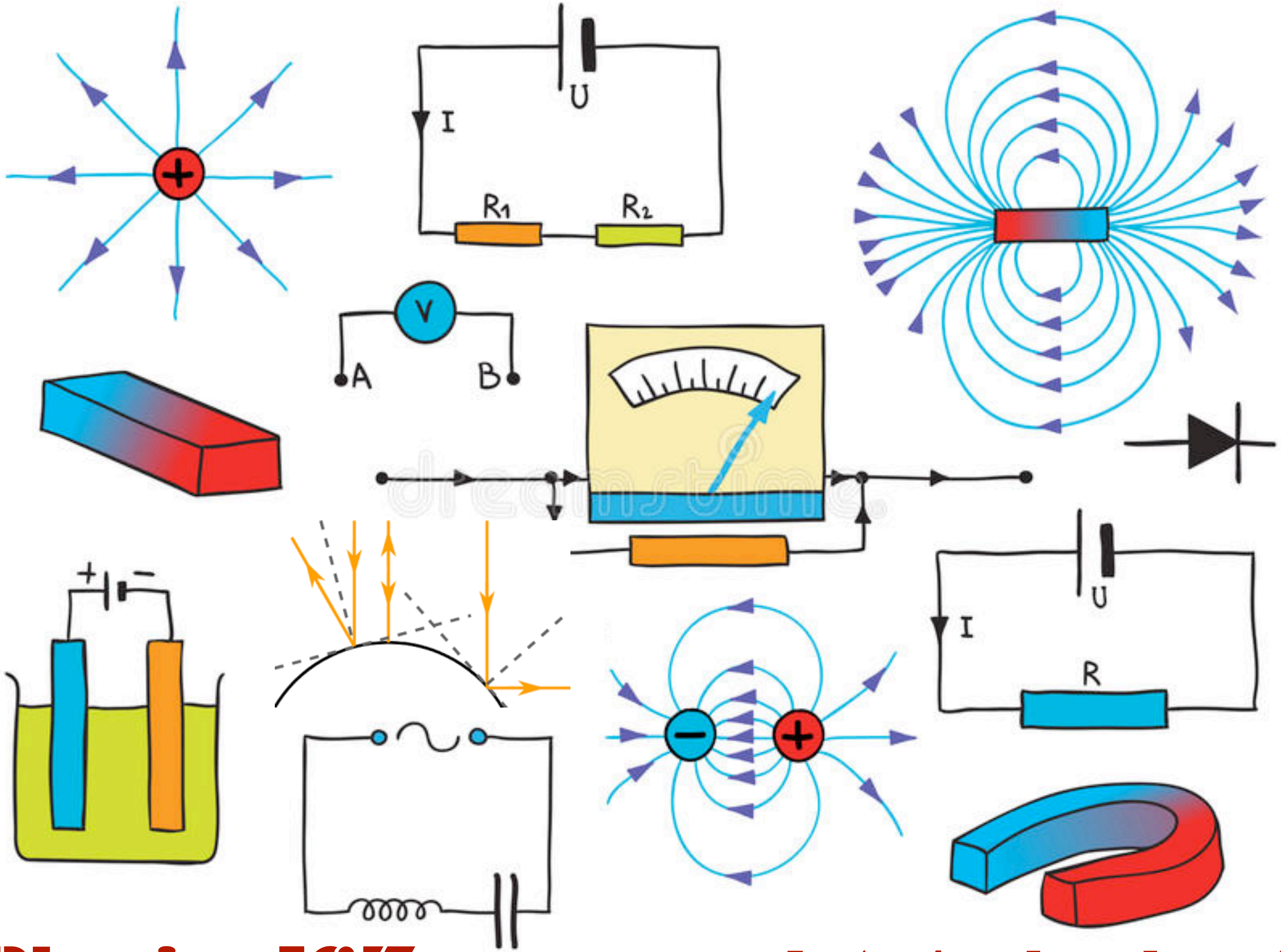


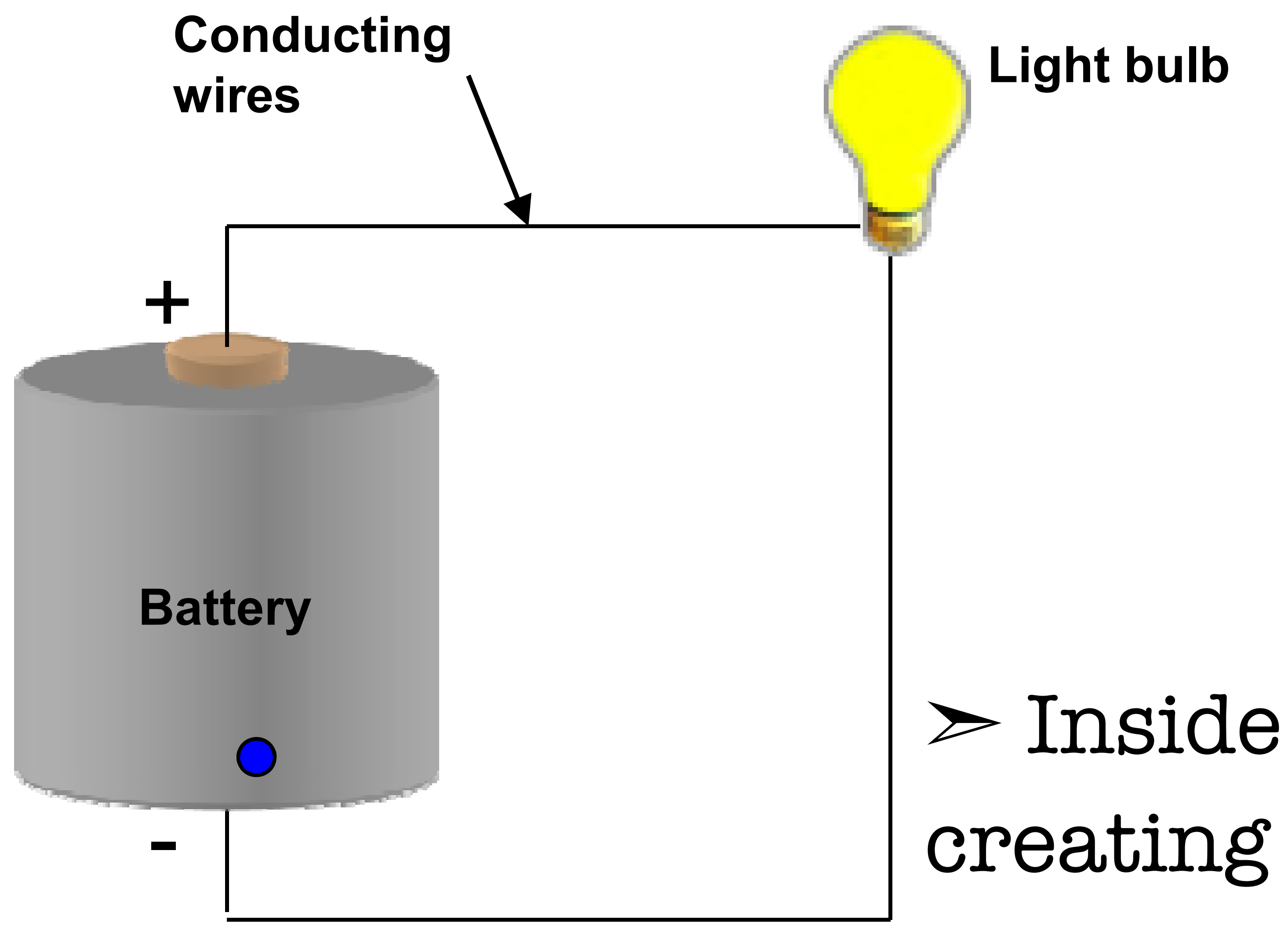
Physics 167

Luis Anchordoqui

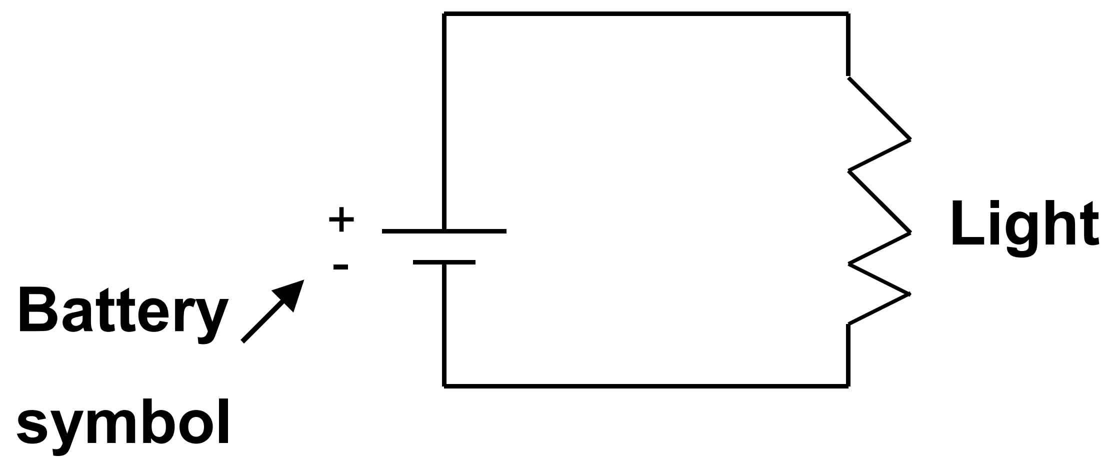
Electromotive Force

➢ Every electronic device depends on circuits

➢ Electrical energy is transferred from a power source, such as a battery, to a device, say a light bulb



A diagram of this circuit would look like the following



➢ Inside a battery, a chemical reaction separates positive and negative charges, creating a potential difference

➢ This potential difference is equivalent to the battery's voltage, or **emf (ε) electromotive force**

(this is not really a "force" but a potential)

➢ Because of the emf of the battery, an electric field is produced within and parallel to the wires

➢ This creates a force on the charges in the wire and moves them around the circuit

This flow of charge in a conductor is called electrical current (I)

➢ A measure of the current is how much charge passes a certain point in a given time

Electrical Current

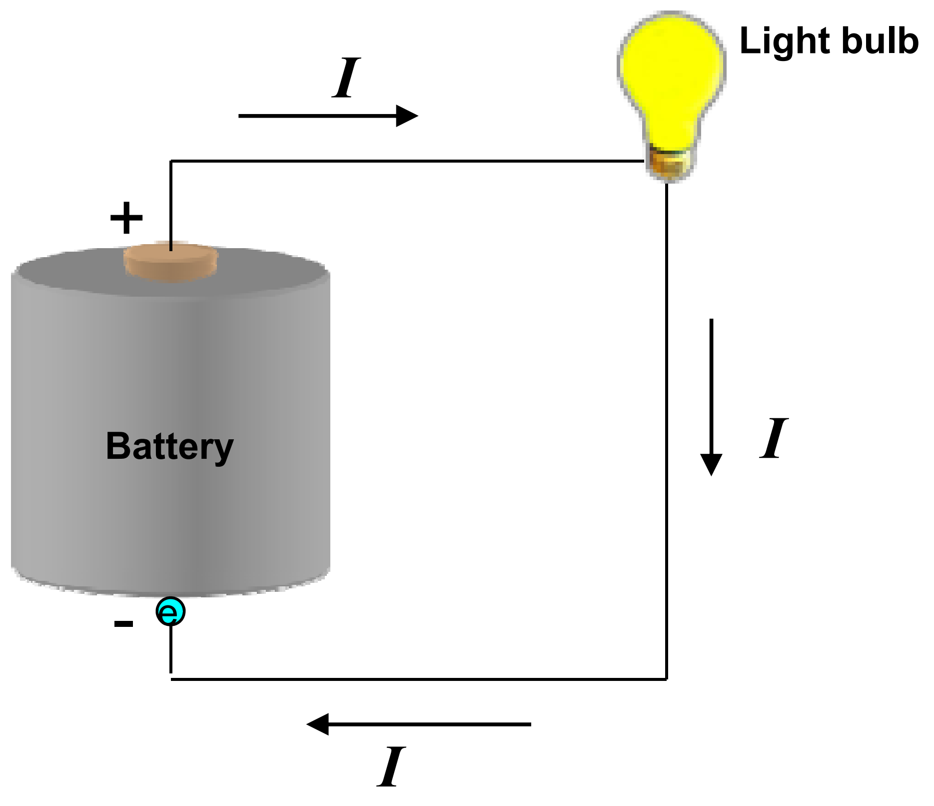
$$I = \frac{\Delta q}{\Delta t}$$

Units?

$$\left[\frac{\text{Charge}}{\text{time}}\right] = \left[\frac{\text{C}}{\text{s}}\right] = [\text{Ampere}] = [\text{A}]$$

➢ If the current only moves in one direction, like with batteries, it's called **Direct Current (DC)**

➢ If the current moves in both directions, like in your house, it's called **Alternating Current (AC)**



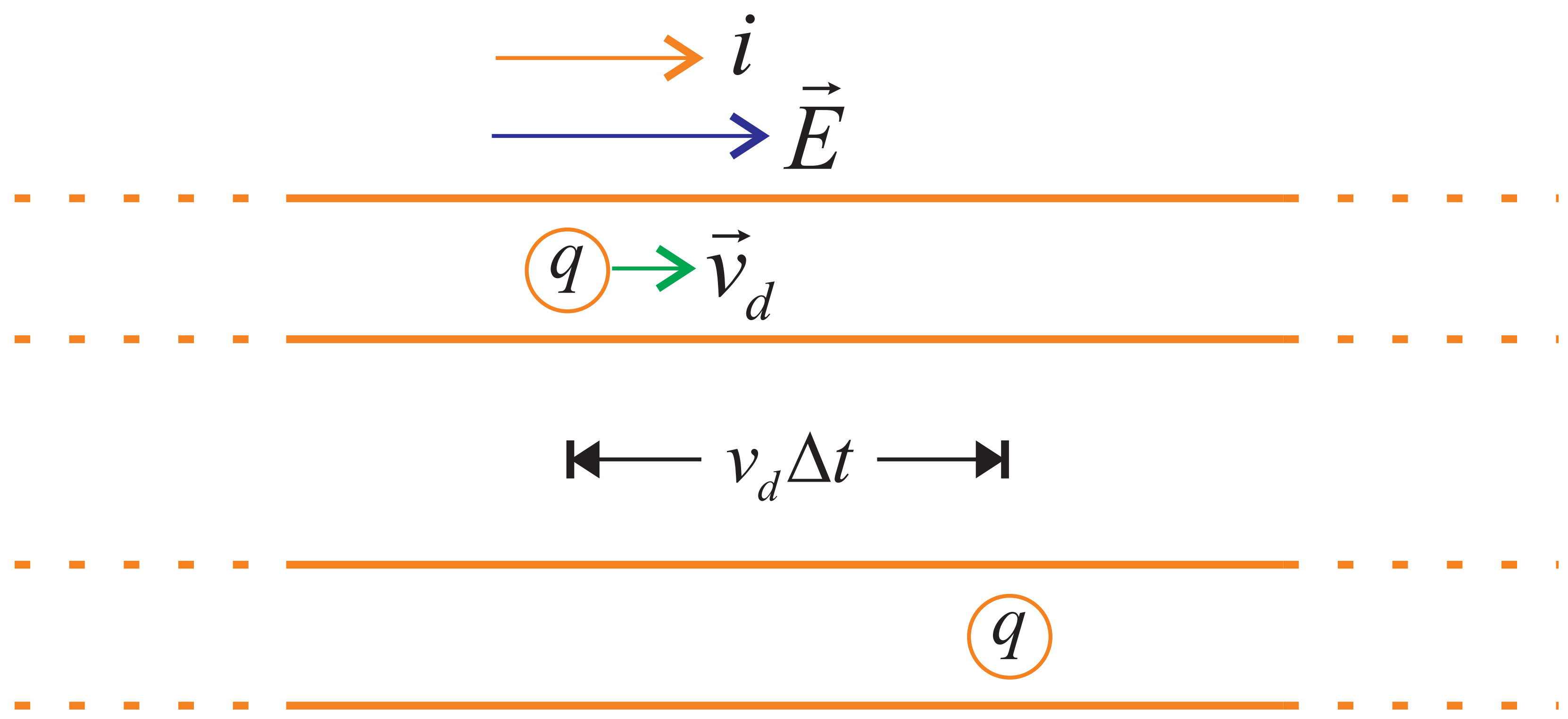
➢ Electric current is due to the flow of moving electrons,

but we will use the **positive conventional current** in the circuit diagrams

➢ So I shows the direction of **positive** charge flow from high potential to low potential

Drift Velocity

➢ Consider a current i flowing through a cross-sectional area A



∴ In time Δt ☛ total charges passing through segment

$$\Delta Q = qA(v_d \Delta t)n$$

q ☛ charge of current carrier

n ☛ density of charge carrier per unit volume

∴ Current $$i = \frac{\Delta Q}{\Delta t} = nqAv_d$$

Current density $$\vec{j} = nq\vec{v}_d$$

Note

- For metals ☛ charge carriers are free electrons inside

$$\therefore \quad \vec{j} = -ne\vec{v}_d \qquad \text{for metals}$$

$\therefore$ Inside metals $\vec{j}$ and $\vec{v}_d$ are in opposite direction

- We define a general property of materials ☛ conductivity (σ)

$$\vec{j} = \sigma \vec{E}$$

Note

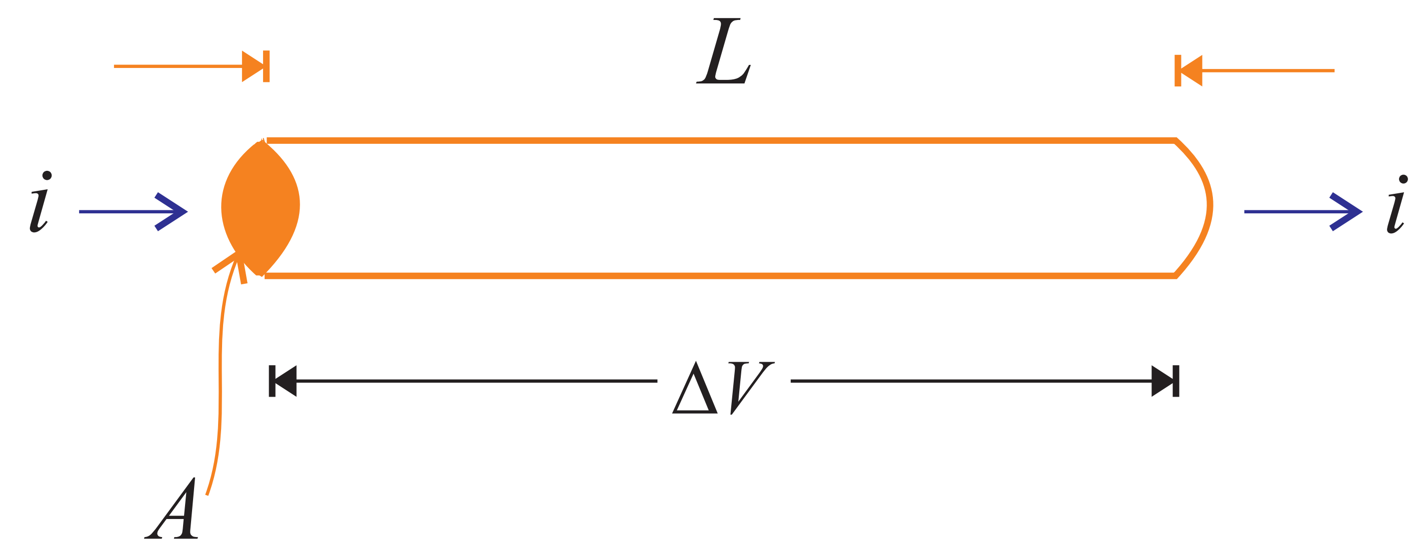
- In general σ is NOT a constant number but rather a **function of position and applied** $\vec{E}$ **-field**
- **Resistivity** (ρ) is more commonly used property defined as $\rho = \dfrac{1}{\sigma}$
- Unit of ρ: Ohm-meter (Ωm) where Ohm (Ω) = Volt/Ampere
- **OHM'S LAW** Ohmic materials have resistivity that are independent of applied electric field

  e.g. metals (in not too high $\vec{E}$ -field)

Example

Consider a **resistor** (ohmic material) of length L and cross-sectional area A

$\therefore$ Electric field inside conductor



$$E = \Delta V / L$$

Current density $j = \dfrac{i}{A}$

$$\therefore \quad \rho = \frac{E}{j}$$

$$\rho = \frac{\Delta V}{L} \cdot \frac{1}{i/A}$$

$$\frac{\Delta V}{i} = R = \rho \frac{L}{A}$$

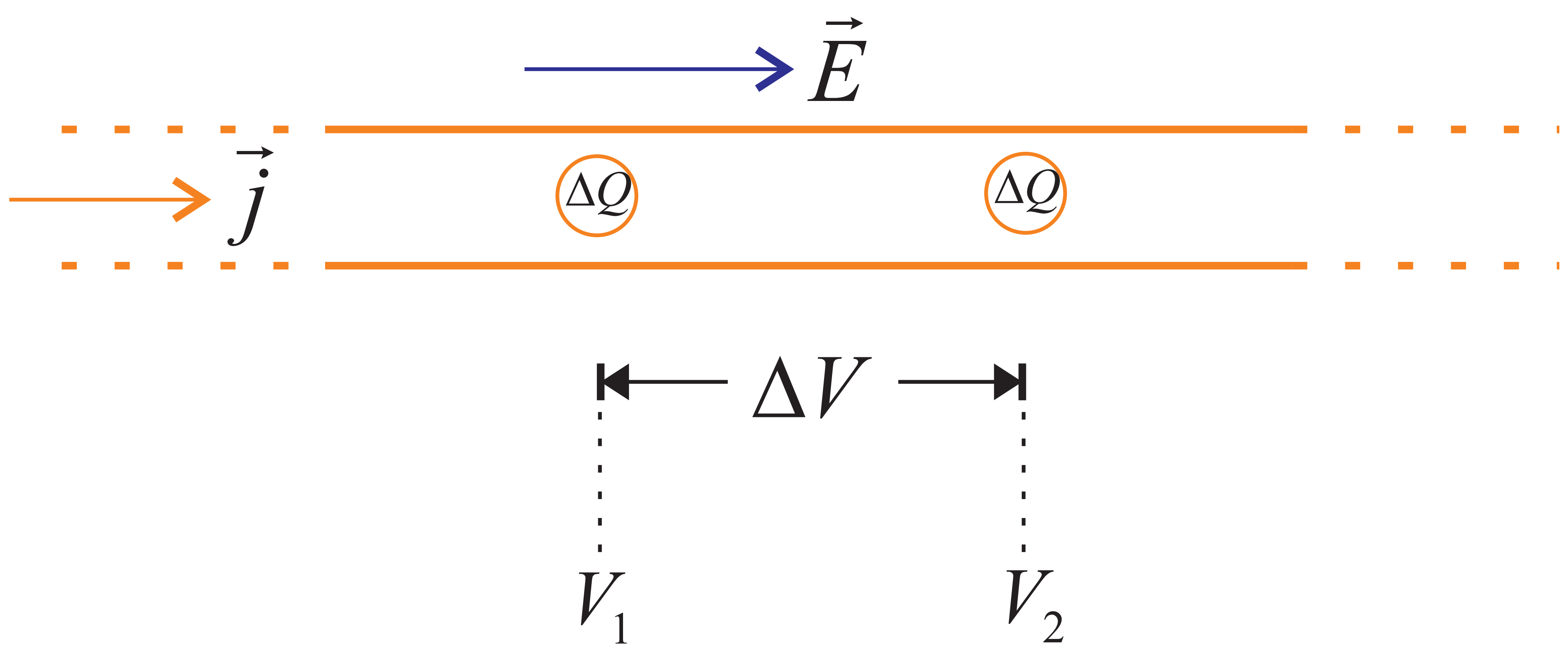
R ☛ **resistance** of conductor

Note

$\Delta V = iR$ is NOT a statement of Ohm's Law but it's just a definition for resistance

Energy In Current

➢ Assuming a charge ΔQ enters with potential V_1 and leaves with potential V_2



∴ Potential energy lost in wire

$$\Delta U = \Delta Q V_2 - \Delta Q V_1$$

$$\Delta U = \Delta Q (V_2 - V_1)$$

∴ Rate of energy lost per unit time

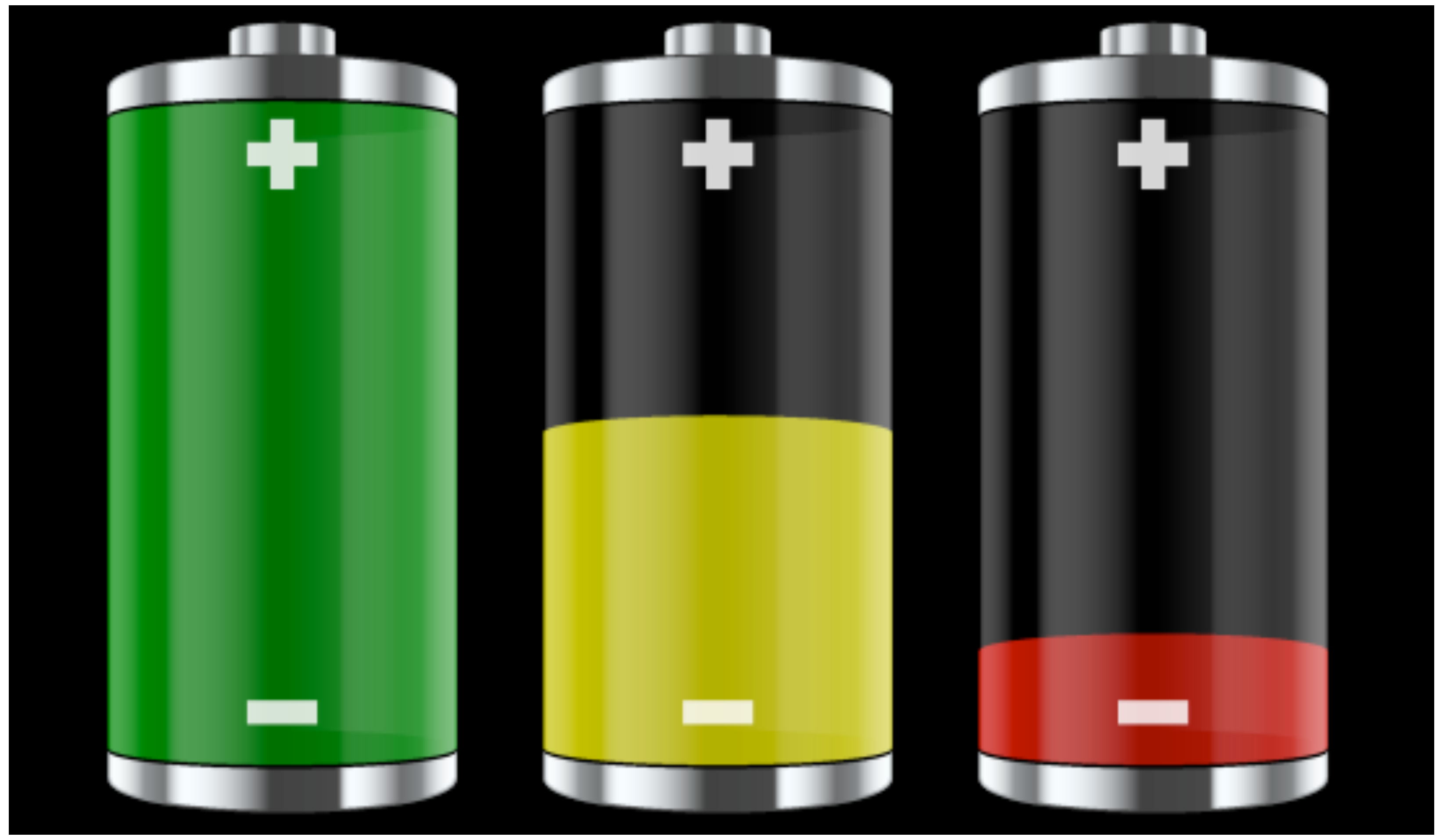
$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}(V_2 - V_1)$$

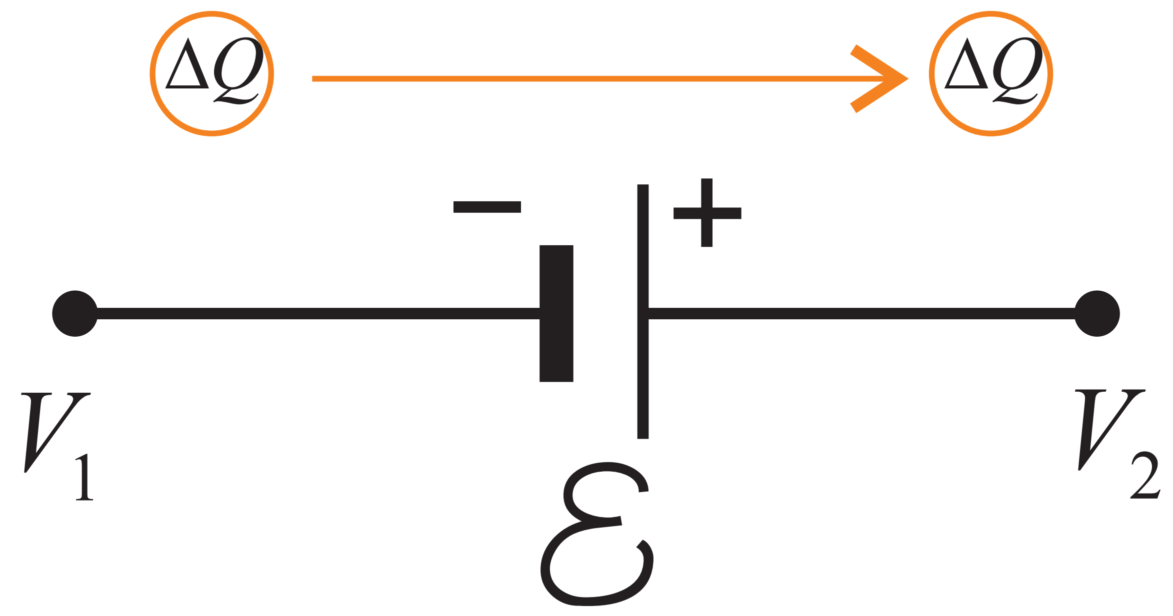
Joule's heating ☛ $P = i \cdot \Delta V =$ Power dissipated in conductor

For a resistor R ➠ $R, \quad P = i^2 R = \dfrac{\Delta V^2}{R}$

DC Circuits

➢ A battery is a device that **supplies electrical energy** to maintain a current in a circuit



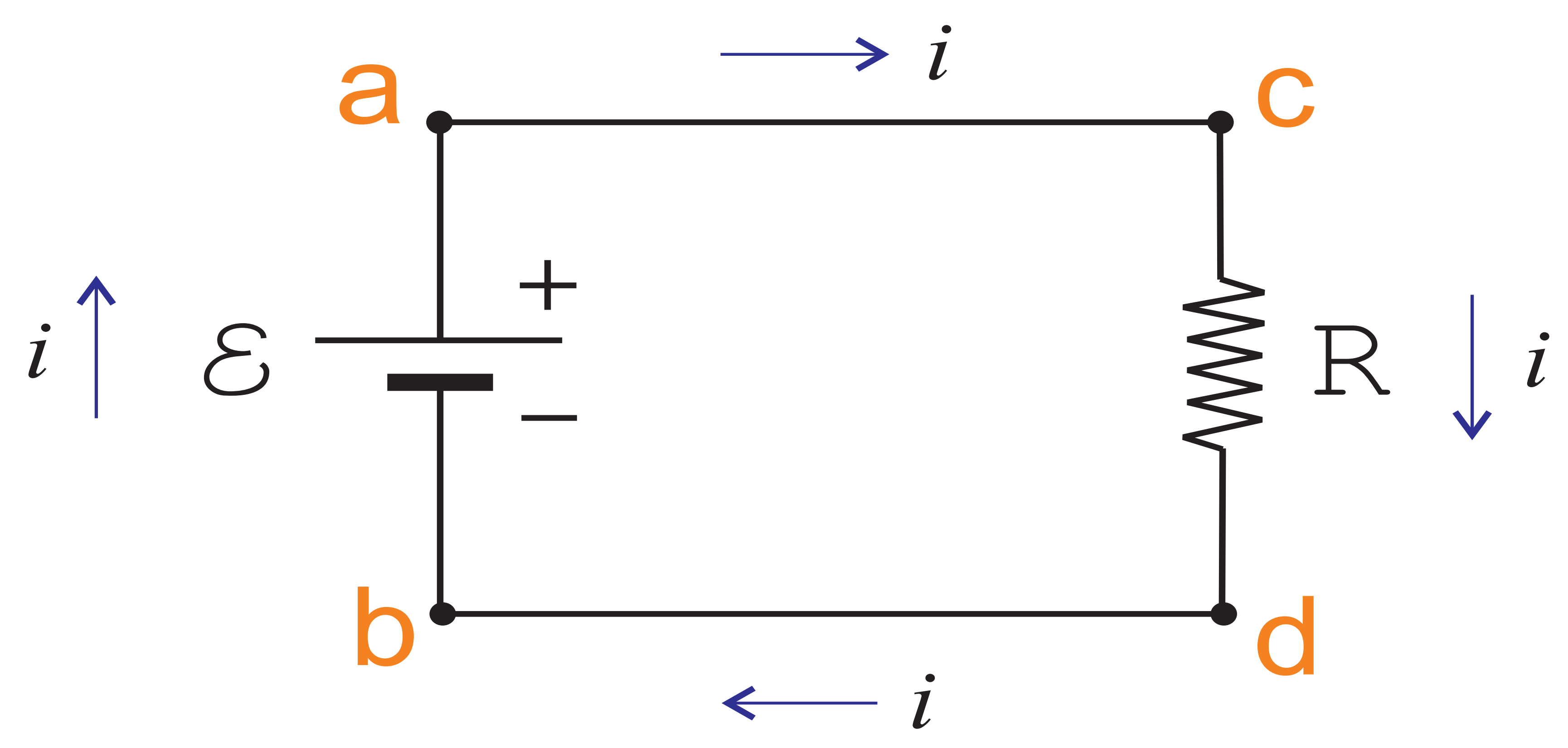


➢ In moving from point 1 to 2 electric potential energy increase by

$$\Delta U = \Delta Q(V_2 - V_1) = \text{Work done by } \mathcal{E}$$

➢ Define $\mathcal{E} = \text{Work done/charge} = V_2 - V_1$

Example



$$V_a = V_c$$
$$V_b = V_d$$

} assuming perfect conducting wires

By Definition

$$V_c - V_d = iR$$
$$V_a - V_b = \mathcal{E}$$
$$\therefore \quad \mathcal{E} = iR \quad \Rightarrow \quad i = \frac{\mathcal{E}}{R}$$

Also ☛ we have assumed zero resistance inside battery

Series Circuits

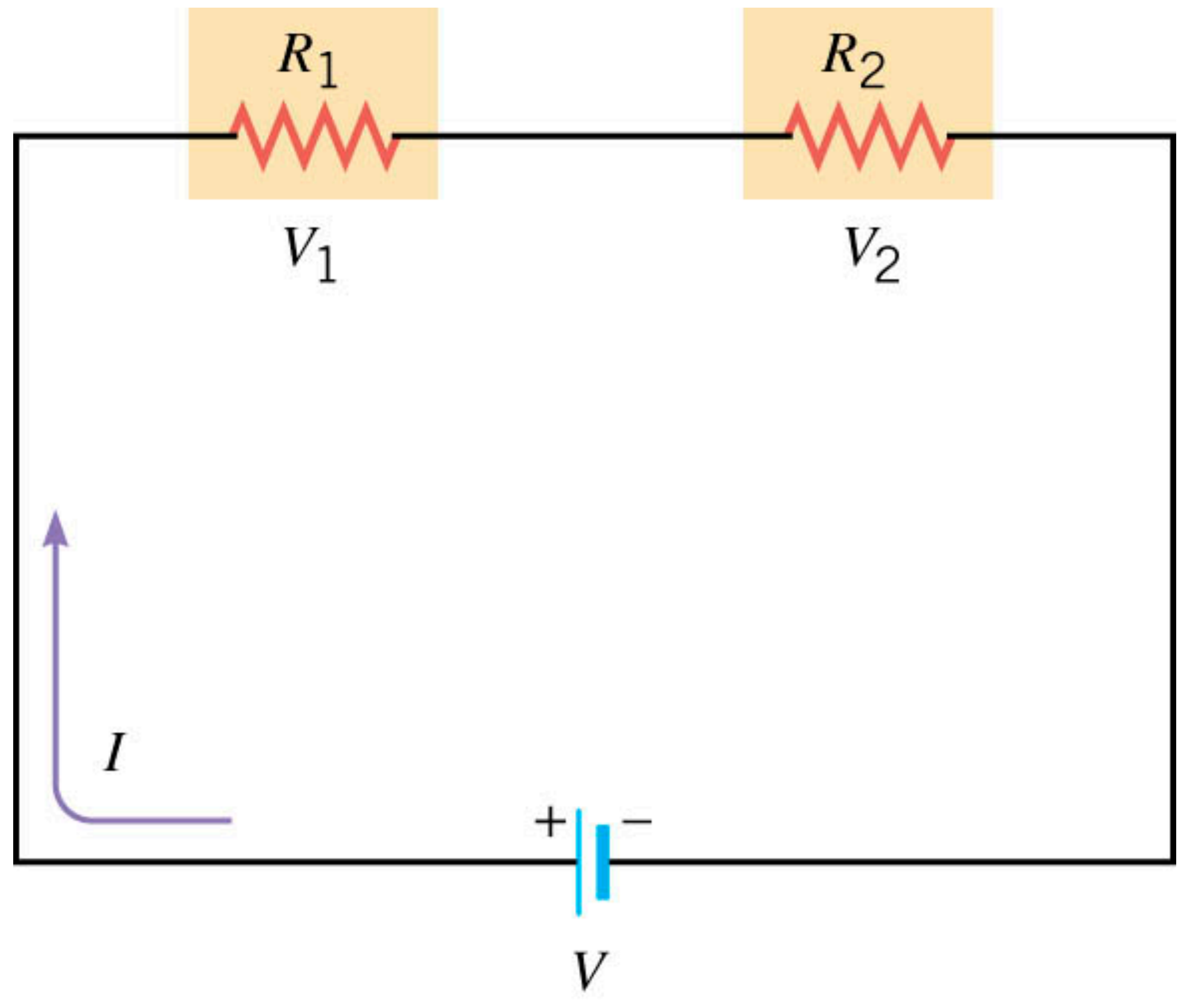
➢ Let's add more than one component to the circuit!

➢ There are several ways to hook these components together

➢ The first way is to wire them together in series

The same current runs through
two components connected in series

V_1 and V_2 are called **voltage drops**



✗ We speak of currents running through resistors, and voltages drops across resistors

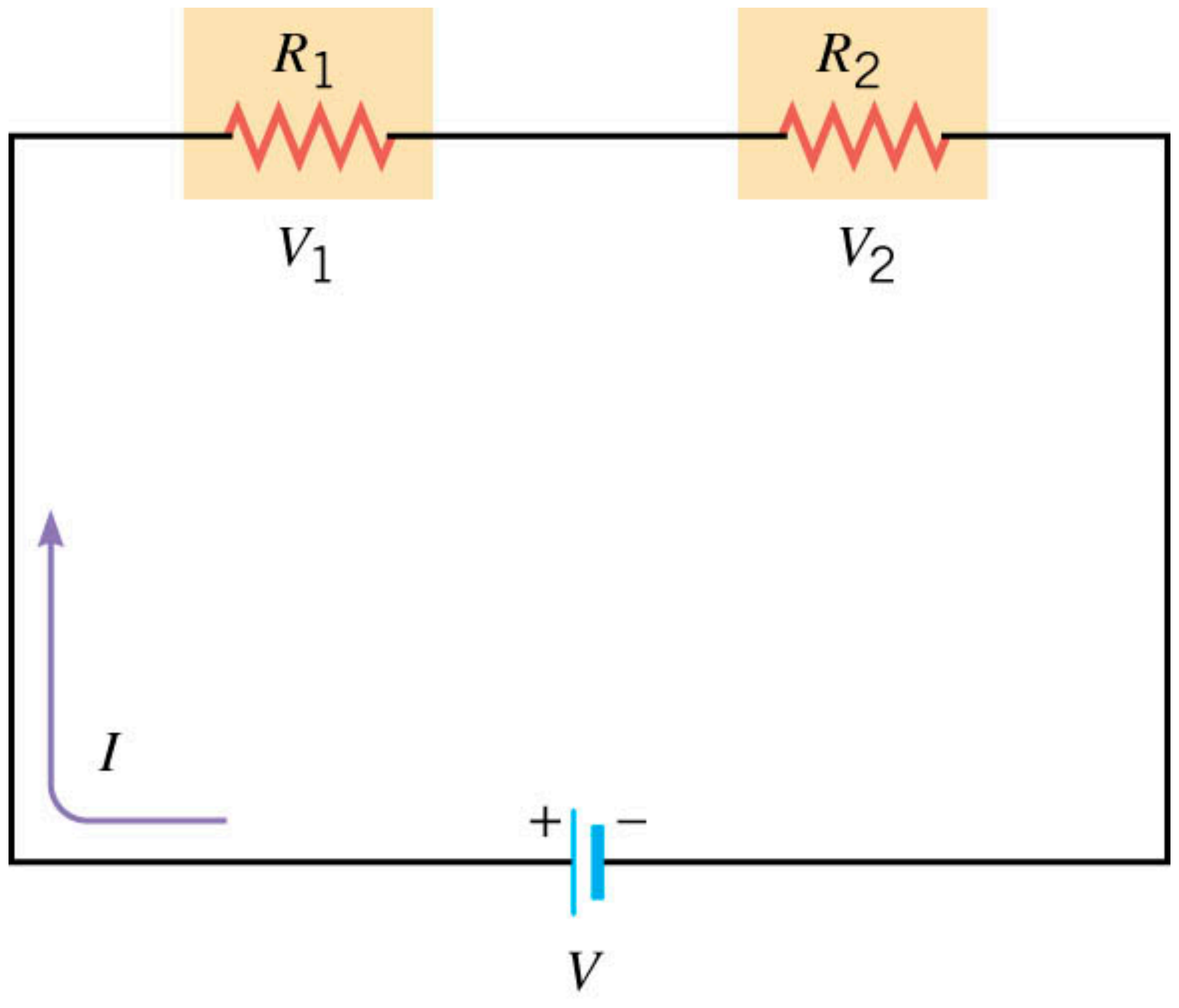
➢ Thus, the **current through** resistor R_1 is I, and the **voltage drop** across R_1 is V_1.

➢ How would we find the net resistance (equivalent resistance, R_{eq}) for resistors connected in series?

➢ For resistors connected in series, sum of voltage drops across all resistors must equal battery voltage

➢ Thus, $V = V_1 + V_2$

➢ But from Ohm's Law $\cancel{I}R_{eq} = \cancel{I}R_1 + \cancel{I}R_2 \Rightarrow R_{eq} = R_1 + R_2$



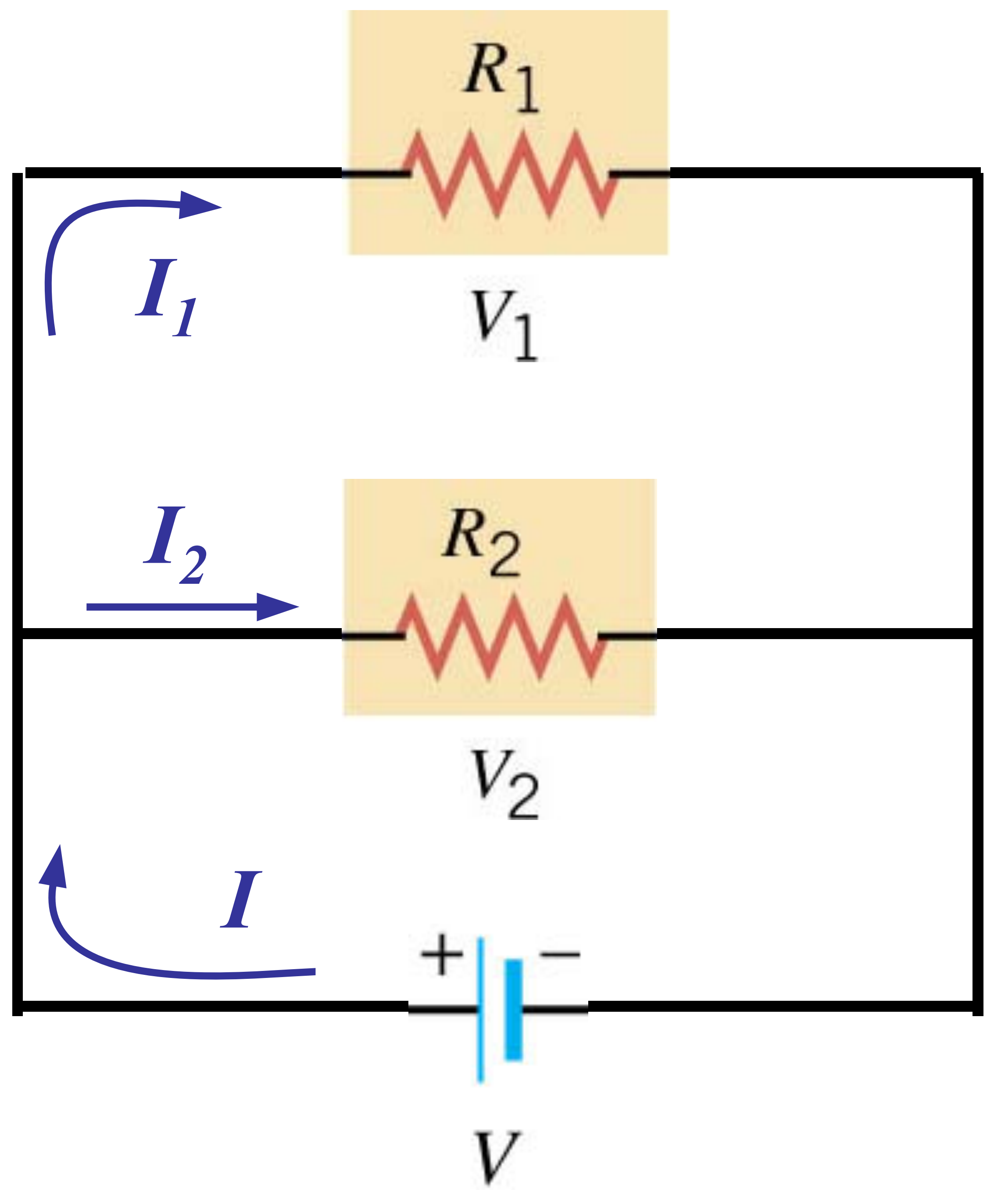
➢ Thus, for resistors wired up in **series**, equivalent resistance is ☛

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

i.e. you just add them!!!

Parallel Circuits

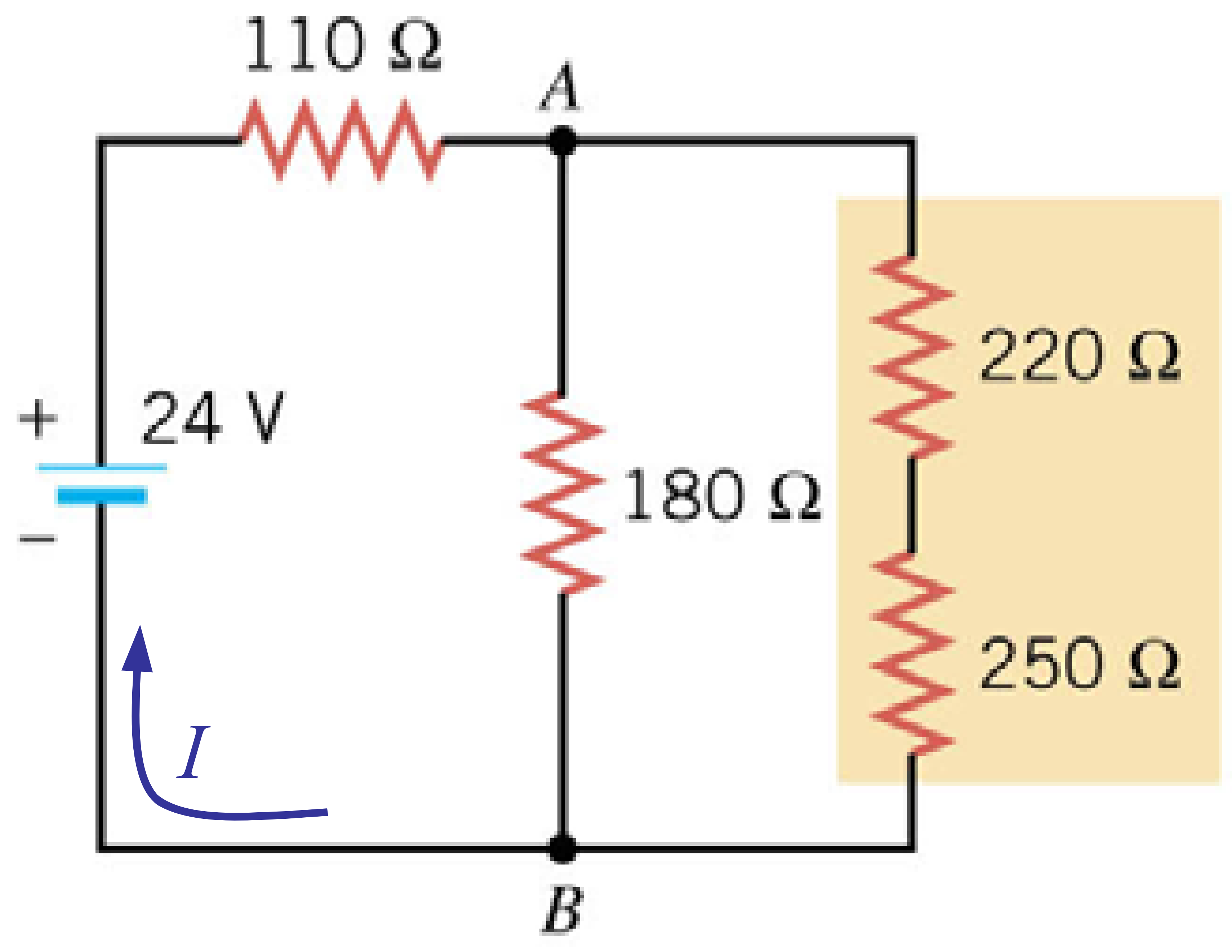
For resistors connected in parallel, the voltage drop across each resistor is the same



- The current through each might be different
- It splits ☛ $I = I_1 + I_2$
- Thus, $V_1 = V_2 = V$
- From Ohm's Law ☛ $R_{eq} = V/I = \frac{V}{I_1 + I_2} = \frac{V}{\frac{V_1}{R_1} + \frac{V_2}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$
- Thus, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$ for resistors in parallel

Series and Parallel Circuits

➢ Now let's hook resistors up both in series and in parallel in the same circuit!



What is the current I in the following circuit?

We need to find the equivalent resistance!

$$R_{eq} = 240\,\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{24}{240} = \boxed{0.1\,\text{A}}$$

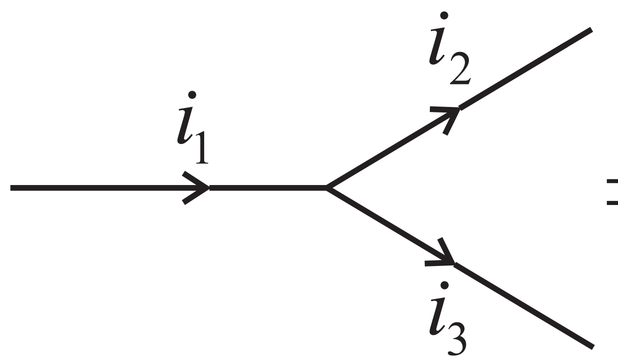
Analysis of Complex Circuits

KIRCHOFF'S LAWS

① First Law (Junction Rule)

Total current entering a junction equal to total current leaving junction

Conservation of charge



$\Rightarrow i_1 = i_2 + i_3$

② Second Law (Loop Rule)

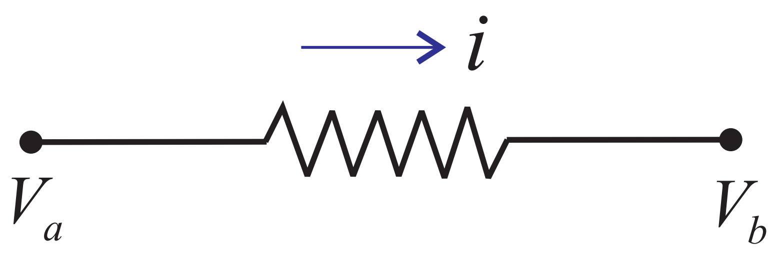
The sum of potential differences around a complete circuit loop is zero

Alternative

Around any closed circuit loop, sum of potential (voltage) drops has to equal sum of potential rises

Convention

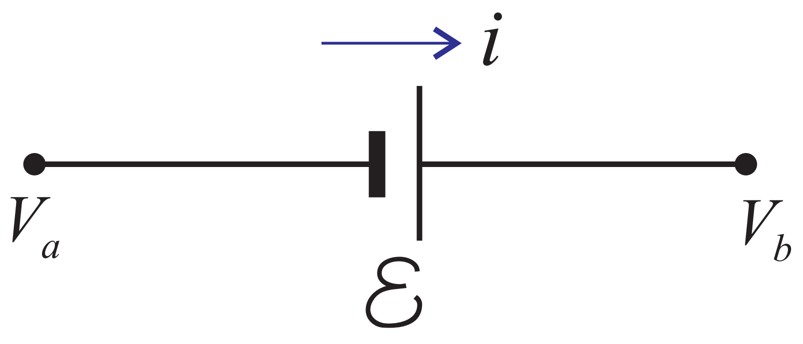
(i)



$V_a > V_b \Rightarrow$ Potential difference = $-iR$

i.e. Potential **drops** across rsistors

(ii)

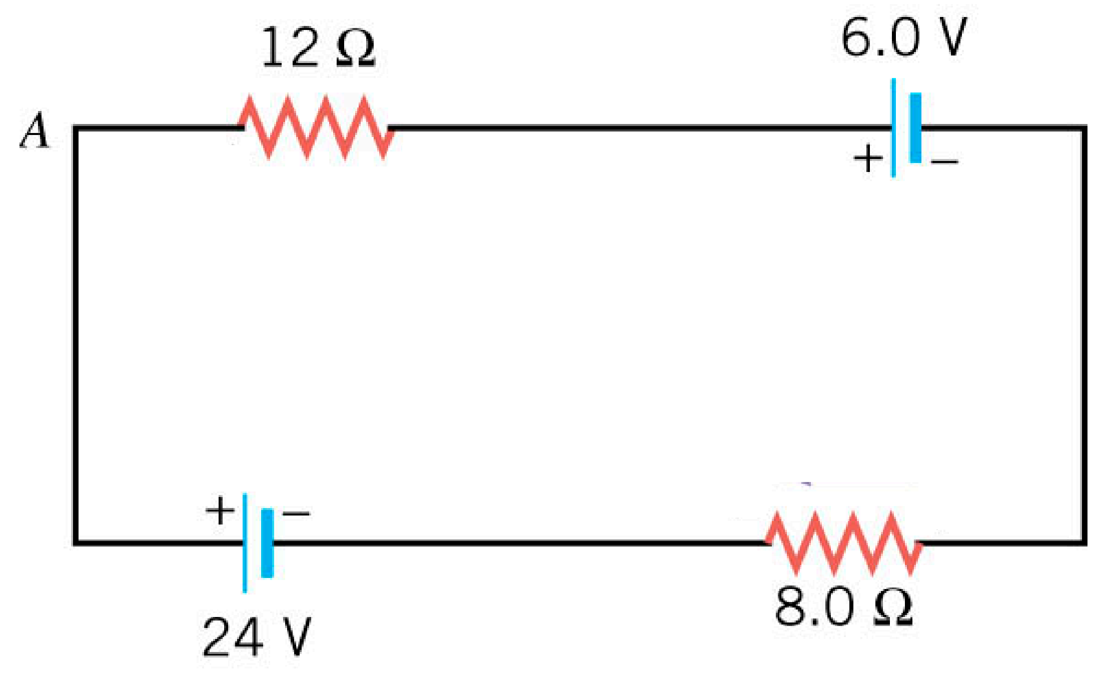


$V_b > V_a \Rightarrow$ Potential difference = $+\varepsilon$

i.e. Potential **rises across** negative plate of battery

Example

We have a closed circuit loop with multiple batteries



What is the current in the circuit?

1- Choose the direction of the current(s) in each loop

2- Label the resistors from + to - in the direction of the current flow

Solution

Start at point A and go around the loop clockwise and make a list of the potential drops and rises as we go all the way around

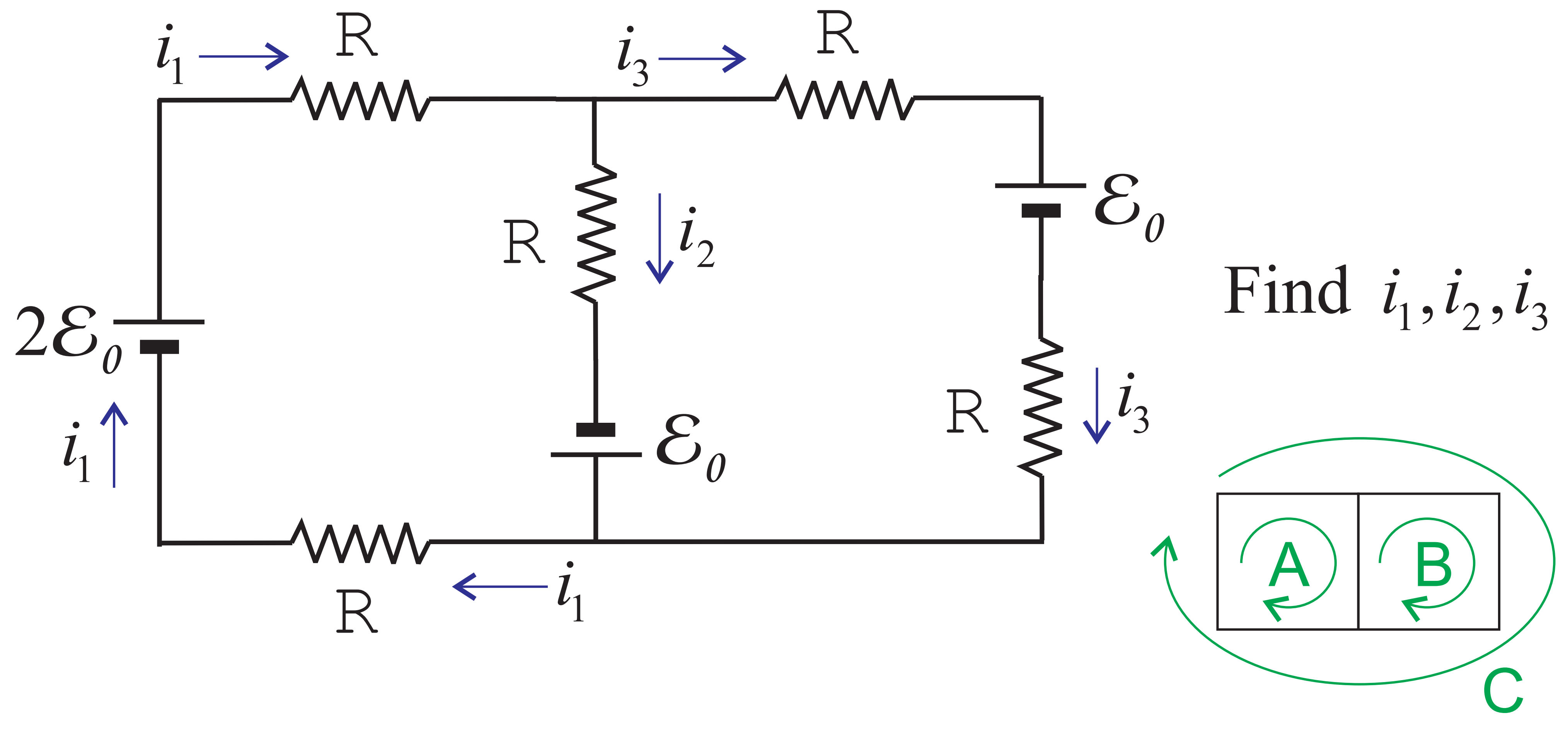
Drops	Rises
$12I$	24
6	
$8I$	

Now apply the loop rule ☛ $\sum$ Drops = $\sum$ Rises

$$\Rightarrow 12\,I + 6 + 8I = 24 \Rightarrow 20\,I = 18$$

$$\Rightarrow I = 0.9\ A$$

Example



Find i_1, i_2, i_3

By junction rule

$$i_1 = i_2 + i_3 \quad \mathbf{1}$$

By loop rule

$$\text{Loop A} \Rightarrow 2\mathcal{E}_0 - i_1R - i_2R + \mathcal{E}_0 - i_1R = 0 \quad \mathbf{2}$$

$$\text{Loop B} \Rightarrow -i_3R - \mathcal{E}_0 - i_3R - \mathcal{E}_0 + i_2R = 0 \quad \mathbf{3}$$

$$\text{Loop C} \Rightarrow 2\mathcal{E}_0 - i_1R - i_3R - \mathcal{E}_0 - i_3R - i_1R = 0 \quad \mathbf{4}$$

BUT ☛ **4 = 2 + 3**

General rule

- Need only 3 equations for 3 current

$$i_1 = i_2 + i_3 \quad \boxed{1}$$

$$3\mathcal{E}_0 - 2i_1R - i_2R = 0 \quad \boxed{2}$$

$$-2\mathcal{E}_0 + i_2R - 2i_3R = 0 \quad \boxed{3}$$

- Substitute 1 into 2

$$3\mathcal{E}_0 - 2(i_2 + i_3)R - i_2R = 0 \quad \boxed{4}$$

$$\Rightarrow \quad 3\mathcal{E}_0 - 3i_2R - 2i_3R = 0$$

- Subtract 3 from 4 , i.e. 4 - 3

$$3\mathcal{E}_0 - (-2\mathcal{E}_0) - 3i_2R - i_2R = 0$$

$$\Rightarrow \quad i_2 = \frac{5}{4} \cdot \frac{\mathcal{E}_0}{R}$$

➢ Substitute i_2 into **3**

$$-2\mathcal{E}_0 + \left(\frac{5}{4} \cdot \frac{\mathcal{E}_0}{R}\right) R - 2 i_3 R = 0$$

$$\Rightarrow \quad i_3 = -\frac{3}{8} \cdot \frac{\mathcal{E}_0}{R}$$

➢ Substitute i_2, i_3 into **1**

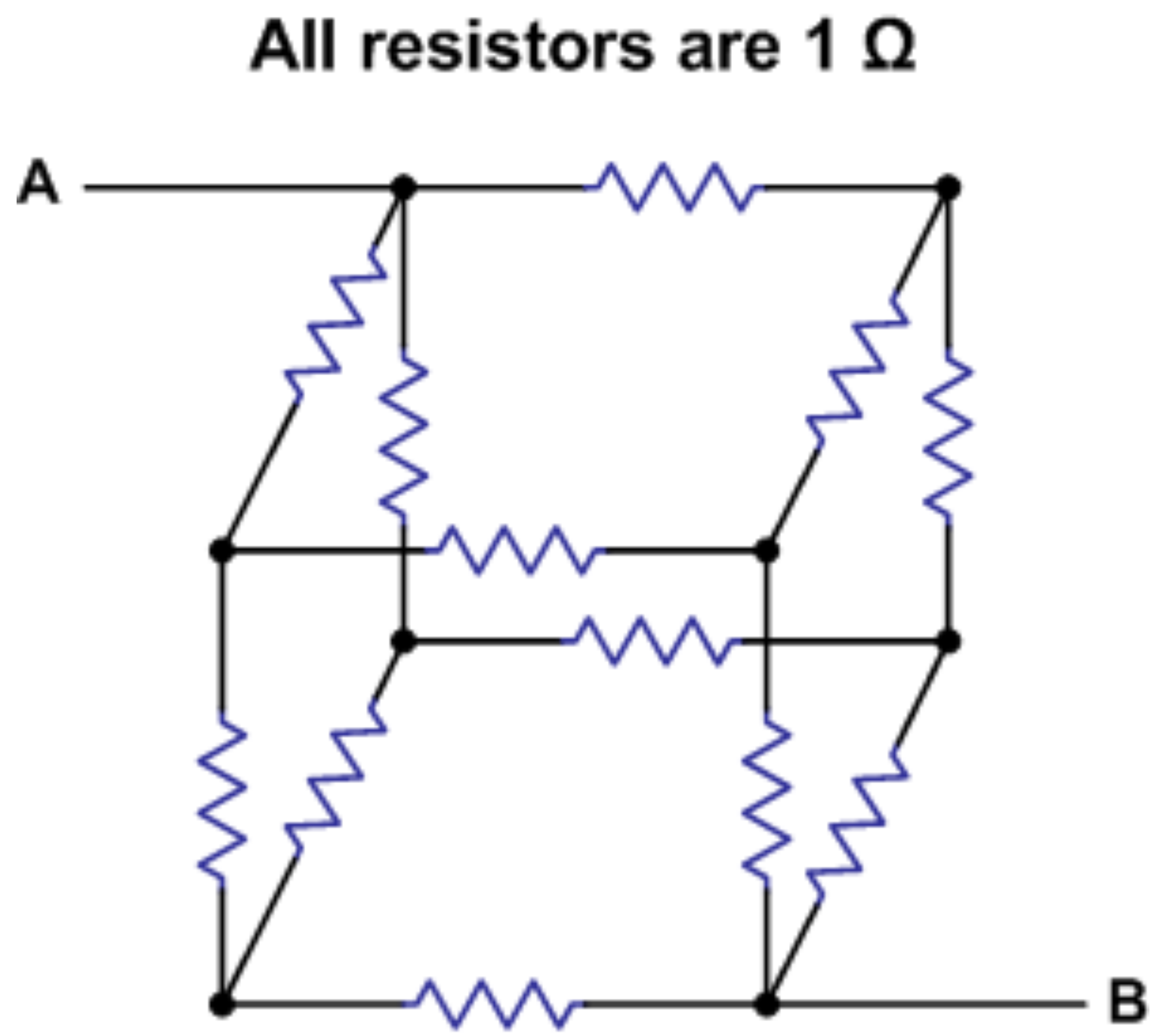
$$i_1 = \left(\frac{5}{4} - \frac{3}{8}\right) \frac{\mathcal{E}_0}{R} = \frac{7}{8} \cdot \frac{\mathcal{E}_0}{R}$$

Note

A **negative** current means that it is flowing in **opposite direction** from one assumed

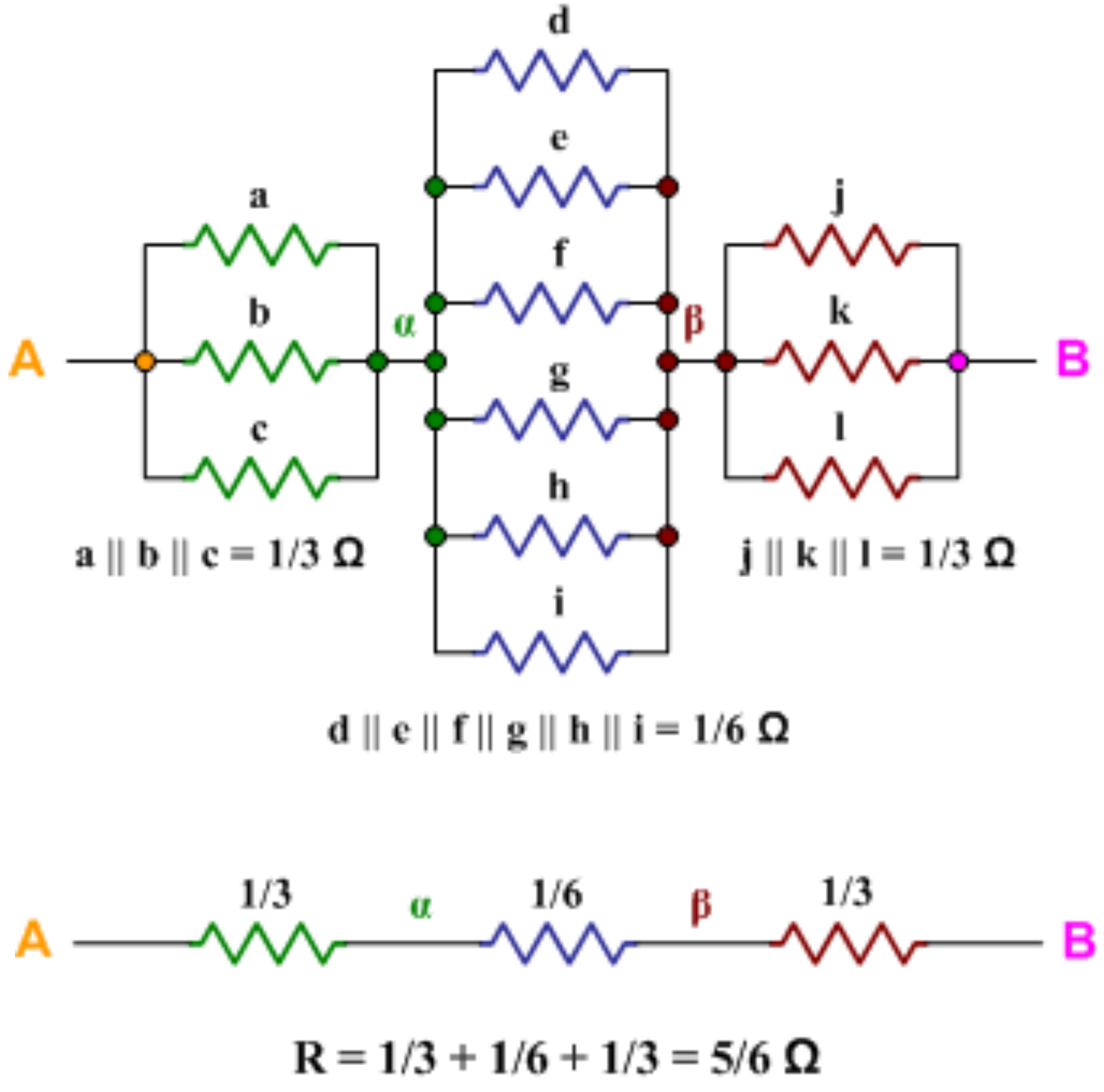
Intuition

- ➢ Each of the 12 edges of a cube contain a 1 Ω resistor



Calculate the equivalent resistance between two opposing corners

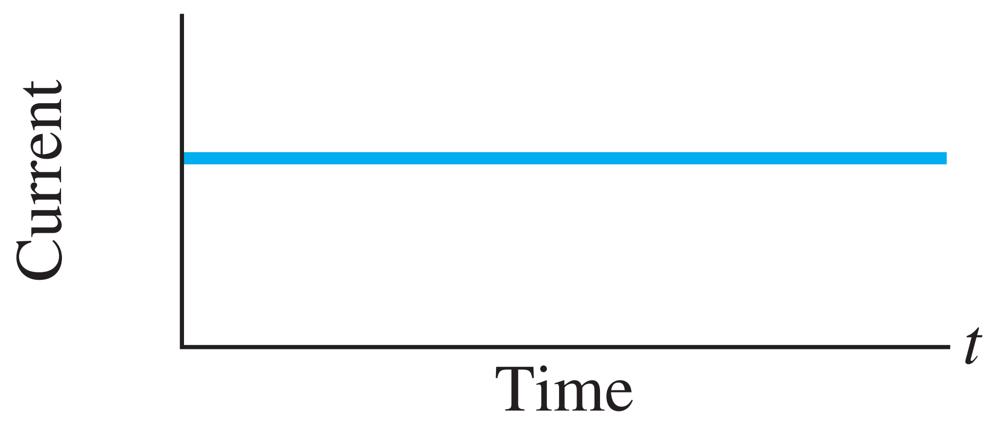
➢ There are two sets of three resistors in parallel in series with one set of six resistors in parallel

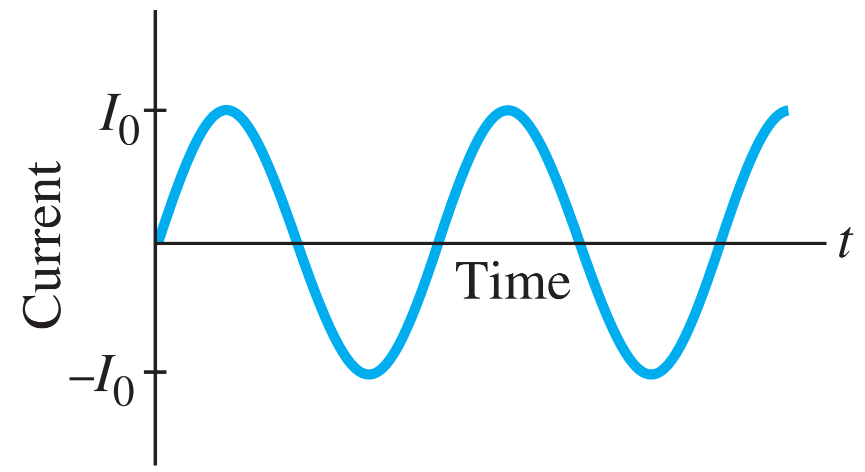


So ☞ you have 1/3 Ω in series with 1/6 Ω in series with 1/3 Ω ➧ which equals 5/6 Ω

Alternating Current

✱ An alternating current reverses direction many times per second and is commonly sinusoidal





Voltage produced by an ac electric generator is sinusoid ☛ $V = V_0 \sin(2\pi f t) = V_0 \sin(\omega t)$

Potential V oscillates between + V_0 and – V_0 and V_0 is referred to as the **peak voltage**

Frequency f is the number of complete oscillations made per second

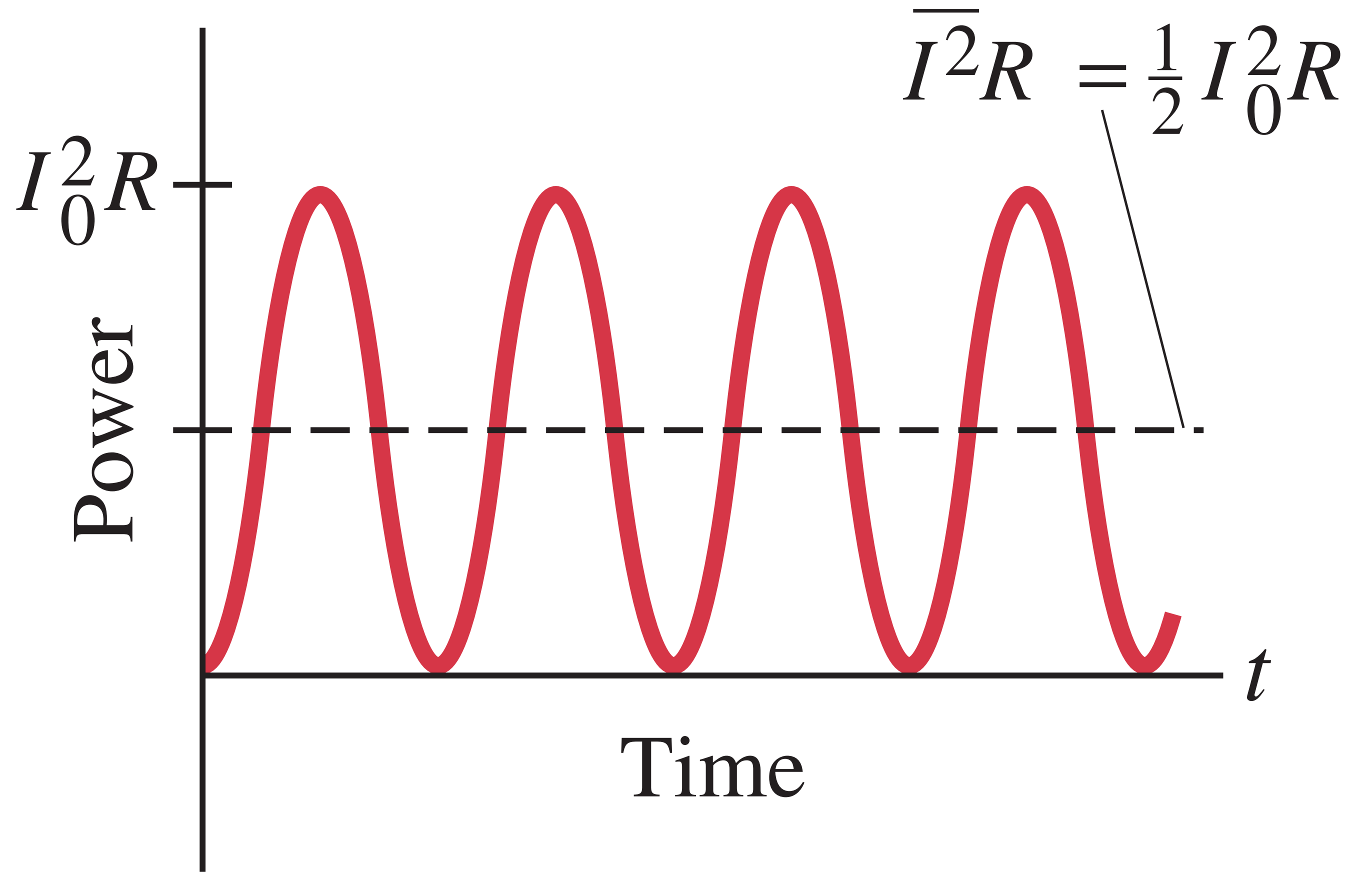
$V = IR$ works also for AC

If a voltage V exists across a resistance R ☛ current I through the resistance is ☛ $I = \frac{V}{R} = \frac{V_0}{R}\sin(\omega t) = I_0 \sin(\omega t)$

$I_0 = V_0/R$ ☛ **peak current**

Current is considered positive when electrons flow in one direction and negative when they flow in opposite direction

➢ Power transformed in a resistance R at any instant is ☛ $P = I^2 R = I_0^2 R \sin^2(\omega t)$



➢ A graph of $\cos^2(\omega t)$ versus time is identical to that for $\sin^2(\omega t)$ except that the points are shifted (by 1/4 cycle) on the time axis

➢ Value of $\cos^2(\omega t)$ and $\sin^2(\omega t)$ averaged over one or more full cycles. will be the same

➢ From the trigonometric identit $\sin^2\alpha + \cos^2\alpha = 1$ and $\overline{\sin^2(\omega t)} + \overline{\cos^2(\omega t)} = 2\,\overline{\sin^2(\omega t)} = 1$

we can write $\therefore \overline{\sin^2(\omega t)} = \dfrac{1}{2} \Rightarrow \overline{P} = \dfrac{1}{2} I_0^2 R$

since ☛ $P = V^2/R = (V_0^2/R)\sin^2(\omega t) \Rightarrow \overline{P} = \dfrac{1}{2}\dfrac{V_0^2}{R}$

➢ The root-mean square (rms) of effective values are defined as

$$I_{\text{rms}} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

$$V_{\text{rms}} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707 V_0$$

➢ The average power can be rewritten as

$$\overline{P} = I_{\text{rms}} V_{\text{rms}}$$

$$\overline{P} = \frac{1}{2} I_0^2 R = I_{\text{rms}}^2 R$$

$$\overline{P} = \frac{1}{2} \frac{V_0^2}{R} = \frac{V_{\text{rms}}^2}{R}$$

➢ In US And Canada ☛ $V_{\text{rms}} = 120\ \text{V}$

➢ In Argentina, Europe, Australia ☛ $V_{\text{rms}} = 240\ \text{V}$

Hair Dryer

(a) Calculate the resistance and the peak current in a 1500-W hair dryer connected to a 120-V AC line.

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{1500\ \text{W}}{120\ \text{V}} = 12.5\ \text{A}.$$

Then

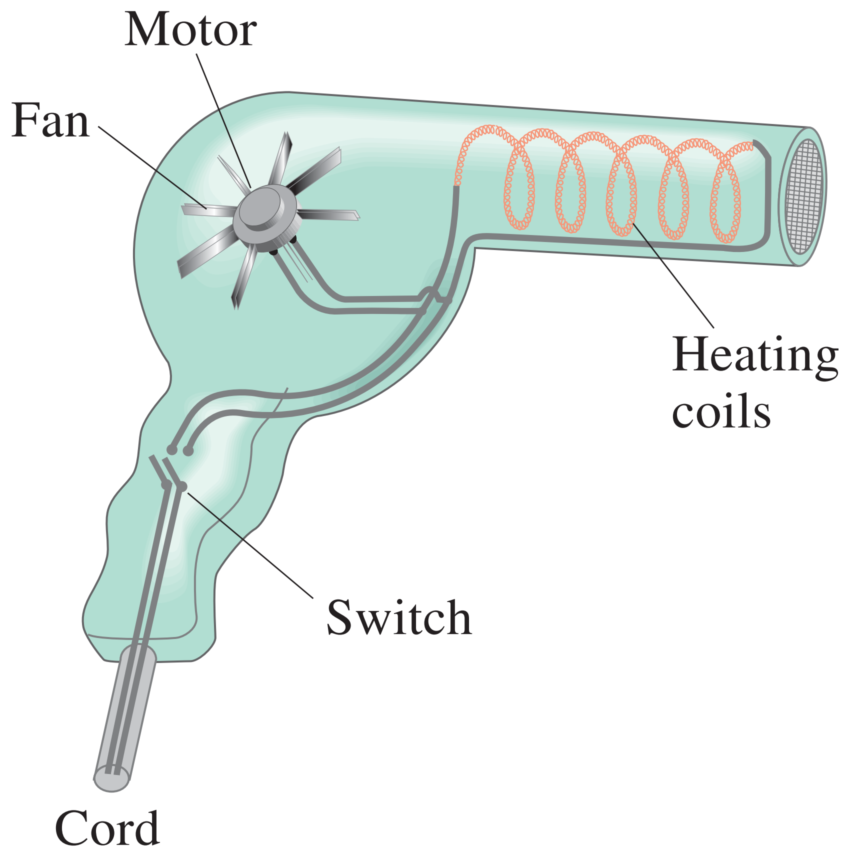
$$I_0 = \sqrt{2}\, I_{\text{rms}} = 17.7\ \text{A}.$$

The resistance is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\ \text{V}}{12.5\ \text{A}} = 9.6\ \Omega.$$

The resistance could equally well be calculated using peak values:

$$R = \frac{V_0}{I_0} = \frac{170\ \text{V}}{17.7\ \text{A}} = 9.6\ \Omega.$$



(b) What happens if it is connected to a 240-V AC line in Britain?

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{(240\ \text{V})^2}{(9.6\ \Omega)} = 6000\ \text{W}.$$

This is four times dryer's power rating and would undoubtedly melt heating element or wire coils of motor

Electron Speed in Wire

- ➢ A copper wire 3.2 mm in diameter carries a 5.0-A current
- ➢ Determine the drift velocity of the free electrons
- ➢ Assume that one electron per Cu atom is free to move (the others remain bound to the atom)
- ➢ To find the drift velocity we first determine the number *n* of free electrons per unit volume
- ➢ Since we assume there is one free electron per atom ☛ number density of free electrons *n* is the same as the number of C_u atoms per unit volume
- ➢ The atomic mass of C_u is 63.5u ☛ so. $m_{\text{Cu}} = 63.5$ g of C_u contains Avogadro's number of free electrons ☛

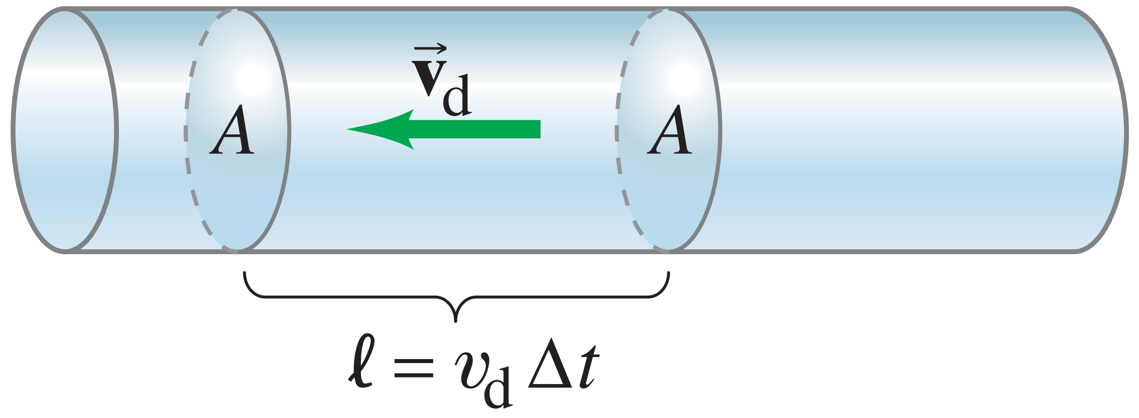
$$N = 6.02 \times 10^{23}$$

- ➢ To find the volume $V = \rho_{\text{Cu}}/m_{\text{Cu}}$ of this amount of copper we use the mass density of copper ☛

$$\rho_{\text{Cu}} = 8.9 \times 10^3 \ \text{kg/m}^3$$

$$n = \frac{N}{V} = \frac{N}{m_{\text{Cu}}/\rho_{\text{Cu}}} = \left(\frac{6.02 \times 10^{23} \ \text{electrons}}{63.5 \times 10^{-3} \ \text{kg}}\right) 8.9 \times 10^3 \ \frac{\text{kg}}{\text{m}^3} = 8.4 \times 10^{28} \ \text{m}^{-3}$$

➢ Cross-sectional area of wire is ☛ $A = \pi r^2 = \pi(1.6 \times 10^{-3}\ \mathrm{m})^2 = 8 \times 10^{-6}\ \mathrm{m}^2$



➢ Recall $\Delta Q = neAv_d\Delta t$ ☛ $I = \dfrac{\Delta Q}{\Delta t} = neAv_d$

➢ Finally ☛ drift velocity has magnitude

$$v_d = \frac{I}{neA} = \frac{5.0\ \mathrm{A}}{(8.4 \times 10^{28}\ \mathrm{m}^{-3})(1.6 \times 10^{-19}\ \mathrm{C})(8 \times 10^{-6}\ \mathrm{m}^2)}$$
$$= 4.6 \times 10^{-5}\ \mathrm{m/s} \approx 0.05\ \mathrm{mm/s}$$

Drift Speed of Electrons and Electric Current

But why is time for light to come on so short when electrons move so slowly?

Because electrons do not travel from the switch to the light to make it glow

In fact there are already plenty of electrons in light for light to turn on ☛ something just has to make those electrons move

$\vec{E}$ cause charges to move ☛ current starts as quickly as field spreads through wire (close to speed of light in the material)

The greater ΔV the stronger $\vec{E}$ and the faster charges end up moving

This is why higher voltage creates more current (Ohm's law!)

Spreading of field is still slow enough for those delays to matter in telecommunications ☛ but not so much for light switches!


aNy