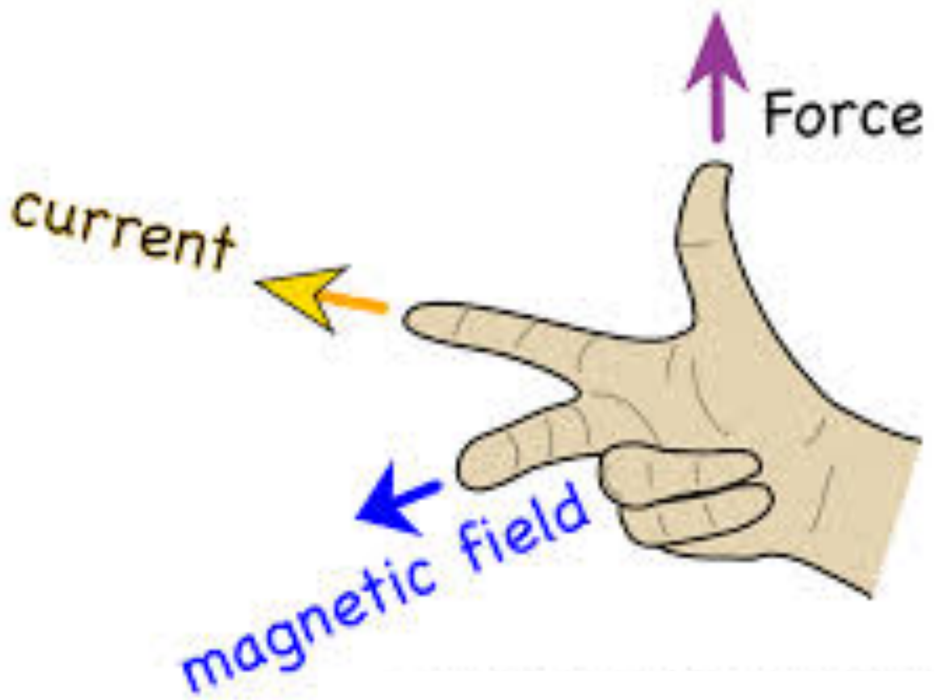
1 gauss = 1×10^{-4} T

Earth's magnetic field is ~ 0.5 gauss

Direction of Force on a Charge Particle in a Magnetic Field

Use Right Hand Rule 1 (RHR-1)

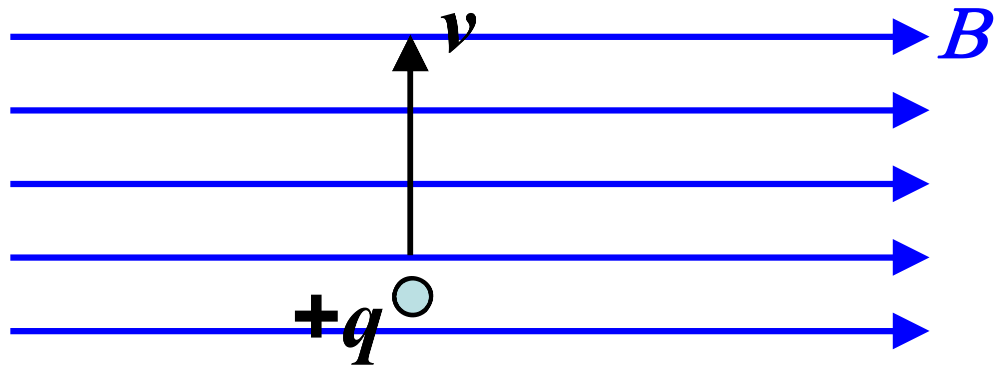


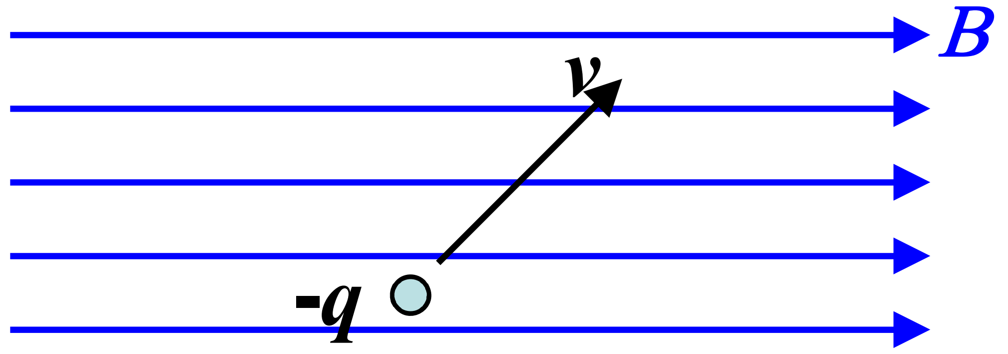


$$F = q\nu B \sin\theta$$

- ➢ This is the procedure to follow when charge is positive
- ➢ If charge is negative, do everything exactly the same, but then reverse direction of force at end

Examples





Force? **Force is into the page ⊗**

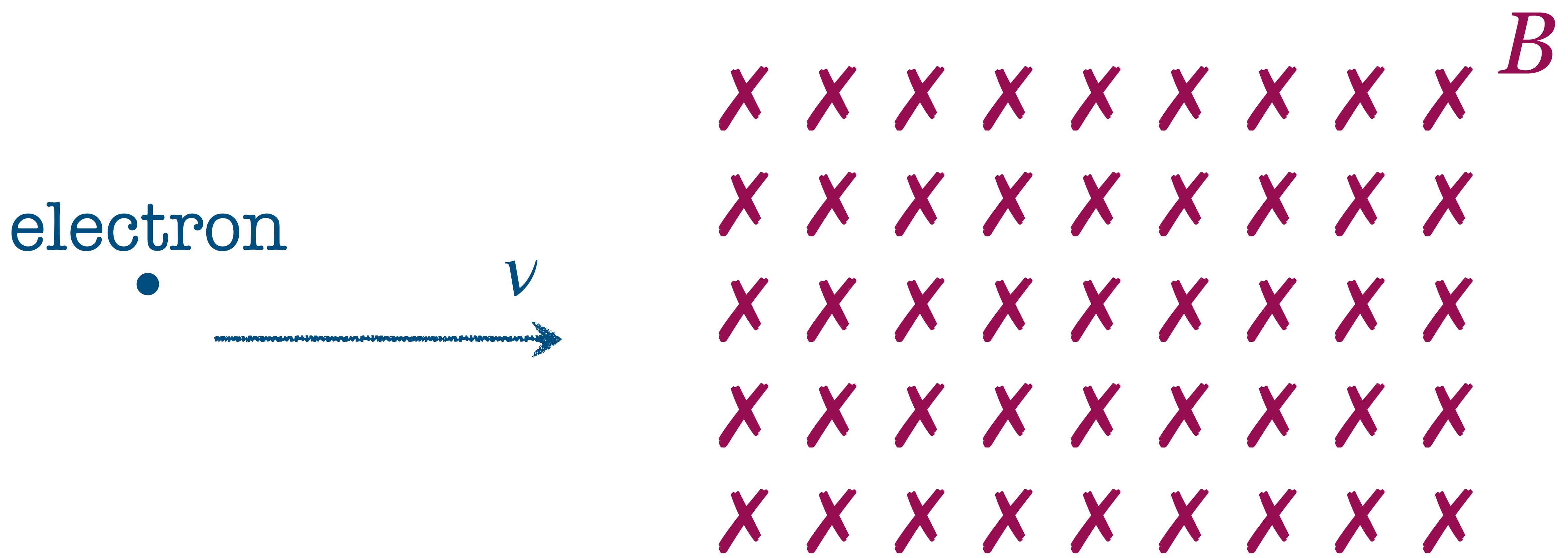
Force? **Force is out of the page ⊙**

Question

An electron moving with speed v = 1.5 x 10^4 m/s from left to right enters a region of space where a uniform magnetic field of magnitude 7.5 T exist everywhere into the page

What direction is the force on the electron?

1. Left
2. Right
3. Up
4. Down
5. Into the page
6. Out of the page



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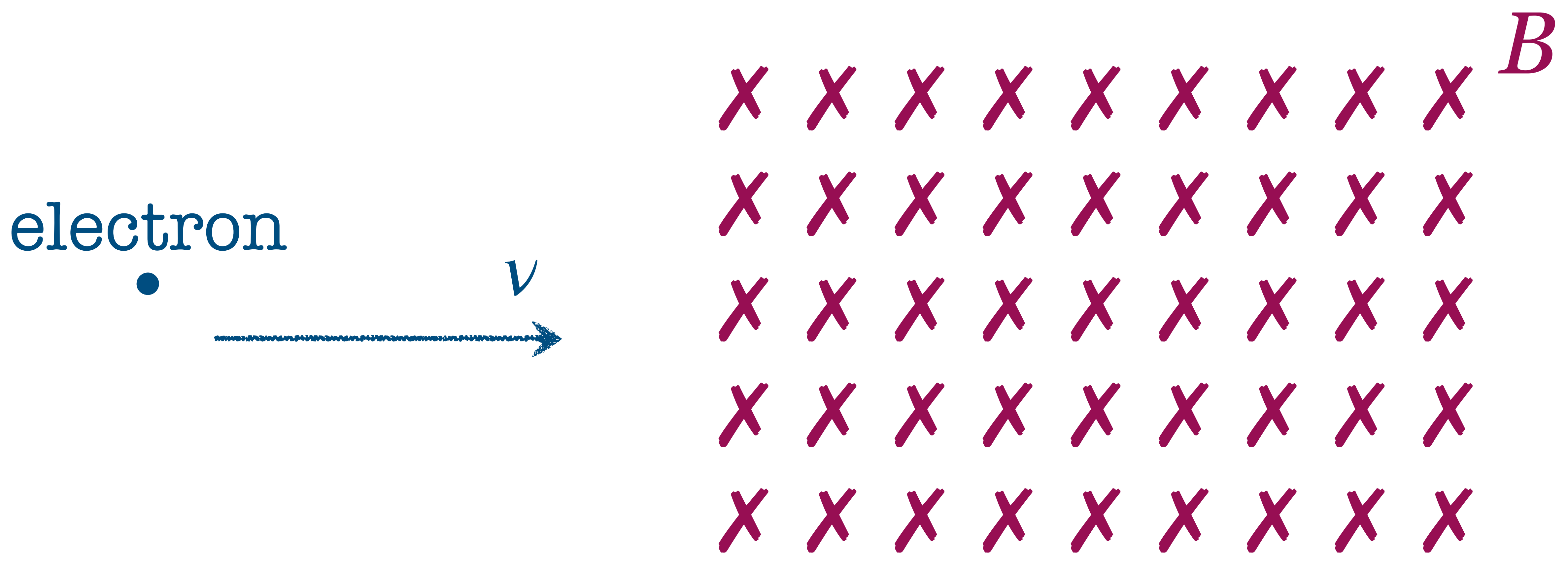
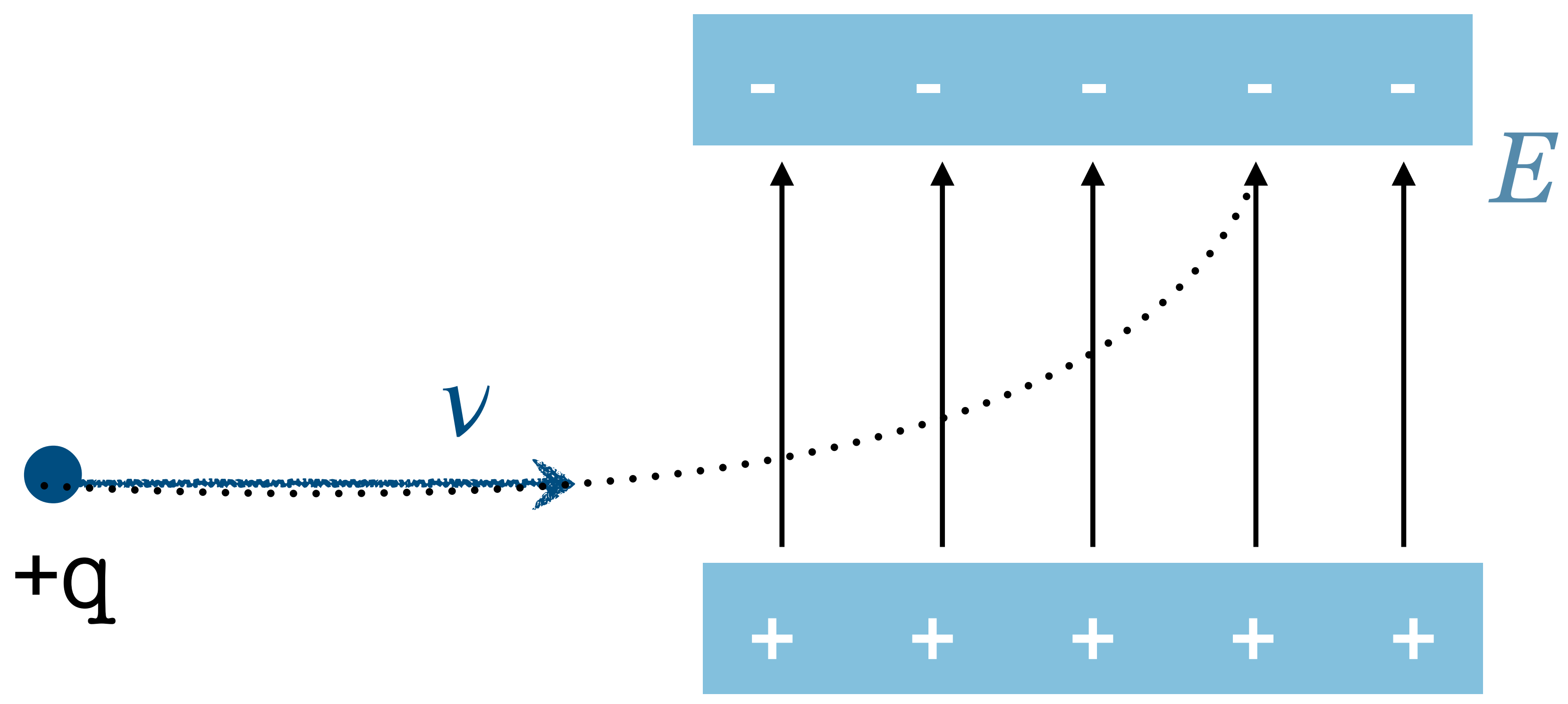


Motion of a Charge Particle in Electric and Magnetic Fields

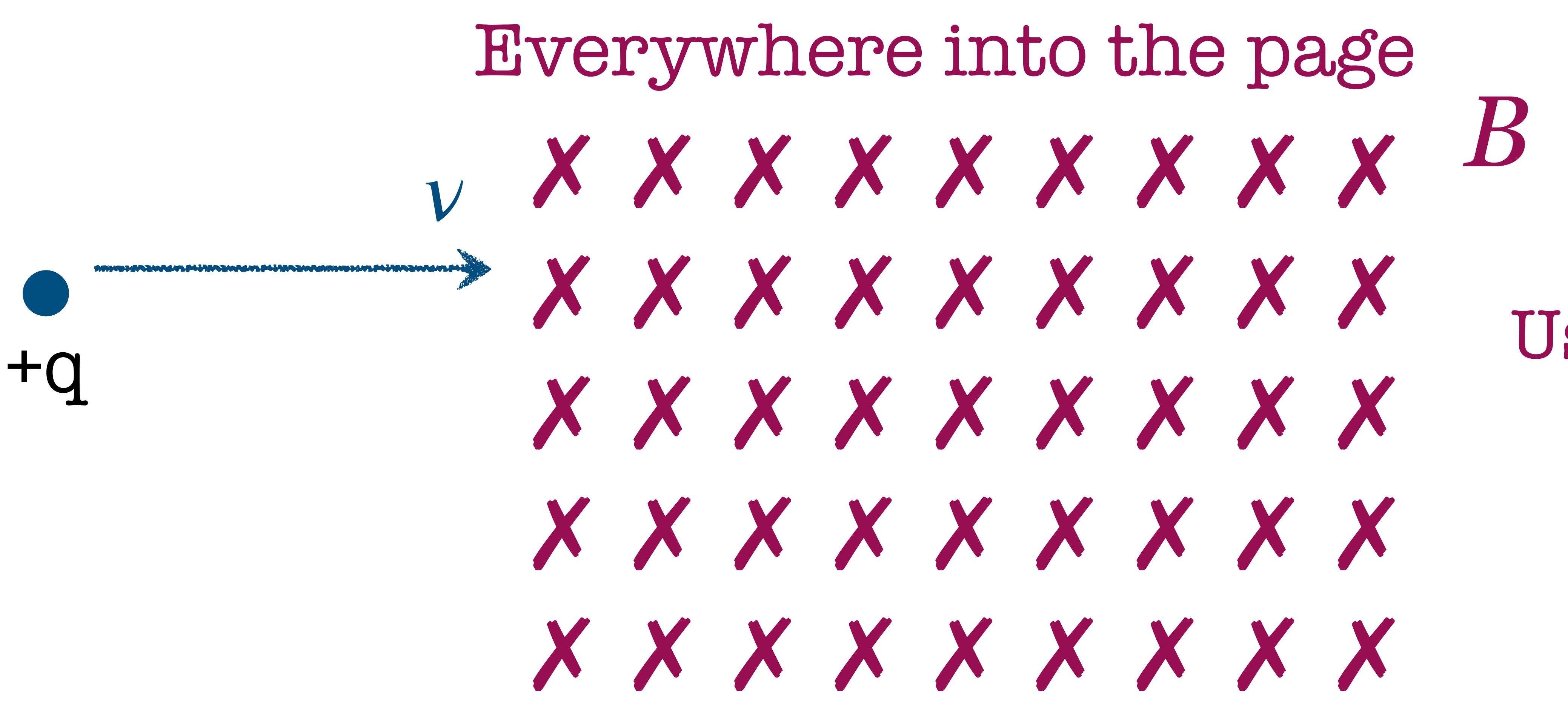
The force on a charged particle in an **electric field** is directed along the field, either parallel or antiparallel



$$F = qE$$

$$\vec{F} \parallel \vec{E}$$

The force on a charged particle in a **magnetic field** is always at right angles to the velocity and field,



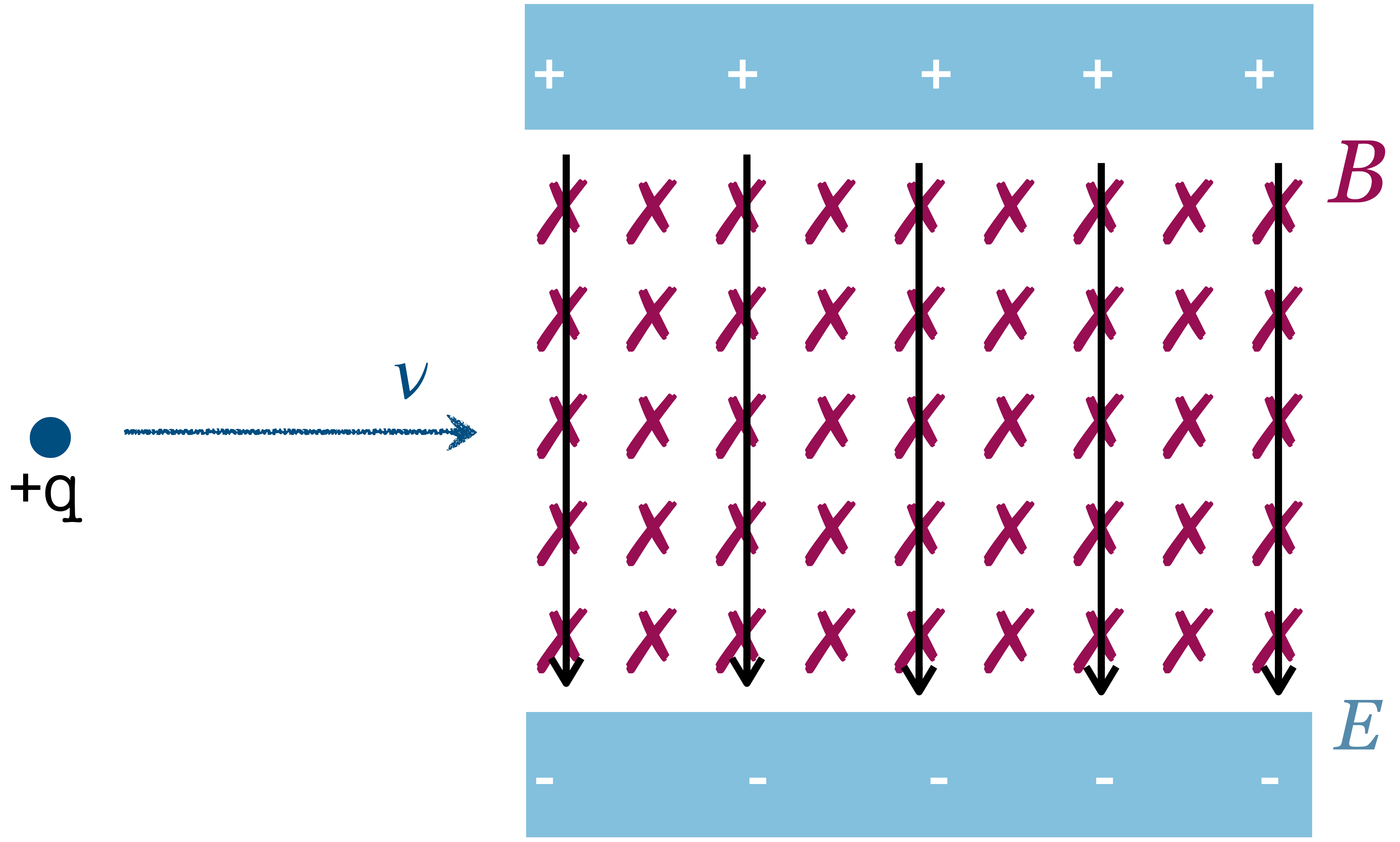
$$F = q\nu B \sin\theta$$

Use RHR-1 to show that the force on the particle is initially upward

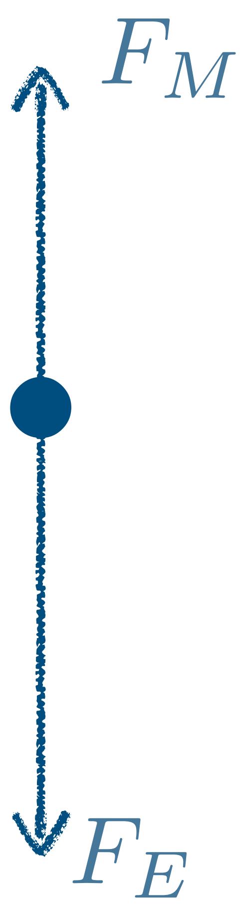
$\vec{F} \perp \vec{\nu}$ and $\vec{F} \perp \vec{B}$

➢ Let's use both fields at the same time

➢ Keep the magnetic field the same, but reverse the direction of the electric field



The force on the positive charge due to the electric field will now be down, and the force on the charge due to the magnetic field (RHR-1) will be up



By adjusting magnitude of E and B,
I can find a combination where $F_M = F_E$
such that net force on charge is zero ☛ charge moves through the fields with no deflection at all!

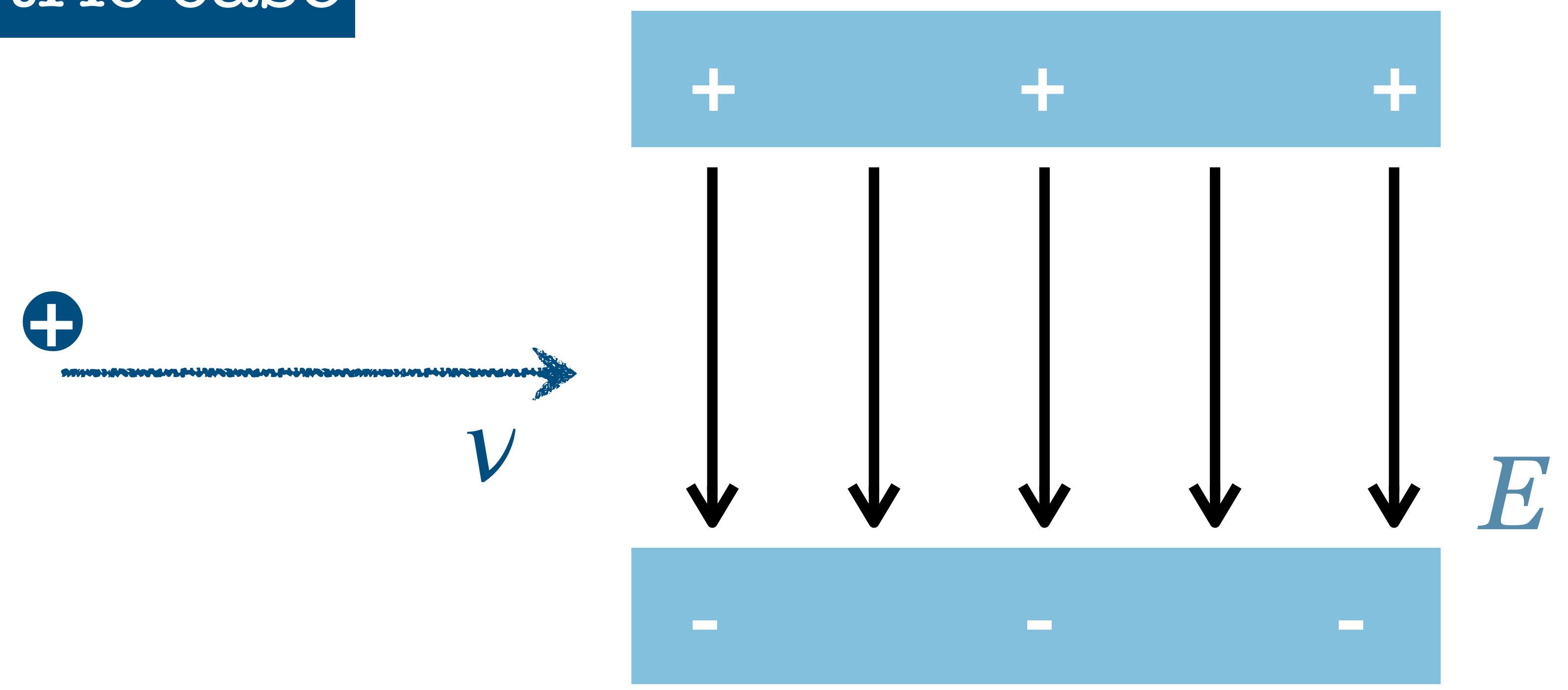
➢ This is call a **velocity selector**

$$F_M = F_E \Rightarrow \cancel{q}vB\cancel{\sin\theta}^{\,1} = \cancel{q}E \Rightarrow \boxed{v = E/B}$$

Work Done By Fields

$W = F \times d$ where F is **along direction of motion** and it is **constant** over displacement

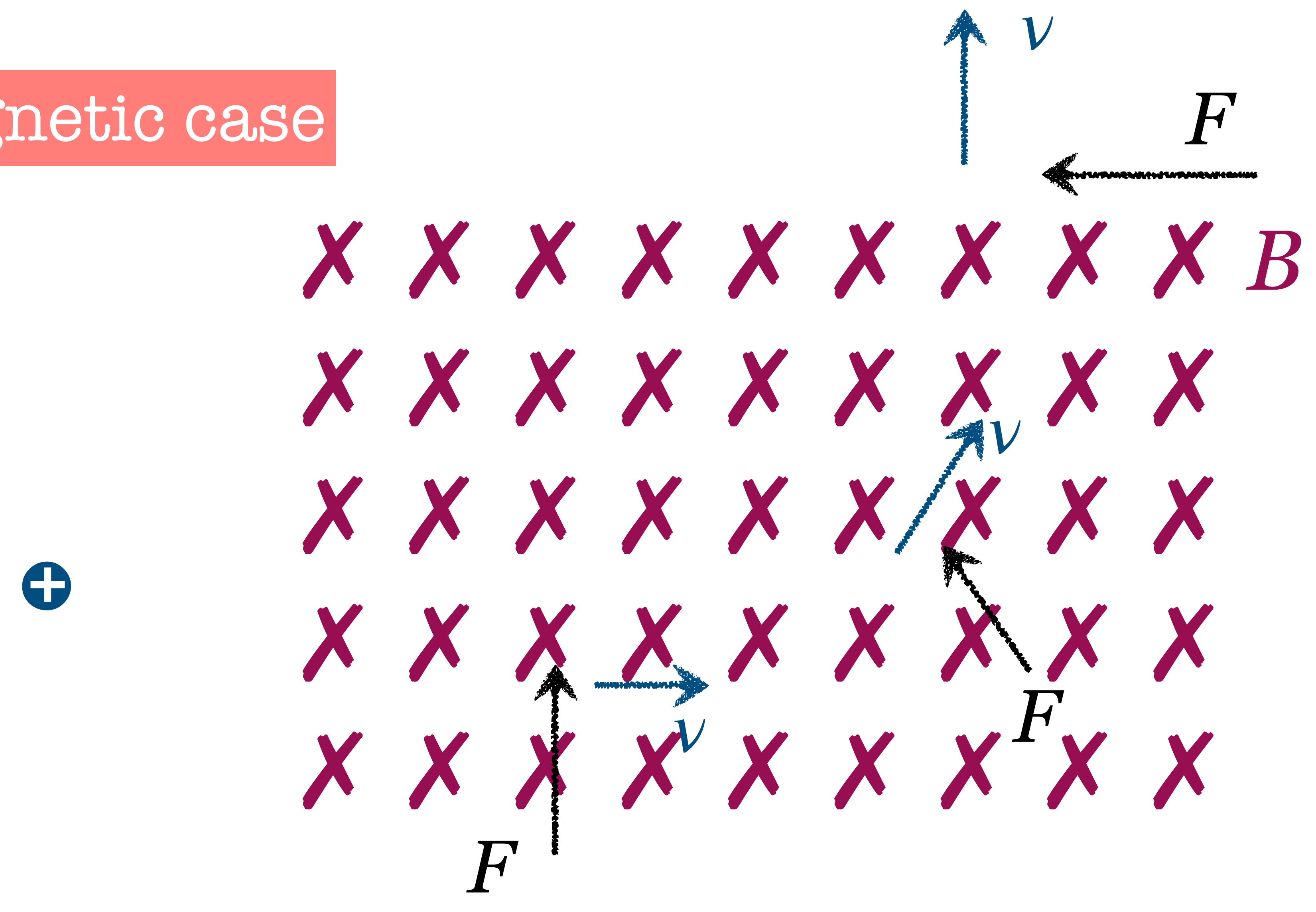
Electric case



- When positive charge enters field, force is downward
- Charge accelerates ☛ it is velocity increases

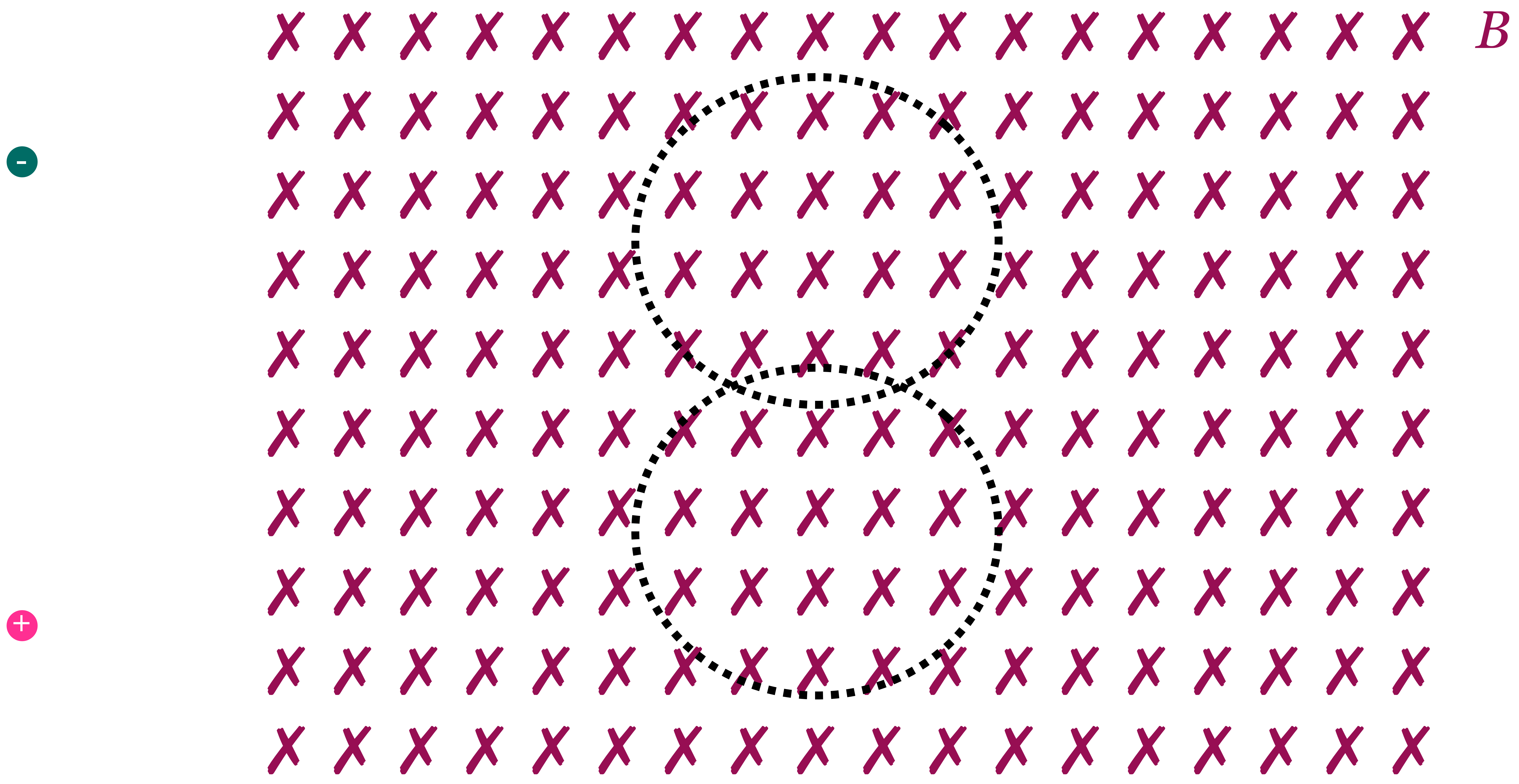
Thus, positive work is done on the charge!

Magnetic case



- When positive charge enters magnetic field, force is initially up
- This bends particle upward, but force changes direction ☛ it must always be perpendicular to v
- This force continues to bend particle around

➢ Keep applying RHR-1 ☛ particle just keeps bending around into a circular path!



➢ Force is always at right angles to velocity, so it is never along the direction of motion

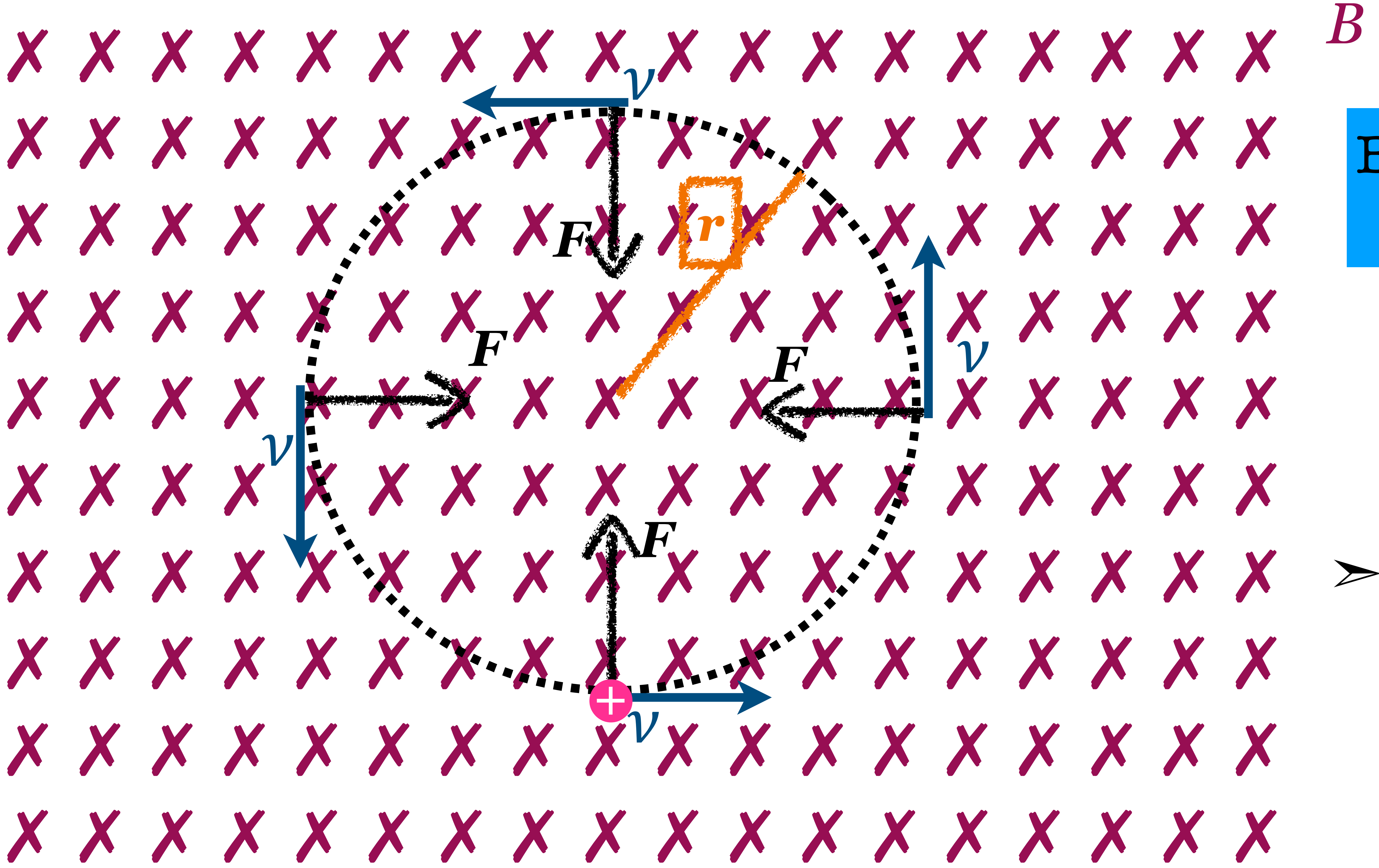
Thus , magnetic force does no work on particle

Lazy B Field!

➢Particle's speed remains constant, but it is direction changes!

Magnetic fields can not speed up or slow down cahrged particles, only change their direction

- Consider again our positively charged particle moving at right angles to a magnetic field
- Velocity is always tangent to particle's trajectory



By RHR-1 ☛ force is always perpendicular to v and directed in toward center of motion

- Whenever we have circular motion, we can identify a **Centripetal Force**

Remember Centripetal force is not a new force, but it is **vector sum of radial forces**

- Here, centripetal force is solely due to magnetic force, thus,

$$F_C = F_M \Rightarrow F_C = qvB\sin\theta \Rightarrow \frac{mv^2}{r} = qvB\sin\theta \Rightarrow r = \frac{mv}{qB}$$

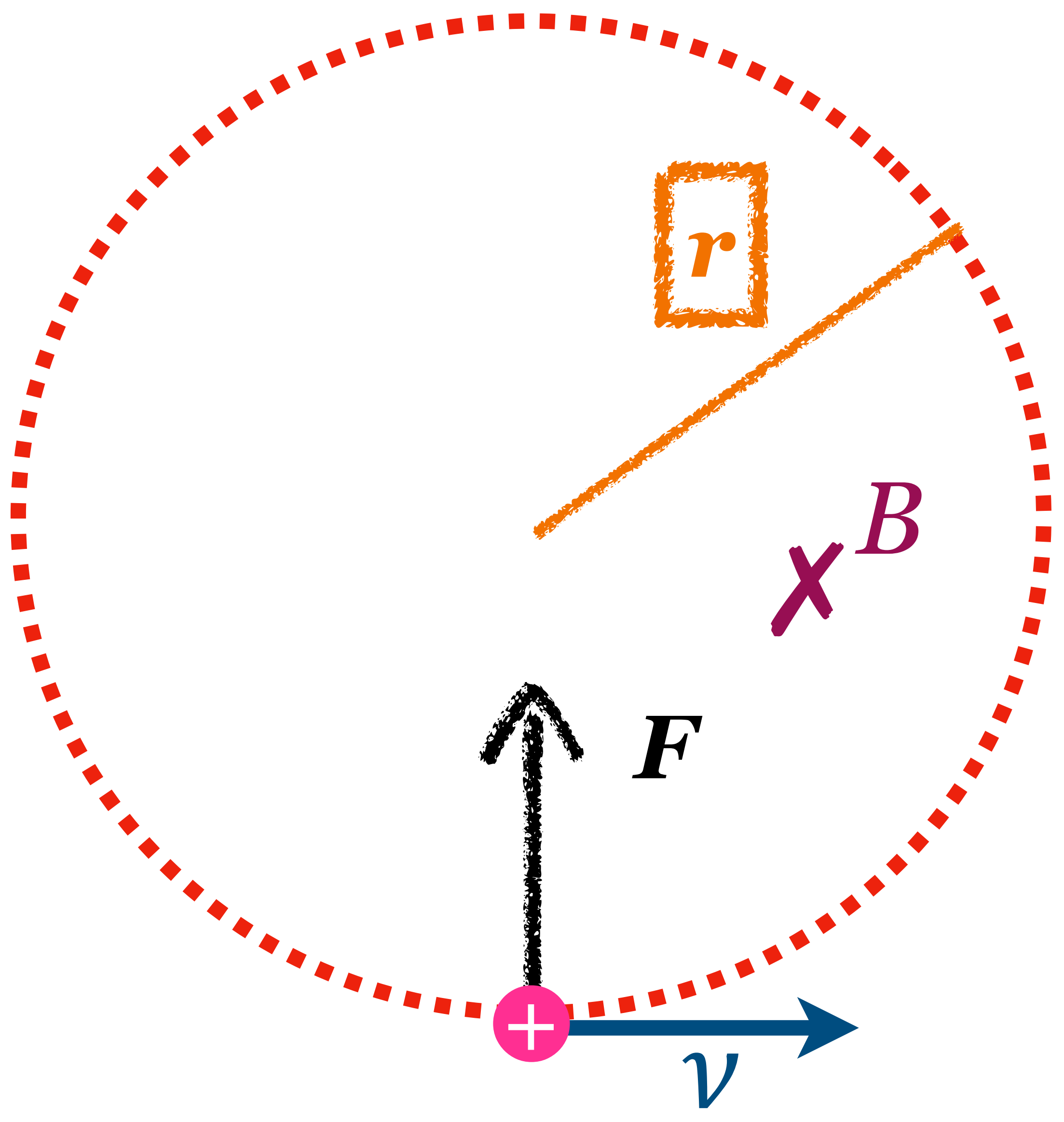
Thus, larger B is, the tighter the circular path (smaller r)

Example

- A beam of protons moves in a circle of radius 0.25m
- Proton moves perpendicular to a 0.3-T magnetic field

(a) What is the speed of each proton?

(b) Determine the magnitude of centripetal force that acts on each proton



(a) $$r = \frac{mv}{qB}$$

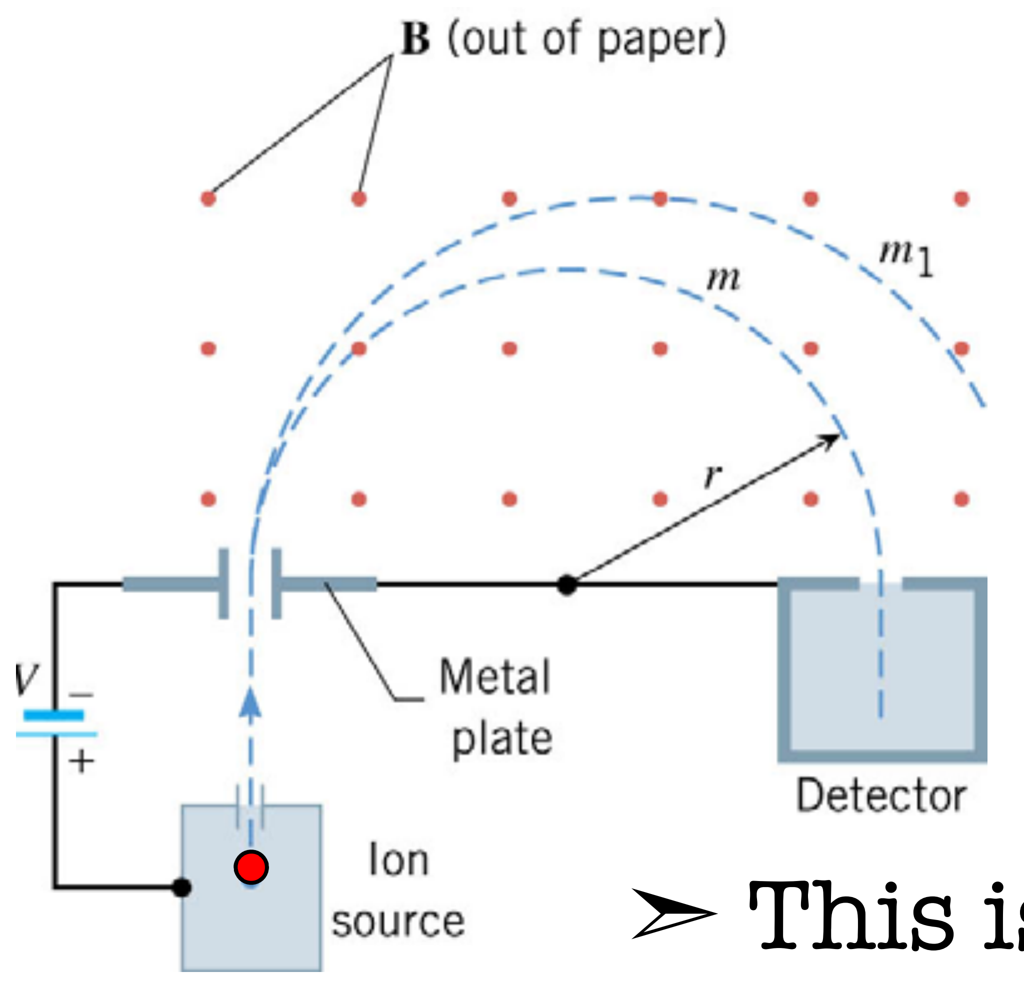
$$v = \frac{qBr}{m} = \frac{eBr}{m_p} = \frac{1.602 \times 10^{-19} C \cdot 0.30T \cdot 0.25m}{1.67 \times 10^{-27}\ kg} = 7.19 \times 10^{6} m/s$$

(b) $$F = qvB \sin\theta \qquad (\theta = 90°)$$

$$F = evB$$

Mass Spectrometer

- ➢ Ionized particles are accelerated by a potential difference V
- ➢ By conservation of energy, we know that this potential energy goes into kinetic energy of particle



$$\Delta K = \Delta U \Rightarrow \frac{1}{2}mv^2 = qV$$

- ➢ Solve this for speed, $v = \sqrt{\dfrac{2qV}{m}}$

- ➢ This is speed particle has when it enters magnetic field

- ➢ It then gets bent into a circular path whose radius is given by previous equation

$r = \dfrac{mv}{qB}$ rearrange $m = \dfrac{qrB}{v}$ Plug in v from here:

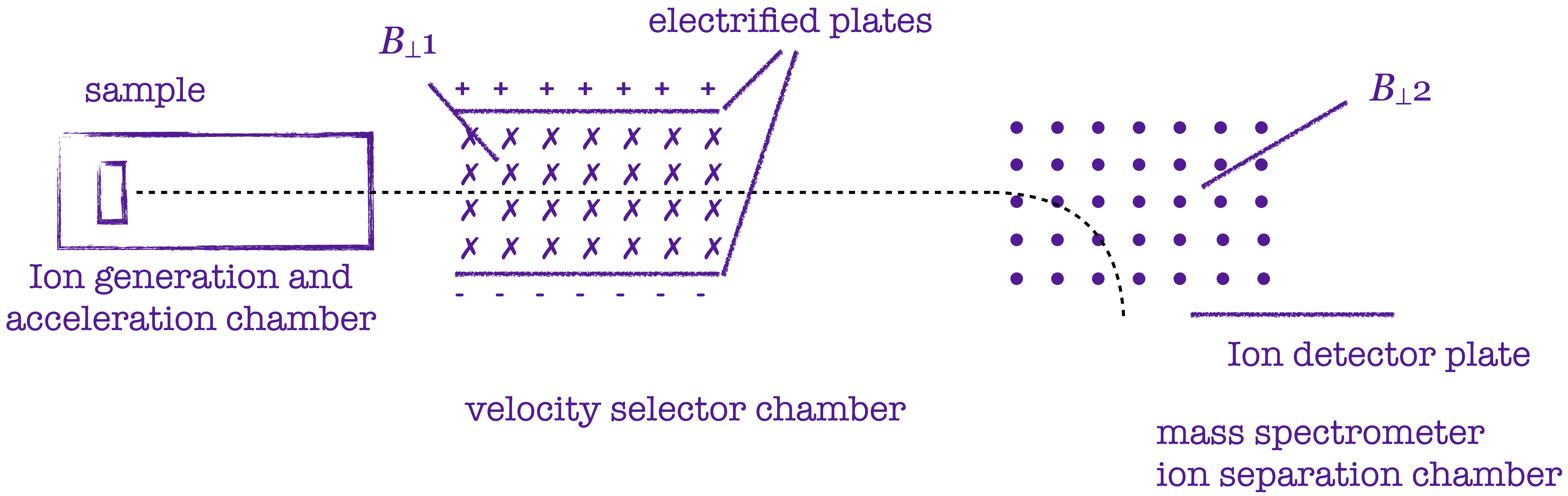
$$m = \left(\frac{qr^2}{2V}\right) B^2$$

- ➢ Thus, mass of deflected ion is proporcional to B^2
- ➢ By changing B, we can select a certain mass for a given radius

Manhattan Project

Mass spectrometers have three bascis parts

1. ion source and accelerator
2. a velocity selector
3. an ion separator



Manhattan Project

- Uranium isotopes, uranium-235 and uranium -239, can be separated using a mass spectrometer
- Uranium-235 isotope travels through a smaller circle and can be gathered at a different point than uranium-239
- During World War II, Manhattan projects was attempting to make an atomic bomb
- Uranium-235 is fissionable but it makes only 0.70% of uranium on Earth
- A large mass spectrometer at Oakridge, Tennessee was used to separate uranium-235 from raw uranium metal
- Uranium-235 and uranium-239 ion, each with a charge of +2 are directed into a velocity selector wich has a magnetic field of 0.250 T and an electric field of 1.25×10^{7} V/m perpendicular to each other
- Ions then pass into a magnetic field of 2.00 mT

What is the radius of deflection for each isotope?

- A charge of +2 means that each ion has two electrons

$$q = 2 \times +1.60 \times 10^{-19}\ C = +3.20 \times 10^{-19}\ C$$

- Since each proton and neutron has a mass of 1.67 x 10^{-27} kg, mass of each isotope is

$$m_{235} = 235 \times (1.67 \times 10^{-27}\ kg) = +3.9245 \times 10^{-25}\ kg$$

$$m_{239} = 239 \times (1.67 \times 10^{-27}\ kg) = +3.9913 \times 10^{-25}\ kg$$

- For velocity selector

$$F_E = F_M$$

$$q\left|\vec{E}\right| = qvB_{\perp 1}$$

$$v = \frac{\left|\vec{E}\right|}{B_{\perp 1}} = \frac{1.25 \times 10^{7}\ V/m}{0.250 T} = 5.00 \times 10^{7}\ m/s$$

- For ion separator

$$F_M = ma$$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB_{\perp}}$$

uranium -235

$$r_{235} = \frac{(3.9245 \times 10^{-25} kg)(5.00 \times 10^{7}\ m/s)}{(3.20 \times 10^{-19}\ C)\ (2.00 \times 10^{-3} T)}$$

$$r_{235} = \mathbf{3.07 \times 10^{4}\ m}$$

uranium -239

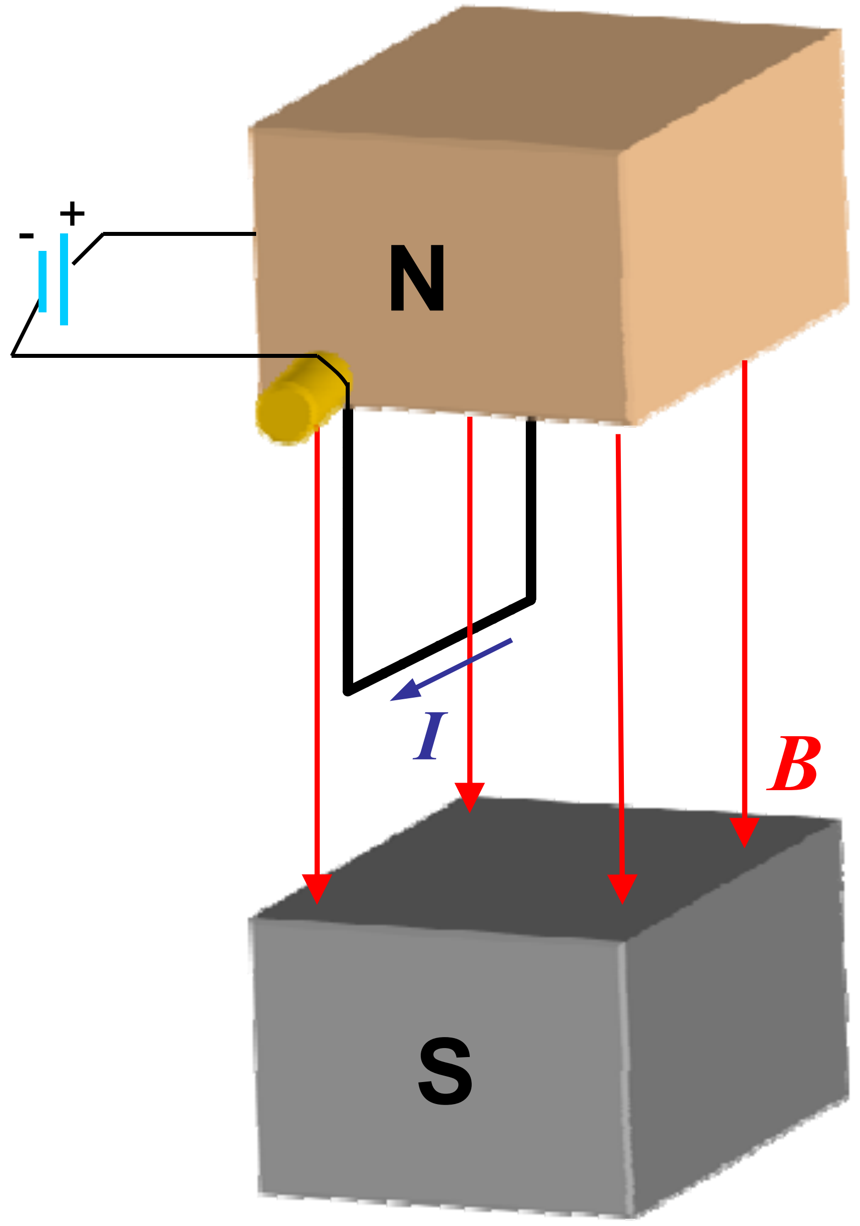
$$r_{239} = \frac{(3.9913 \times 10^{-25} kg)(5.00 \times 10^{7}\ m/s)}{(3.20 \times 10^{-19}\ C)\ (2.00 \times 10^{-3} T)}$$

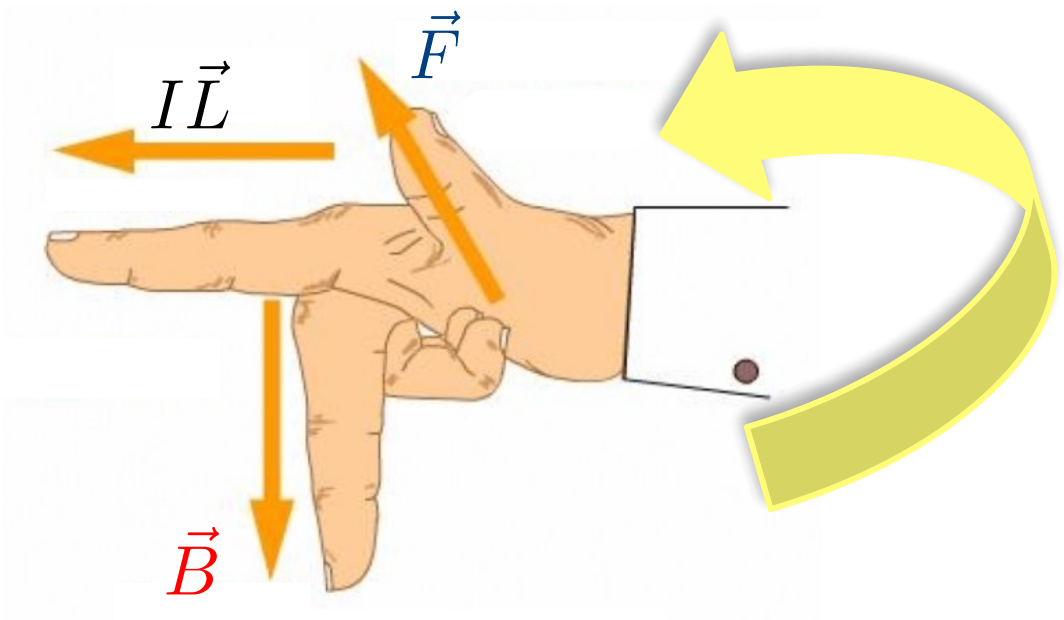
$$r_{239} = \mathbf{3.12 \times 10^{4}\ m}$$

Force on a Current

- Moving charges in a magnetic field experience a force
- A current is just a collection of moving charges, so a current will also feel a force in a magnetic field

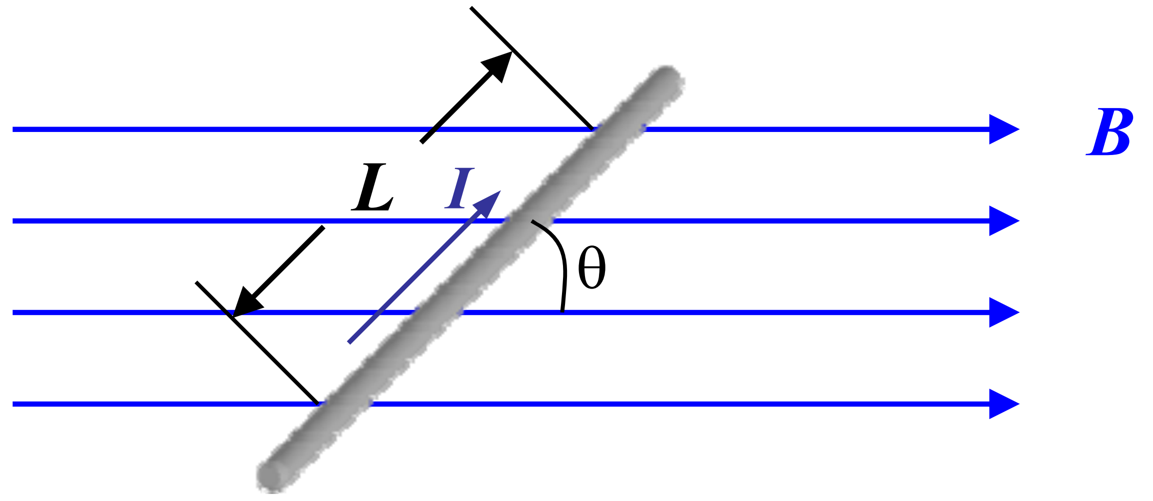
RHR-1 is used to find the direction of the force on a charge moving in a magnetic field, or to find the direction of the force on a current carrying wire in a magnetic field





$$F = ILB \sin\theta$$

- Notice that θ is the angle between the current and the magnetic field
- **Force is maximum when field is perpendicular to the wire!**



- I is the current and L is the length of the wire that is in the field

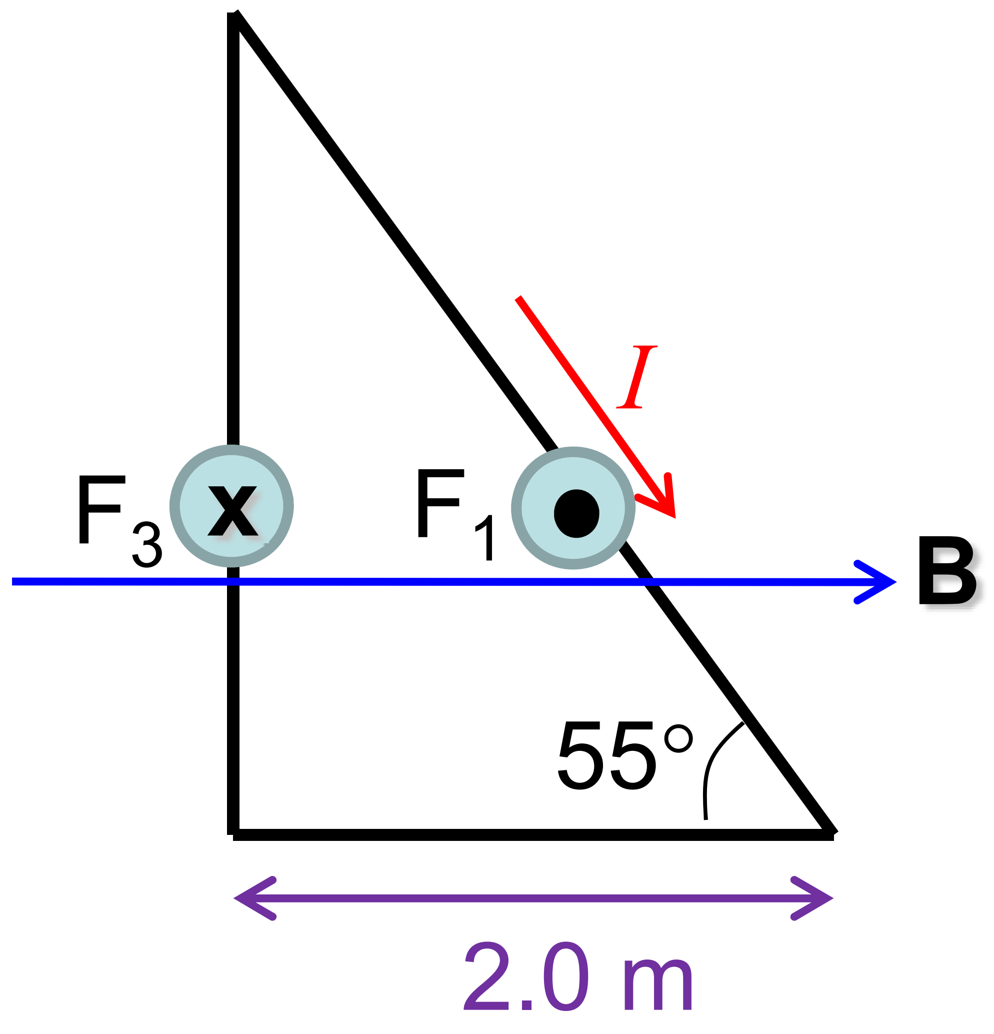
Direction of force? **Into the screen! ⊗**

Example

* A current (I = 5A) runs through a triangular loop and place in a uniform B-field (B = 2T)

(a) Find the force acting on each side of triangle

(b) Determine the net force



(a)

$$F = ILB\sin\theta$$

Magnetic forces act on two side only: L_1 and L_3

$$F_3 = IL_3B\sin 90° = 5A \cdot 2.0m\tan 55° \cdot 2T = 28.56N$$

$$F_1 = IL_1B\sin 55° = 5A \cdot \frac{2.0m}{\cos 55°} \cdot 2T\sin 55° = 28.56N$$

(b) Since $\vec{F}_1 = -\vec{F}_3; \quad \sum \vec{F} = 0$

Magnetic Fields Produced by Currents

- Moving charges experience a force in magnetic fields $F = qvB\sin\theta$
- Current also feel a force in a magnetic field $F = ILB\sin\theta$
- Until 1820 everyone thought electricity and magnetism were completely separate entities
- Then Hans Christian Oersted discovered the following

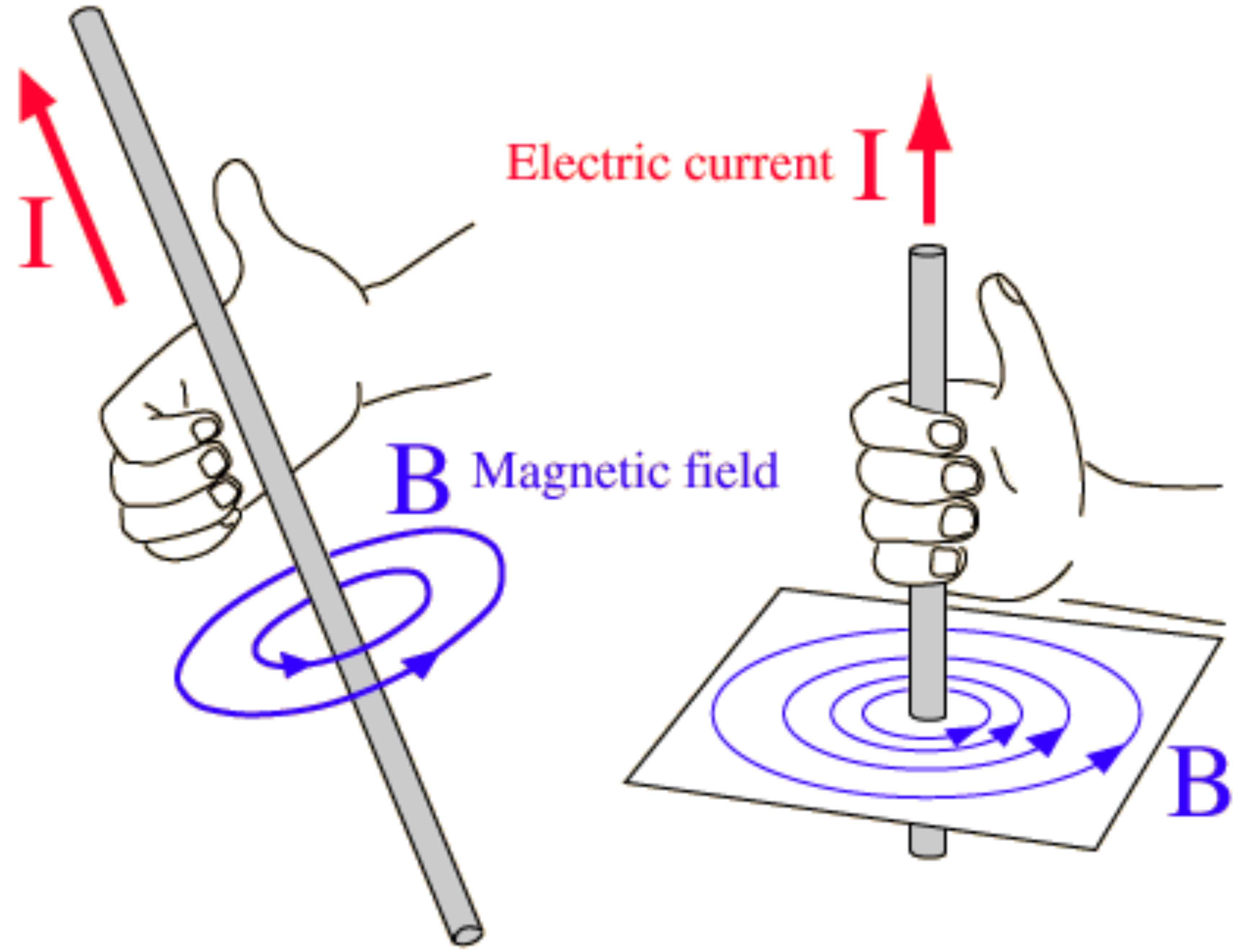
Electric currents create magnetic fields!

A more general statement is that moving charges create magnetic fields

Stationary charges create Electric Fields
Moving charges (constant v) create Magnetic Fields

This discovery helped create the field of **Electromagnetism**

➢ We determine the direction of magnetic field around a long current carrying wire by using **RHR-2**

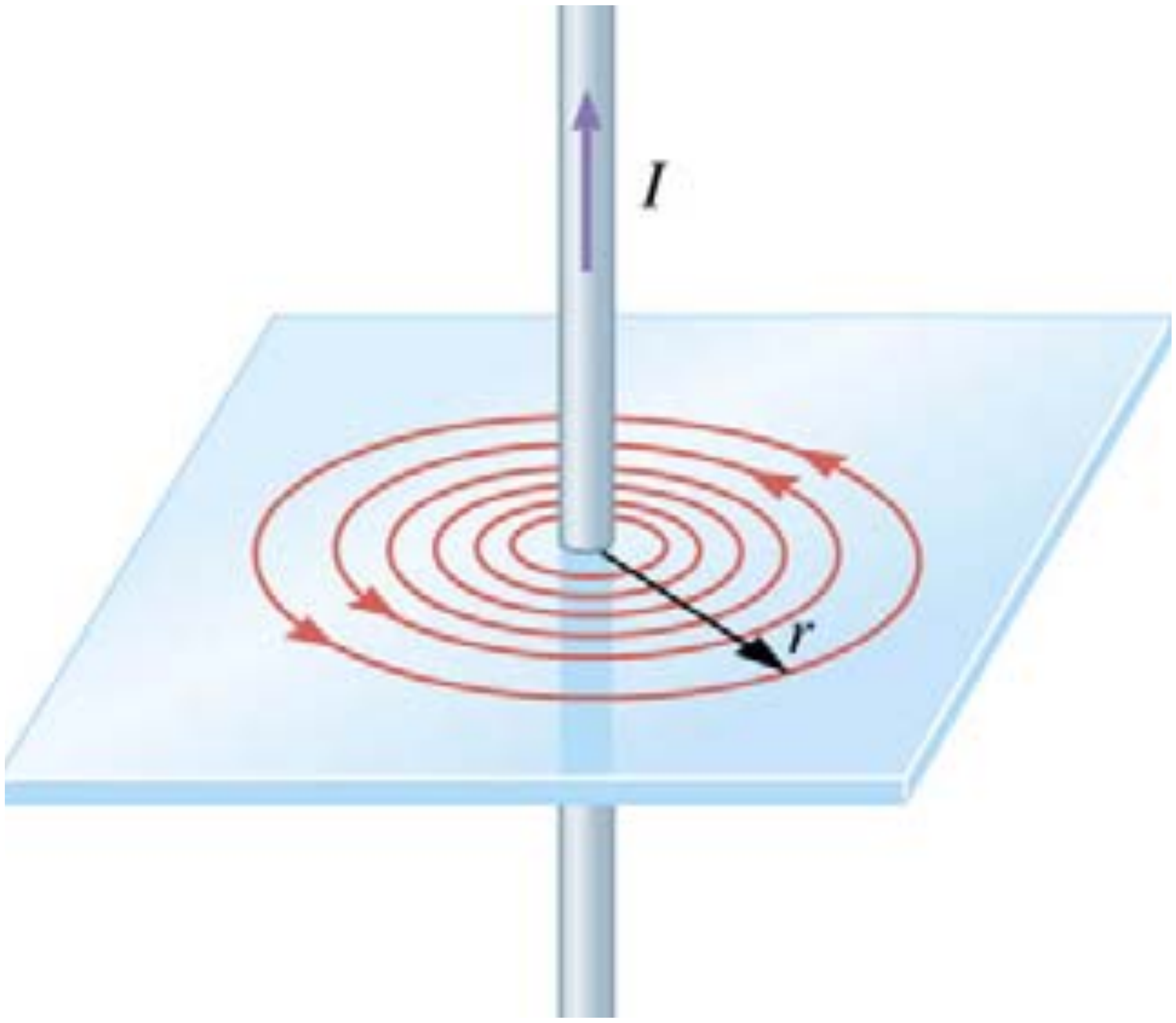


RHR-2: Point the thumb of your right hand in the direction of the current, and your fingers curl around the wire showing the direction of the field lines

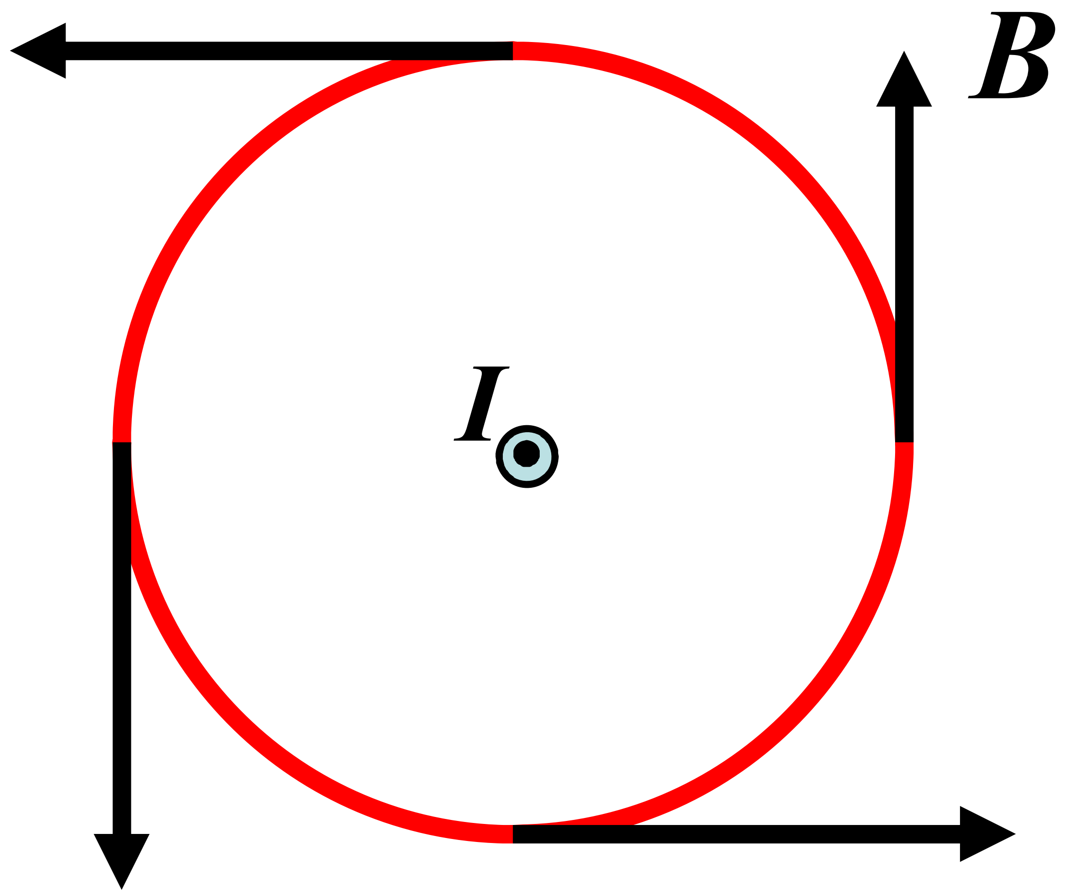
- What do the magnetic field lines look like around along, straight, current-carrying wire?

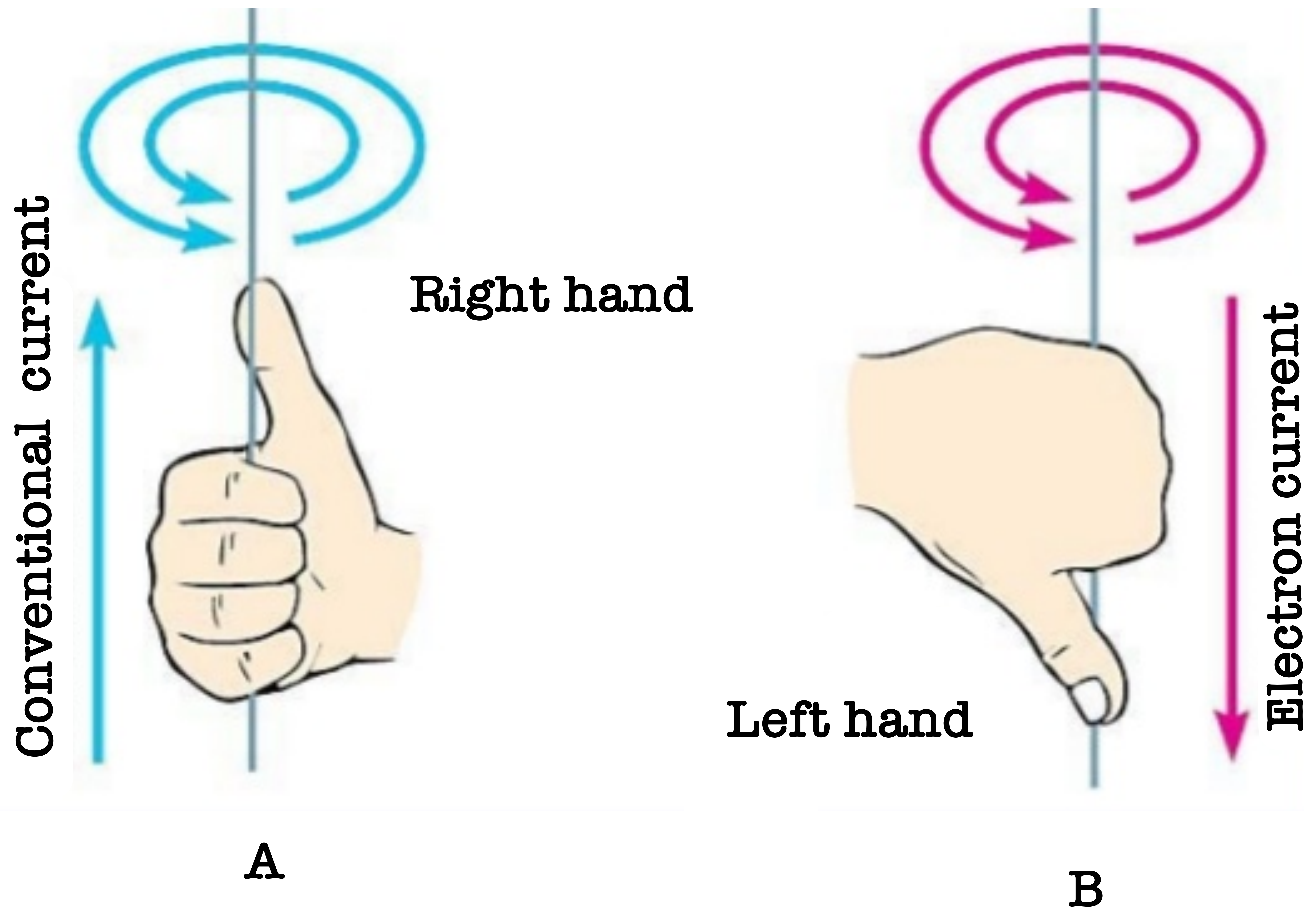
The current produces concentric circular loops of magnetic field around the wire

Remember Magnetic field vector at any point is always tangent to the field line!



The current I is coming out at you





Use (A) a right-hand rule of thumb to determine the direction of magnetic field around a convencional current and (B) a left-hand rule of thumb to determine the direction of a magnetic field around an electron current

Ampère's Law

➢ Consider any (arbitrary) closed path around current

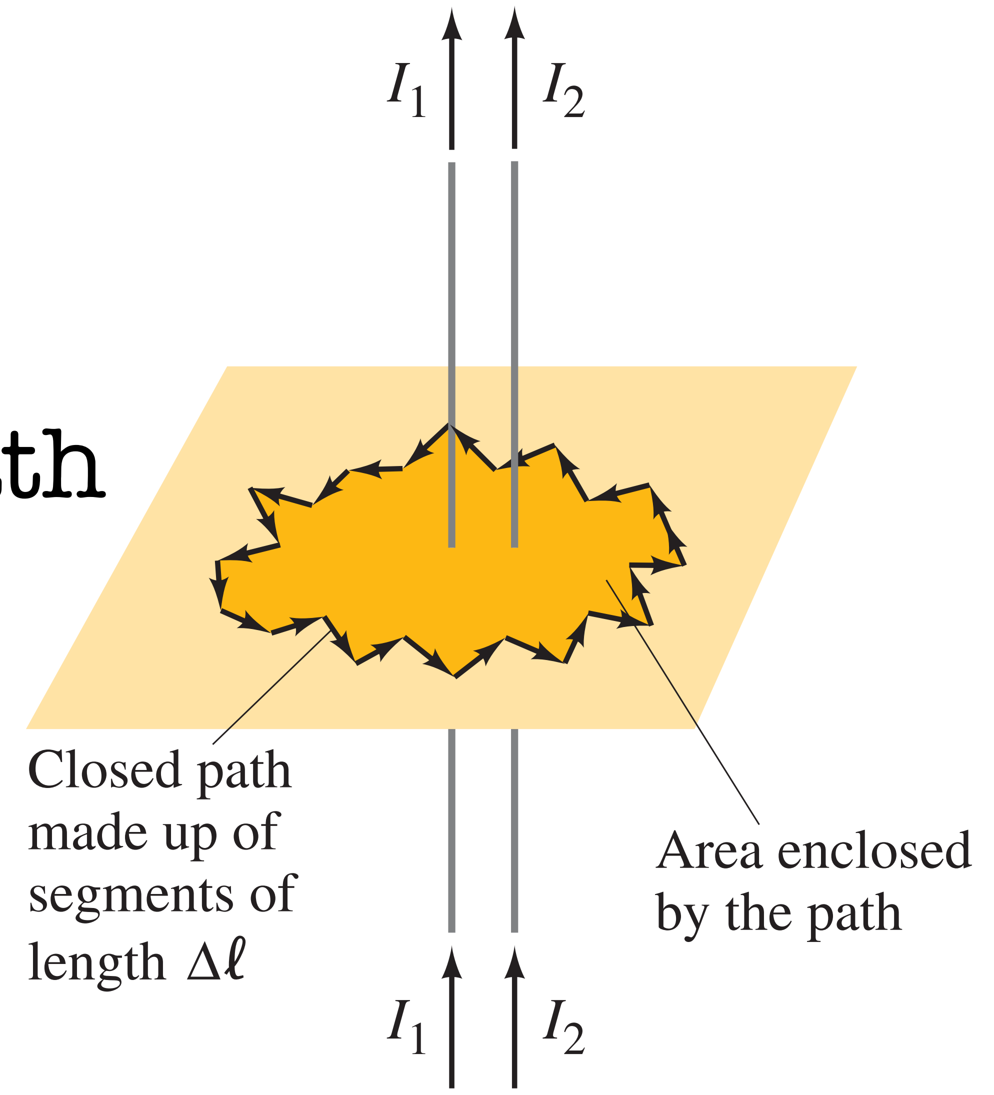
➢ Imagine path being made up of short segments each of length $\Delta\ell$

➢ We take product of length of each segment times the component of magnetic field parallel to that segment

➢ If we now sum all these terms ☛ result equals:

μ_0 times the net current I_{encl} that passes through the surface **enclosed** by path

$$\sum_{\substack{\text{closed}\\\text{path}}} B_{\parallel}\,\Delta\ell = \mu_0 I_{\text{encl}}$$



lengths Δl are chosen small enough so that $B_{\parallel}$ is essentially constant along each length

μ_0 is the permeability of free space ☛ $\mu_o = 4\pi\times10^{-7}\ \dfrac{\text{T}\cdot\text{m}}{\text{A}}$

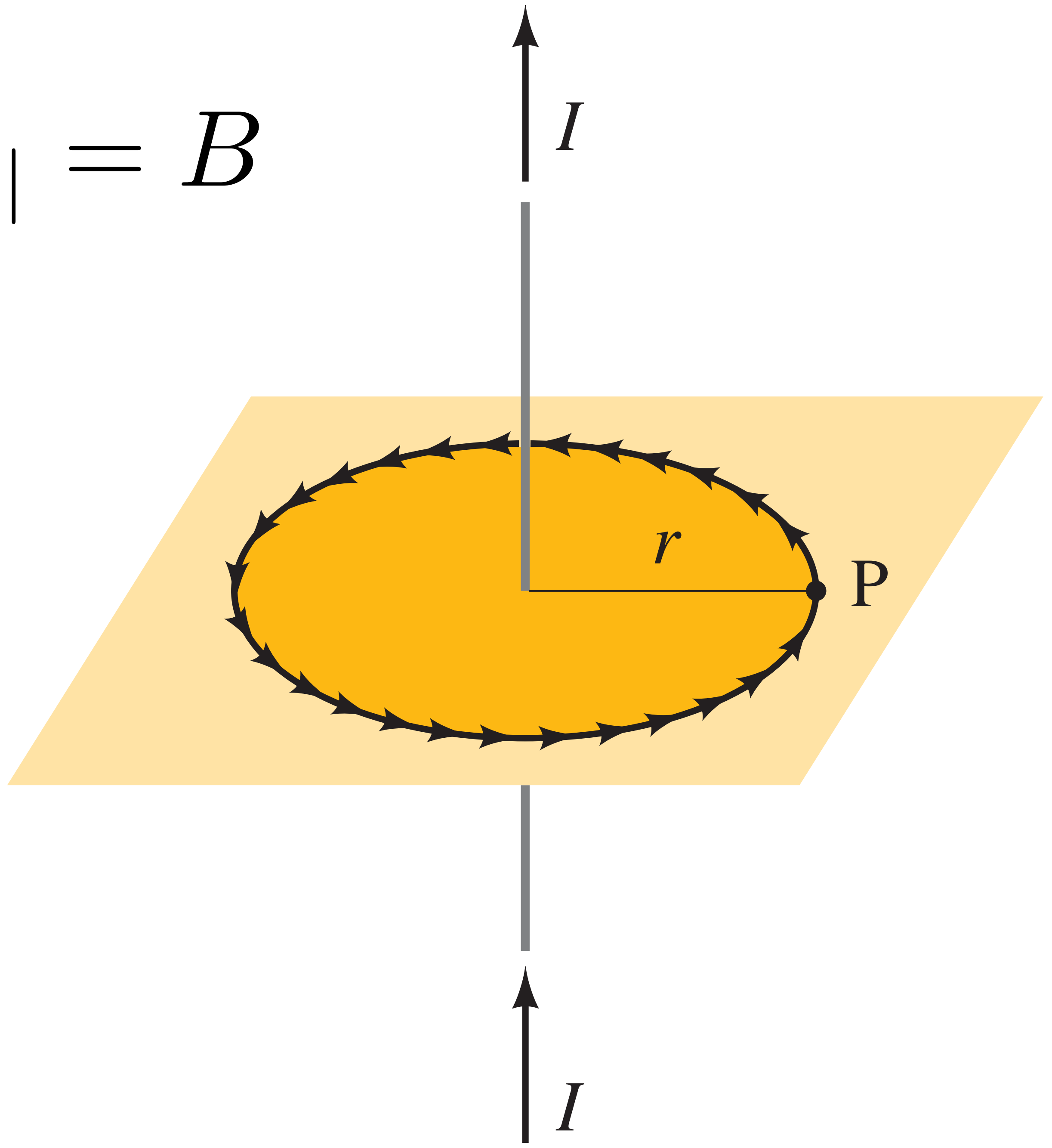
Field Due to a Straight Wire

$$\sum_{\substack{\text{closed}\\ \text{path}}} B_{\|}\ \Delta\ell = \mu_0 I_{\text{encl}}$$

As path to be used we choose circle of radius r ☛ because at any point on this path $\vec{B}$ will be tangent to circle

For any short segment of the circle ☛ $\vec{B}$ will be parallel to that segment so $B_{\|} = B$

Assume we break the circular path down into 100 segments



$$(B\ \Delta\ell)_1 + (B\ \Delta\ell)_2 + (B\ \Delta\ell)_3 + \cdots + (B\ \Delta\ell)_{100} = \mu_0 I_{\text{encl}}$$

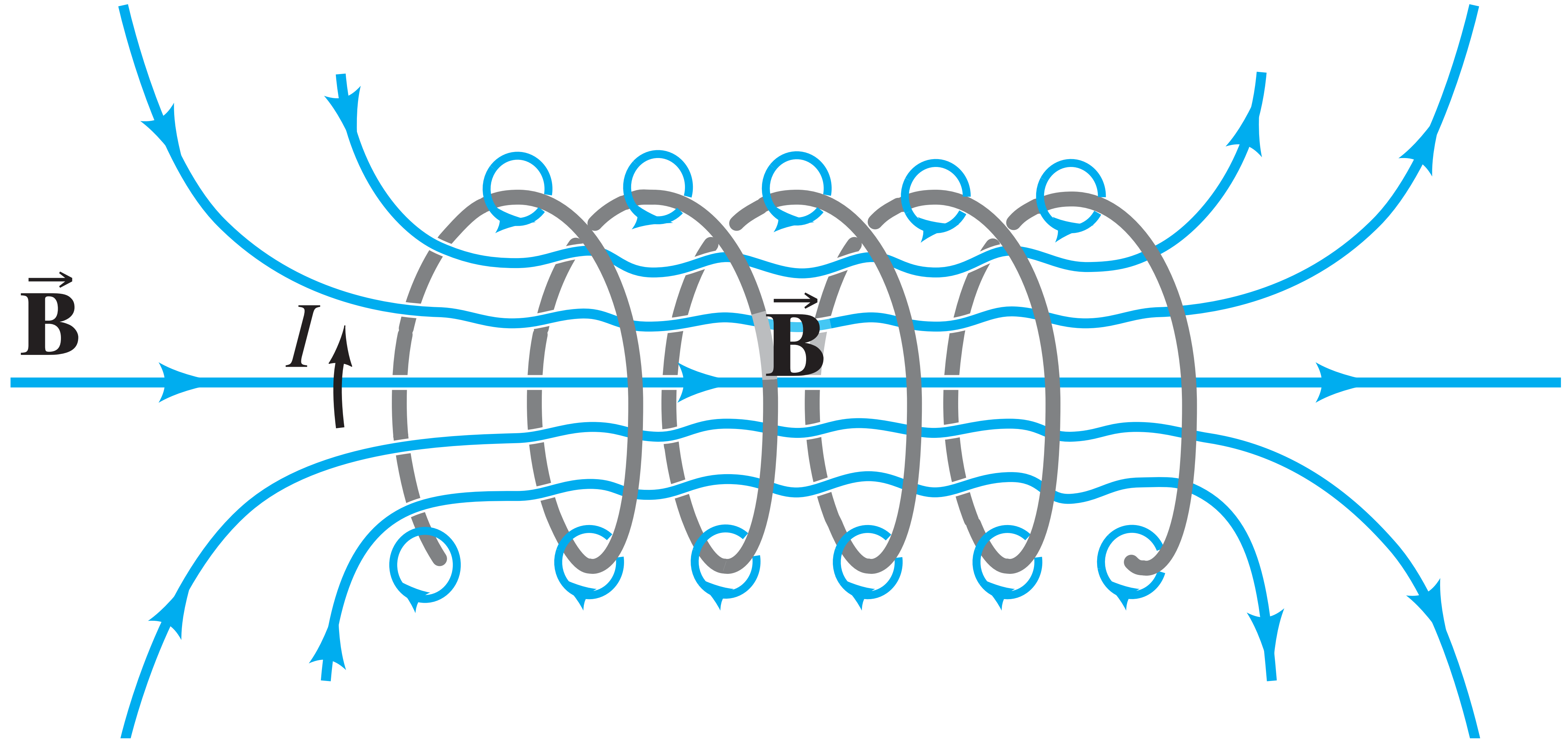
$$B(\Delta\ell_1 + \Delta\ell_2 + \Delta\ell_3 + \cdots + \Delta\ell_{100}) = \mu_0 I_{\text{encl}}$$

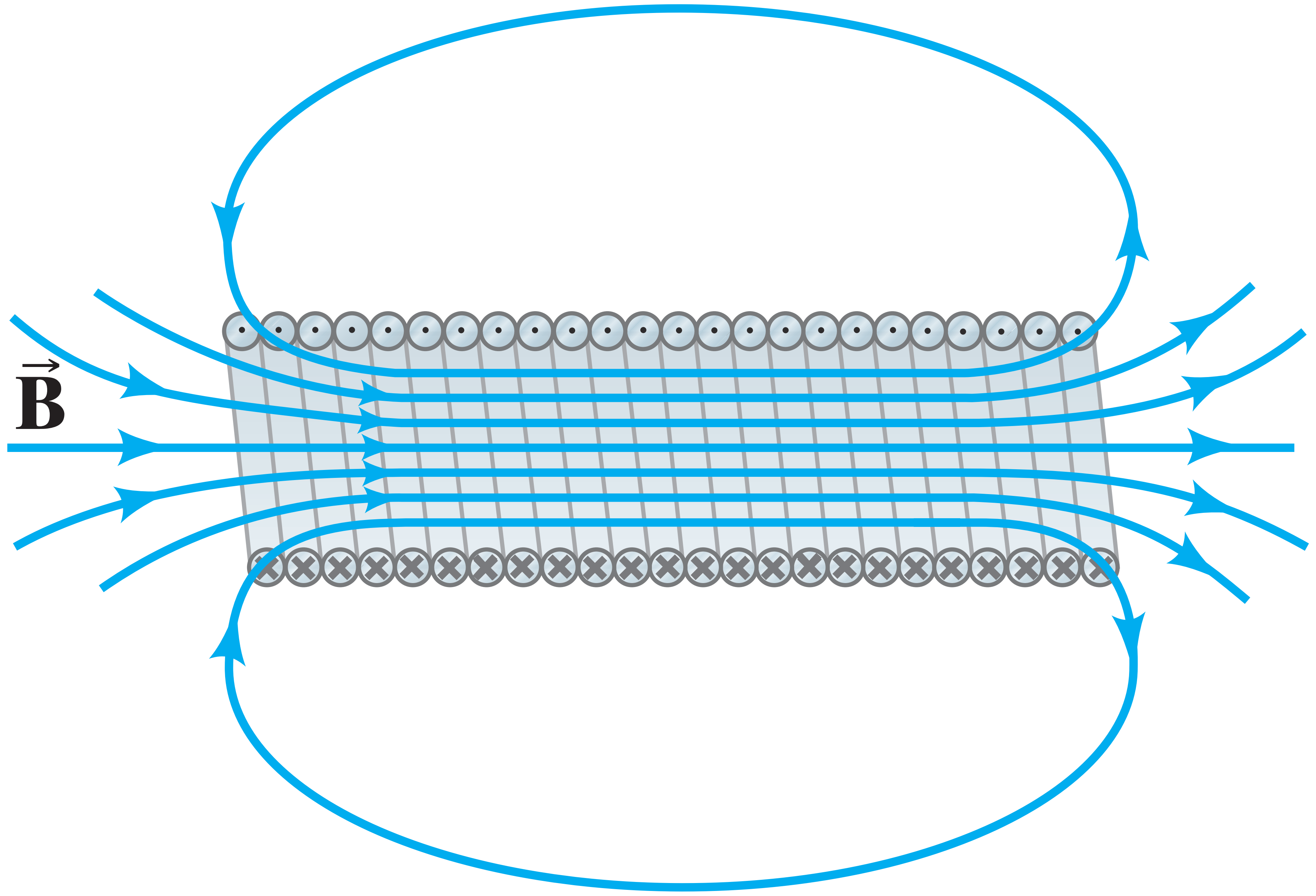
$$B \cdot 2\pi r = \mu_0 I_{\text{encl}}$$

$$B = \frac{\mu_0 I_{\text{encl}}}{2\pi r}$$

Field Inside a Solenoid

➢ Solenoid ☛ long coil of wire with many loops or turns





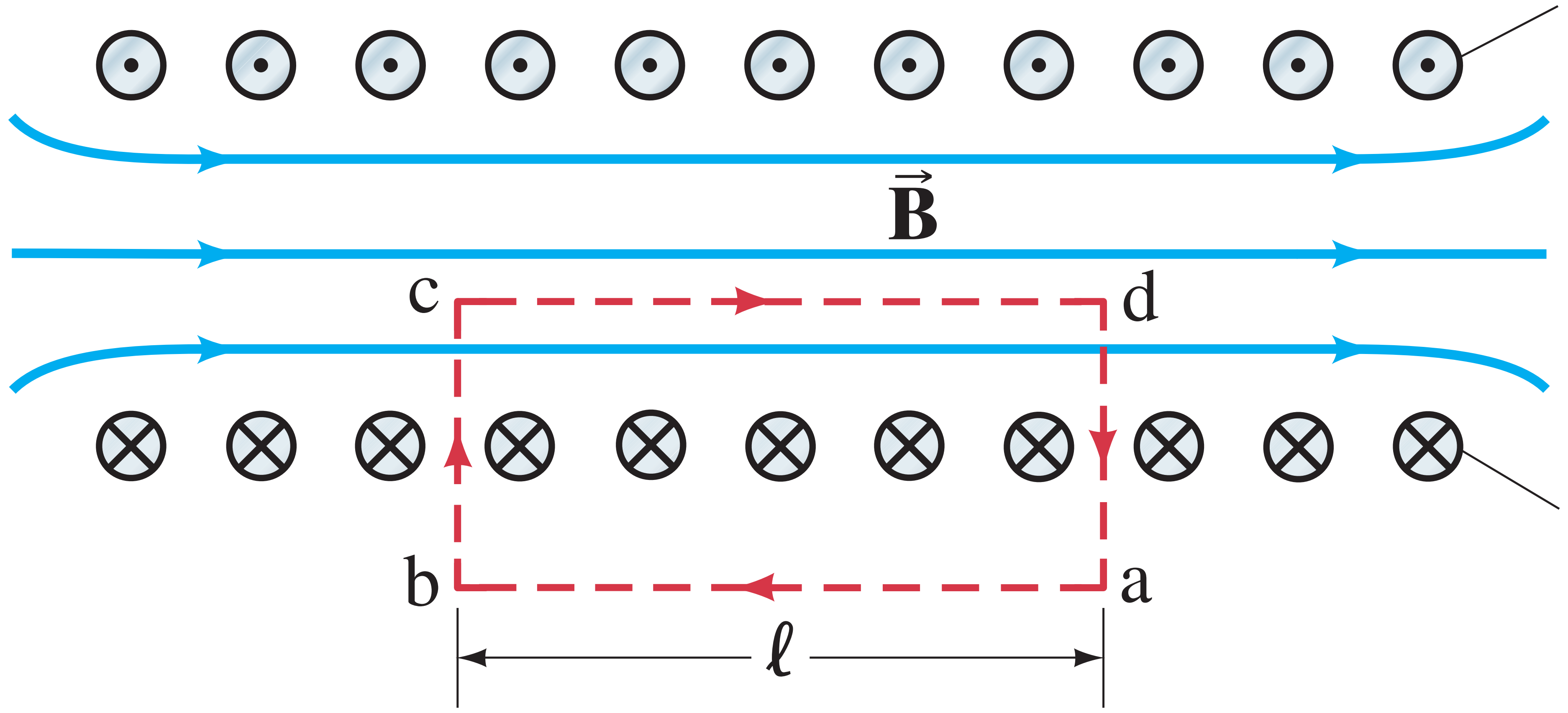
➢ Total field inside solenoid ☛ sum of fields due to each current loop

➢ If solenoid has many loops and they are close together

field inside will be nearly uniform and parallel to solenoid axis except at ends

Field Inside a Solenoid

➢ Close path made up of four straight segments ☛ sides of the rectangle: ab, bc, cd, da



$$(B_{\parallel}\ \Delta\ell)_{ab} + (B_{\parallel}\ \Delta\ell)_{bc} + (B_{\parallel}\ \Delta\ell)_{cd} + (B_{\parallel}\ \Delta\ell)_{da} = \mu_0 I_{\text{encl}}$$

➢ If current I flows in wire of solenoid ☛ total current enclosed by our path abcd is NI

➢ N is number of loops (or turns) Amperian path encircles

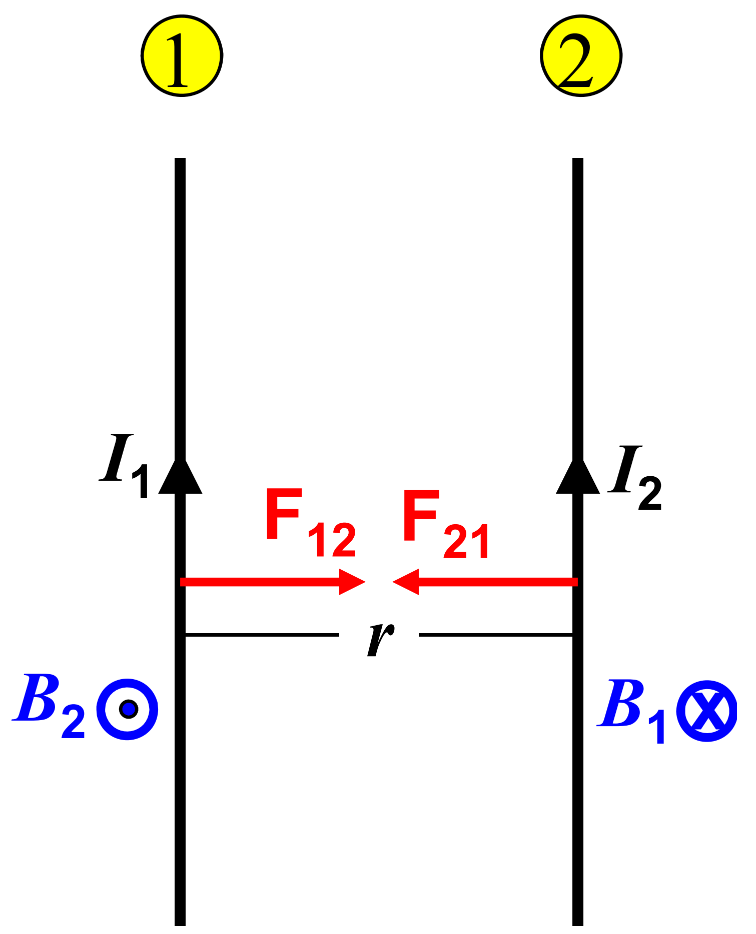
➢ First term in sum ☛ nearly zero because field outside solenoid is negligible compared to field inside

➢ $\vec{B}$ is perpendicular to segments bc and da ☛ these terms are zero too

$$(B_{\parallel}\ \Delta\ell)_{cd} = B\ell \qquad ☛ \qquad B\ell = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{\ell}$$

Magnetic Force between Wires

- So electrical currents create magnetic fields of their own
- This fields can affect the motion of other moving charges or currents
- As an example, let's look at two long parallel wires each carrying a current in the **same direction**



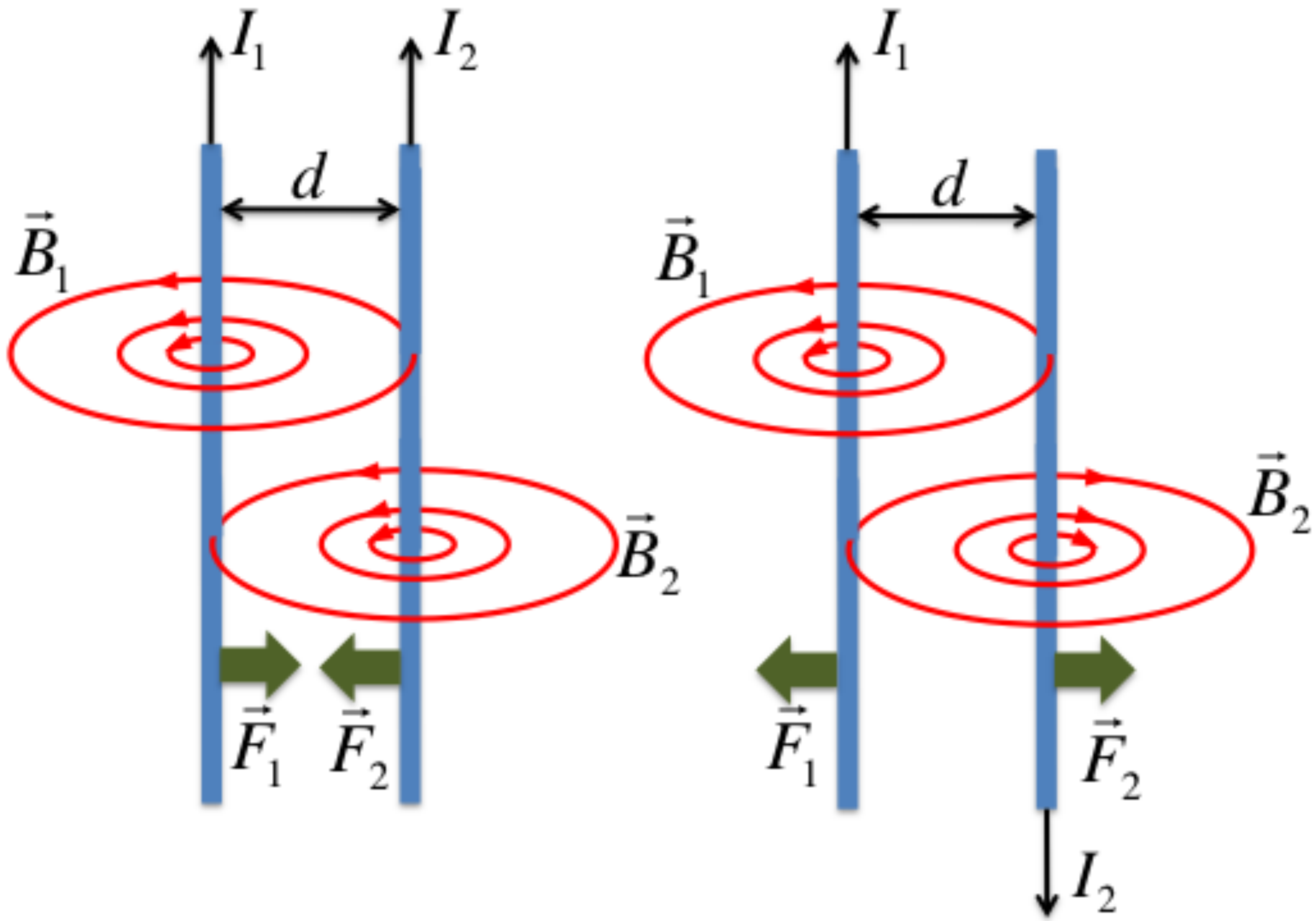
  - **Wire 1 creates a magnetic field that affects wire 2**
  - **Wire 2 creates a magnetic field that affects wire 1**
- Thus, there will be a force on each wire due to magnetic field that other produces

$$F_{12} = I_1 L B_2 \sin\theta_{12}$$
$$F_{21} = I_2 L B_1 \sin\theta_{21}$$

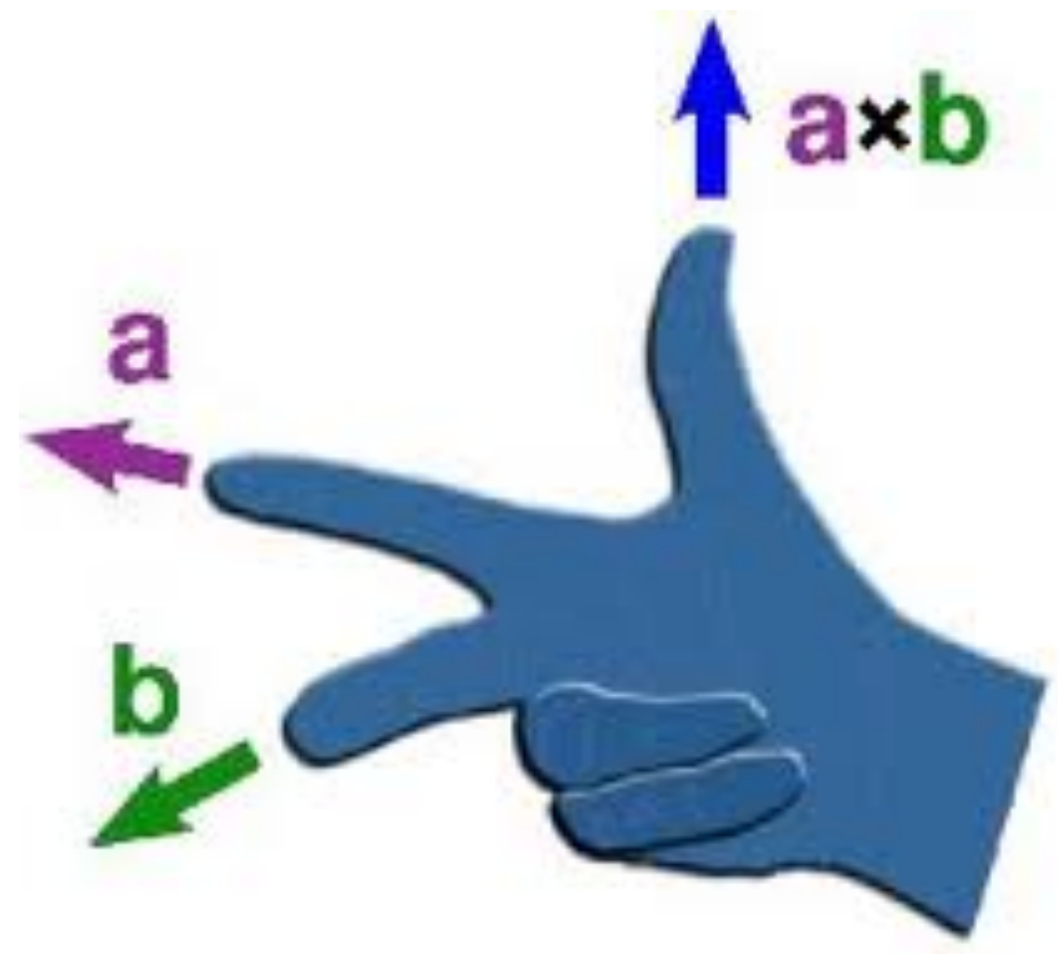
- What is the value of magnetic field (B_1) where I_2 is? $B_1 = \frac{\mu_o I_1}{2\pi r}$
- Likewise $B_2 = \frac{\mu_o I_2}{2\pi r}$

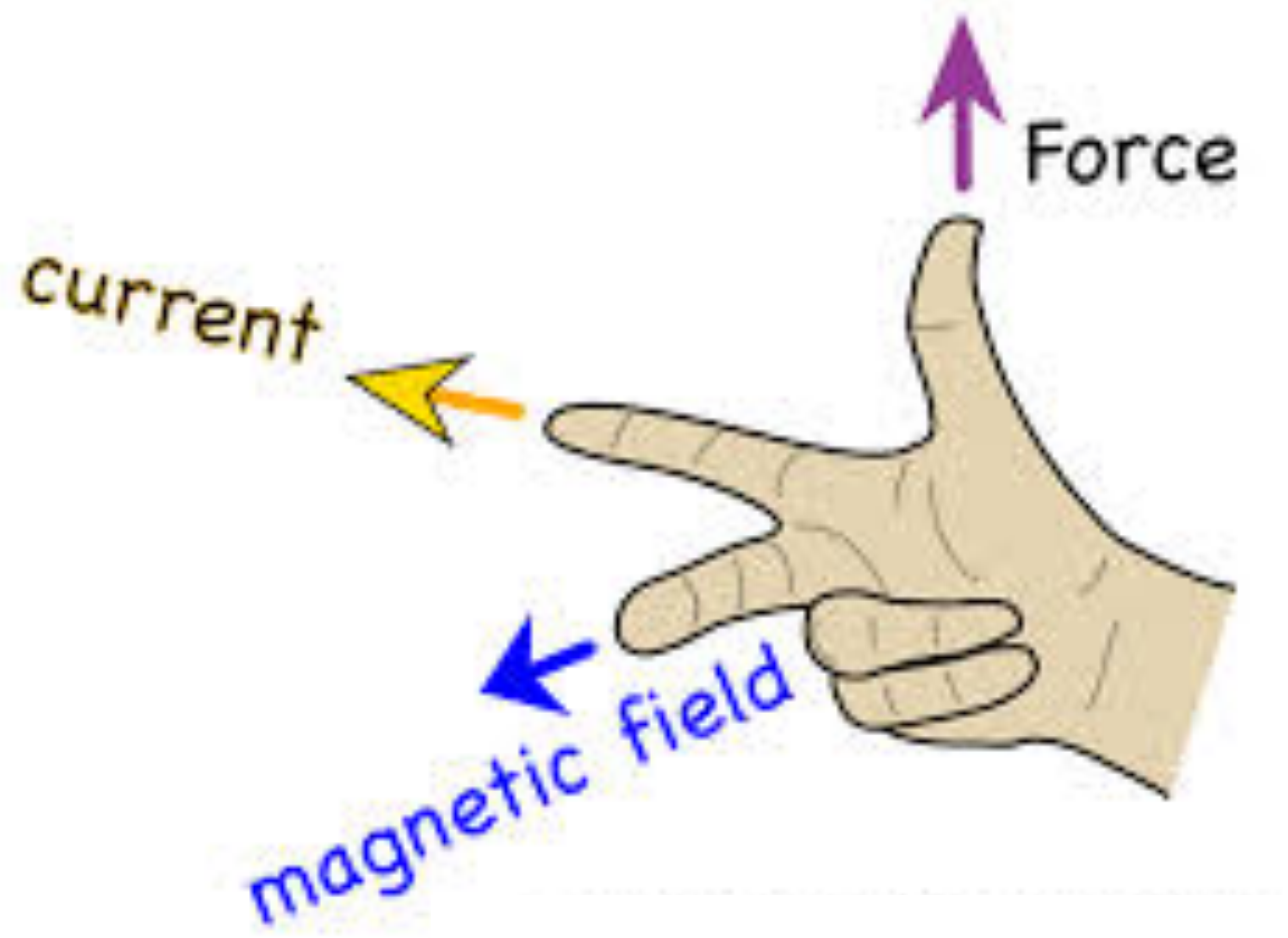
How about directions??

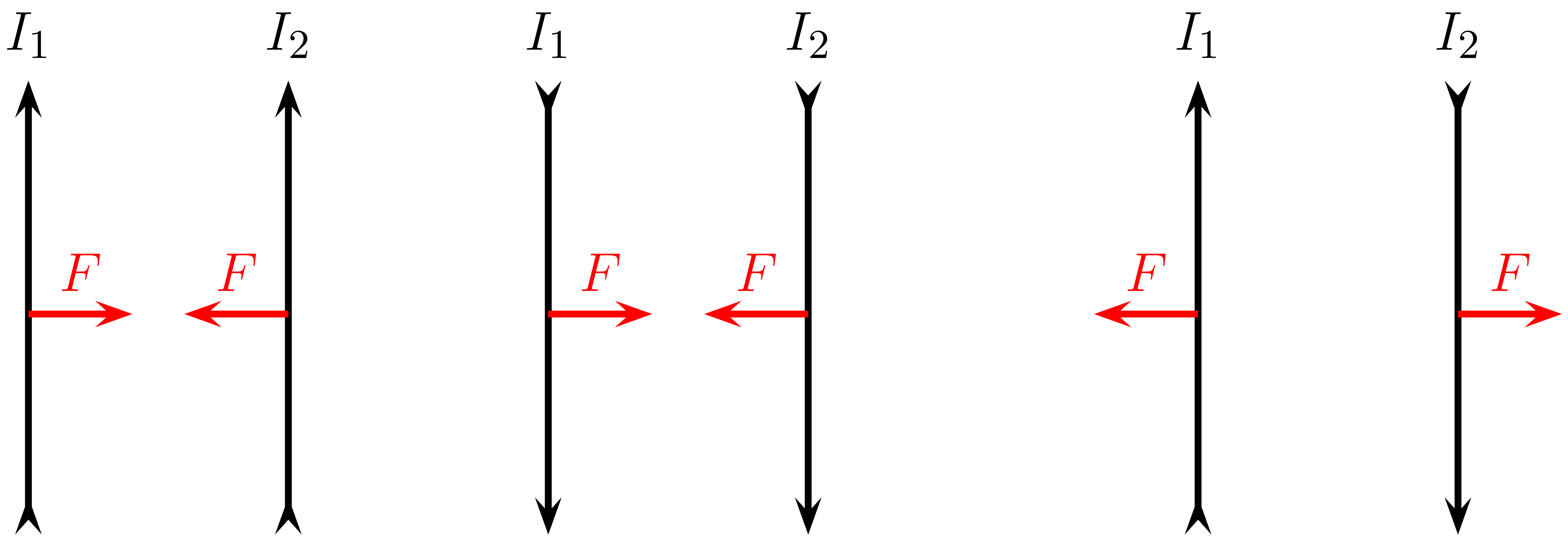
Parallel Currents Attract and Anti-parallel Currents Repel



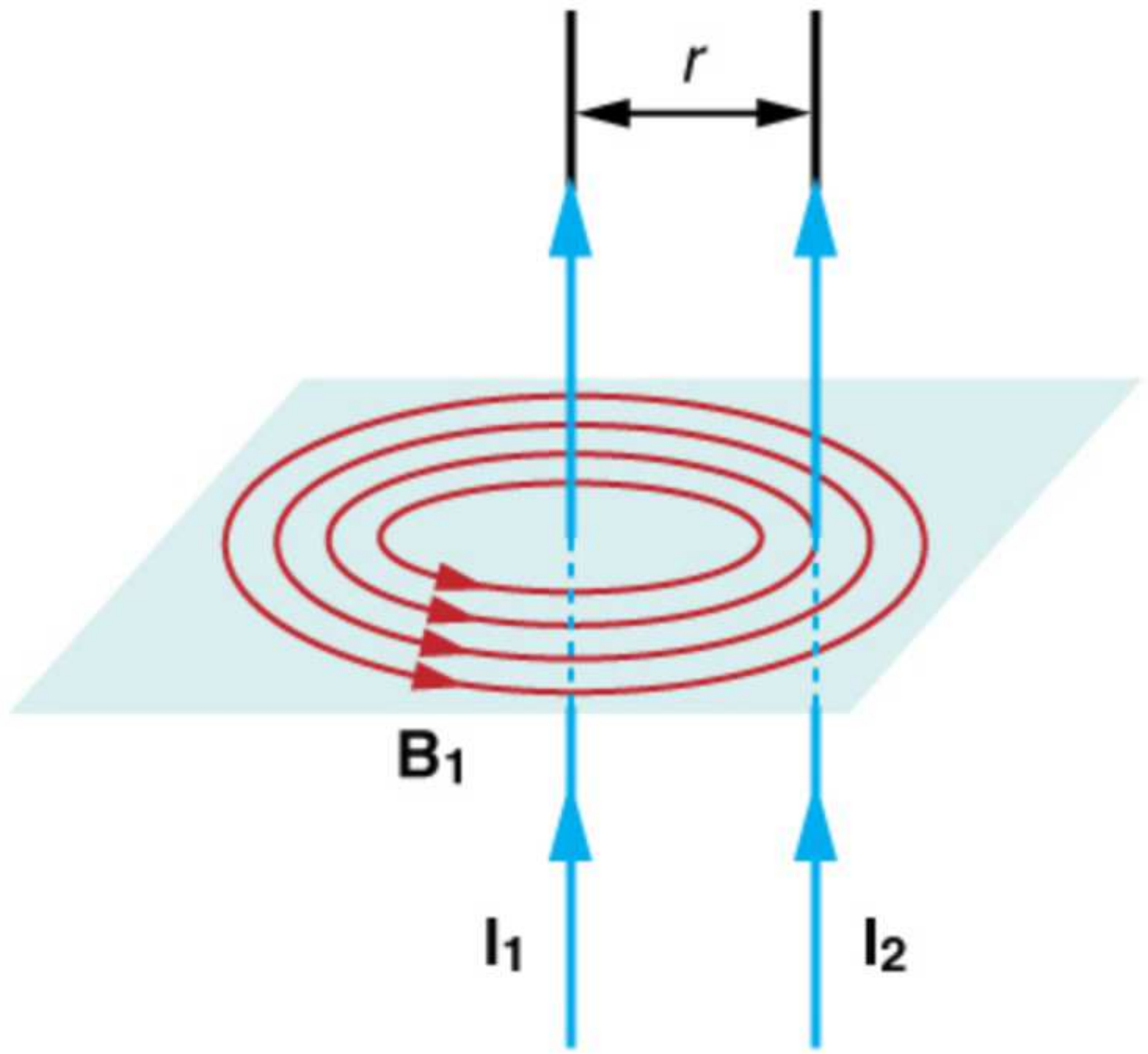
More on Right Hand Rule

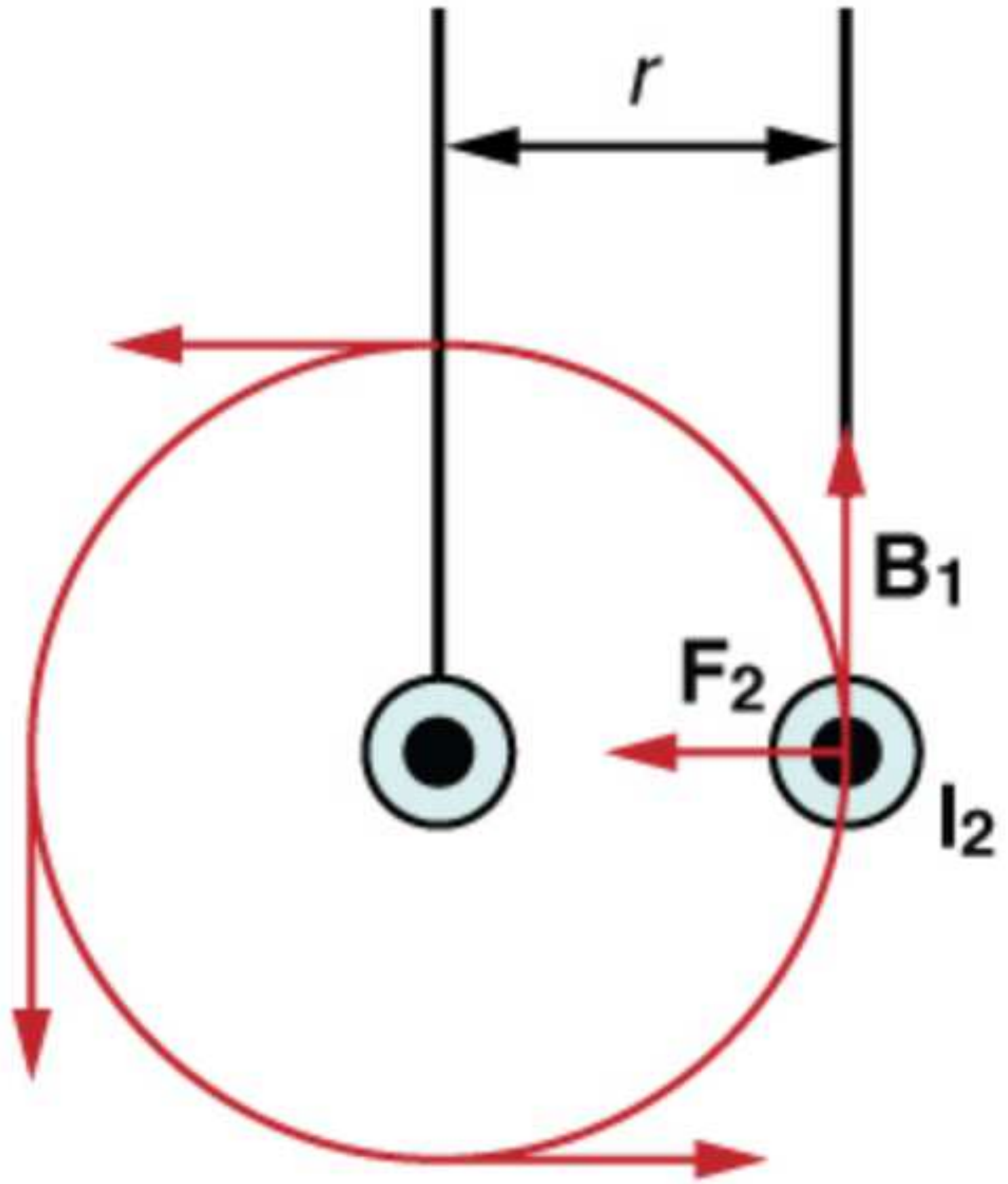




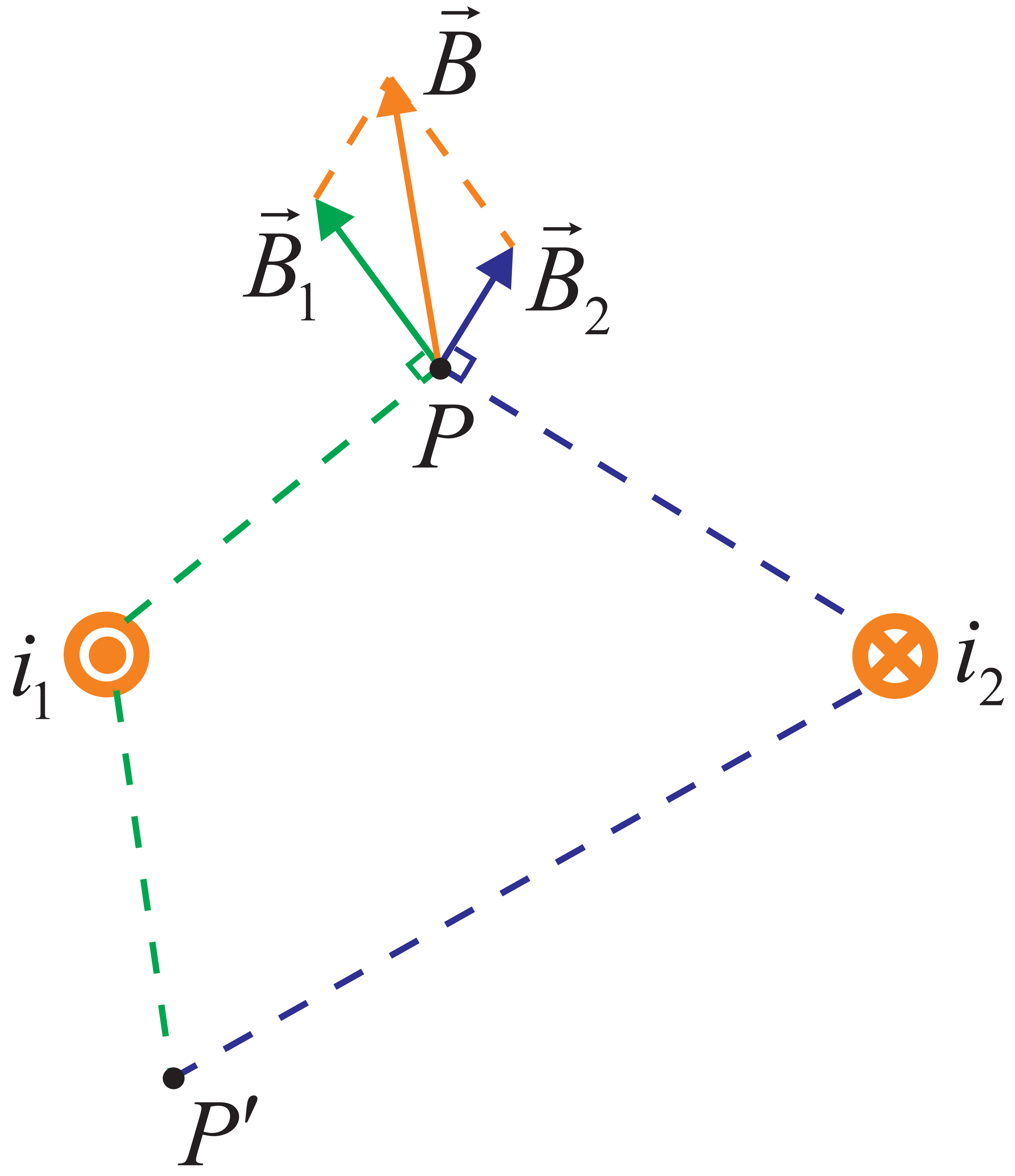


Graphical Explanation of Force's Direction For Currents in Same Direction

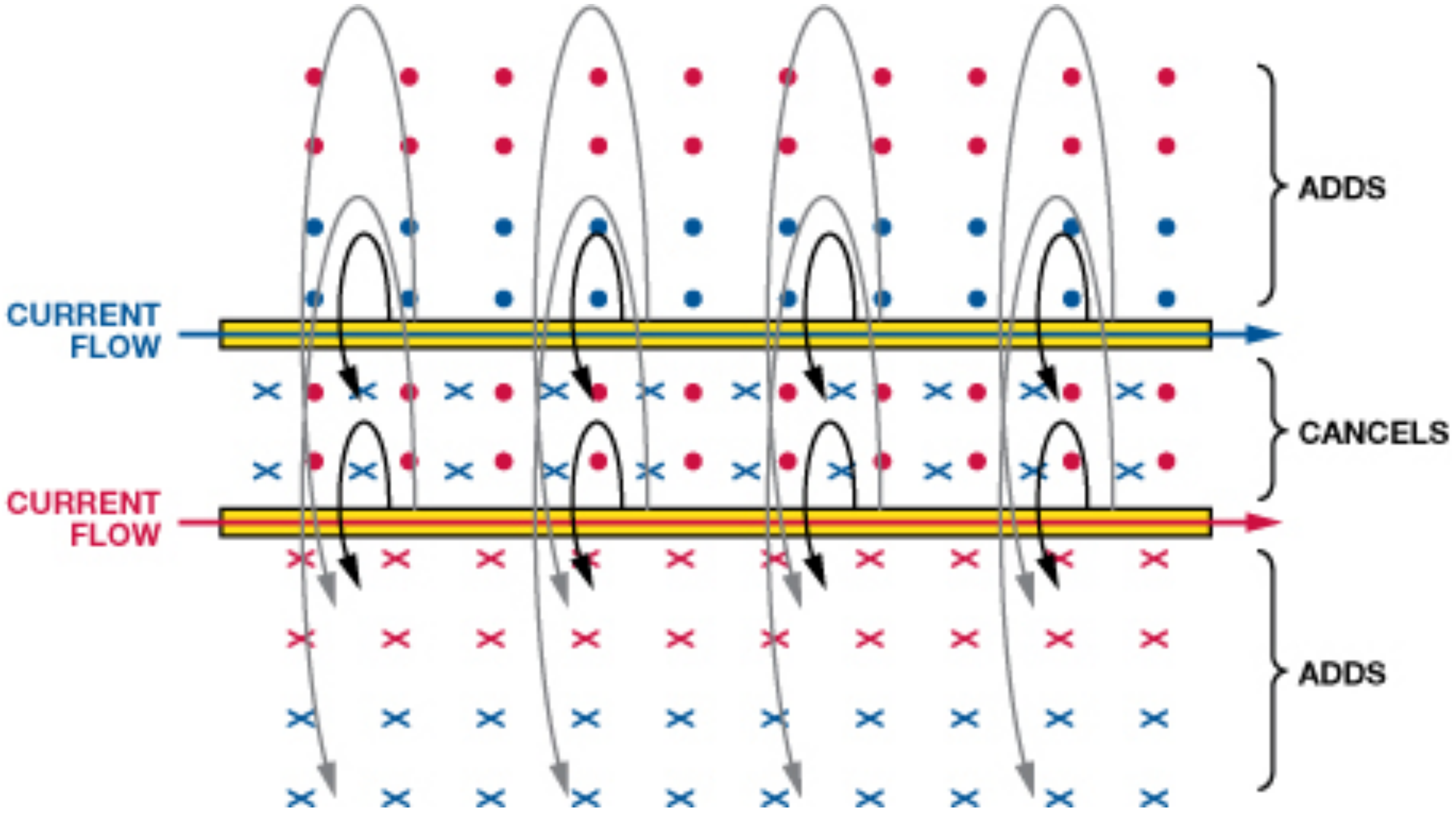


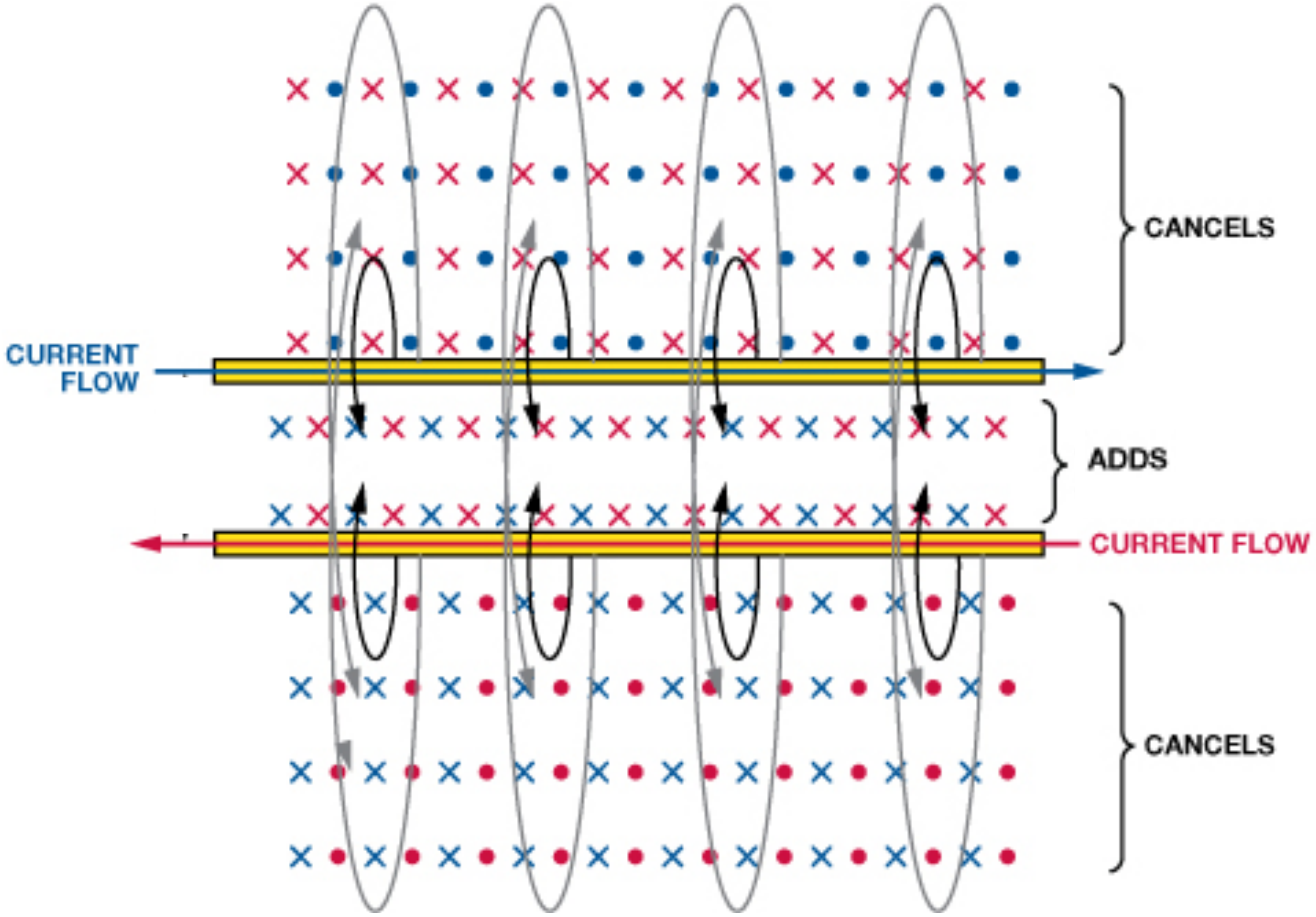


Principle of Superposition



Magnetic field $\vec{B}$ at point P due to individual currents i_1 and i_2 is vector sum of $\vec{B}_1, \vec{B}_2$-fields


ADDS
CURRENT FLOW
CANCELS
CURRENT FLOW
ADDS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