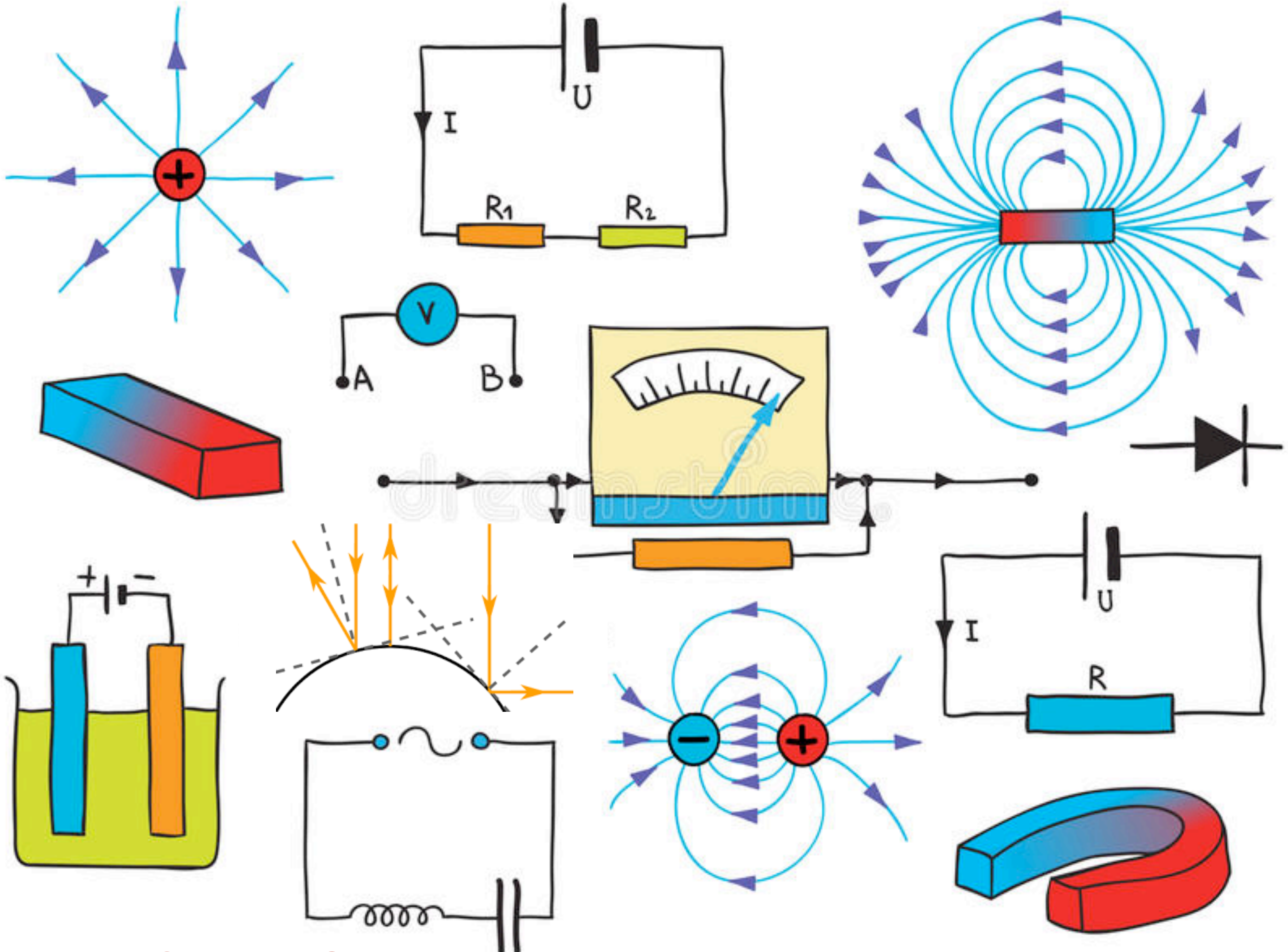


Physics 167

Luis Anchordoqui

Historical Overview

- In 19th century ☞ it was known that:
  - water waves must have medium to move across (water)
  - audible sound waves require medium to move through (e.g. air)
- It was thought that just as in previous examples ☞ light waves require medium called “luminiferous” (light-bearing) “æther”
- If this were the case ☞ as Earth moves in its orbit around Sun flow of æther across Earth could produce detectable “æther wind”
- Unless æther were always stationary with respect to Earth speed of beam of light emitted from source on Earth would depend on magnitude of æther wind and on beam direction
- 1881 Michelson-Morley experiment
  to measure speed of light in different directions
  became most famous failed experiment to date
  and first strong evidence against luminiferous æther

Historical Overview

(1892 -1909)

- To explain nature's apparent conspiracy to hide æther drift
  Lorentz developed theory based on two *ad hoc* hypotheses:
  - Longitudinal contraction of rigid bodies
  - slowing down of clocks (time dilation)

  when moving through æther at speed v ☞ both by $(1 - v^2/c^2)^{1/2}$
- This would so affect every aparatus designed to measure
  æther drift as to neutralize all expected effects

(1898)

- Poincare arugued that æther might be unobservable
  and suggested concept would be thrown aside as useless
  BUT he continued to use concept in later papers of 1908

(1905)

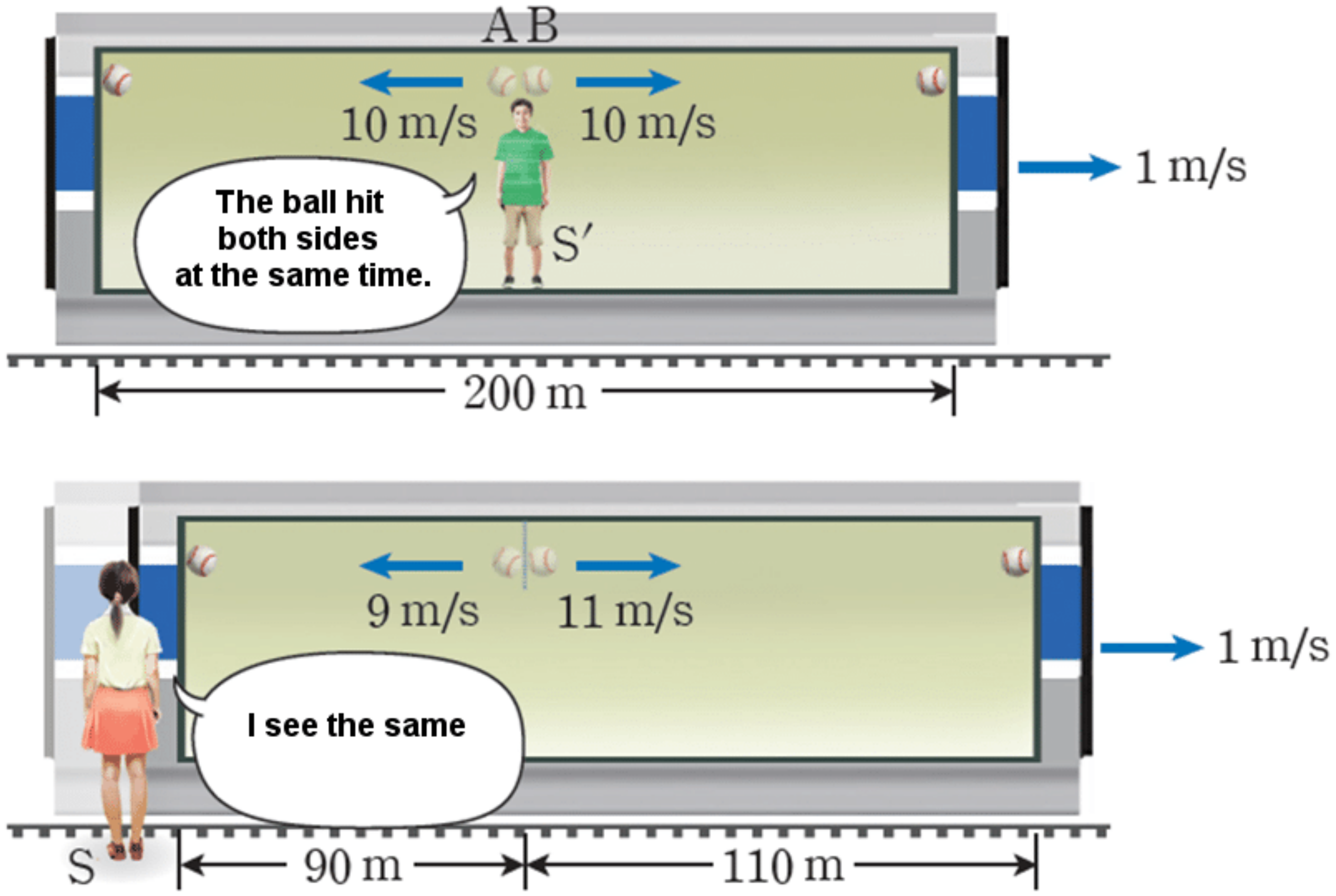
- Einstein advanced principle of relativity

Einstein Postulates

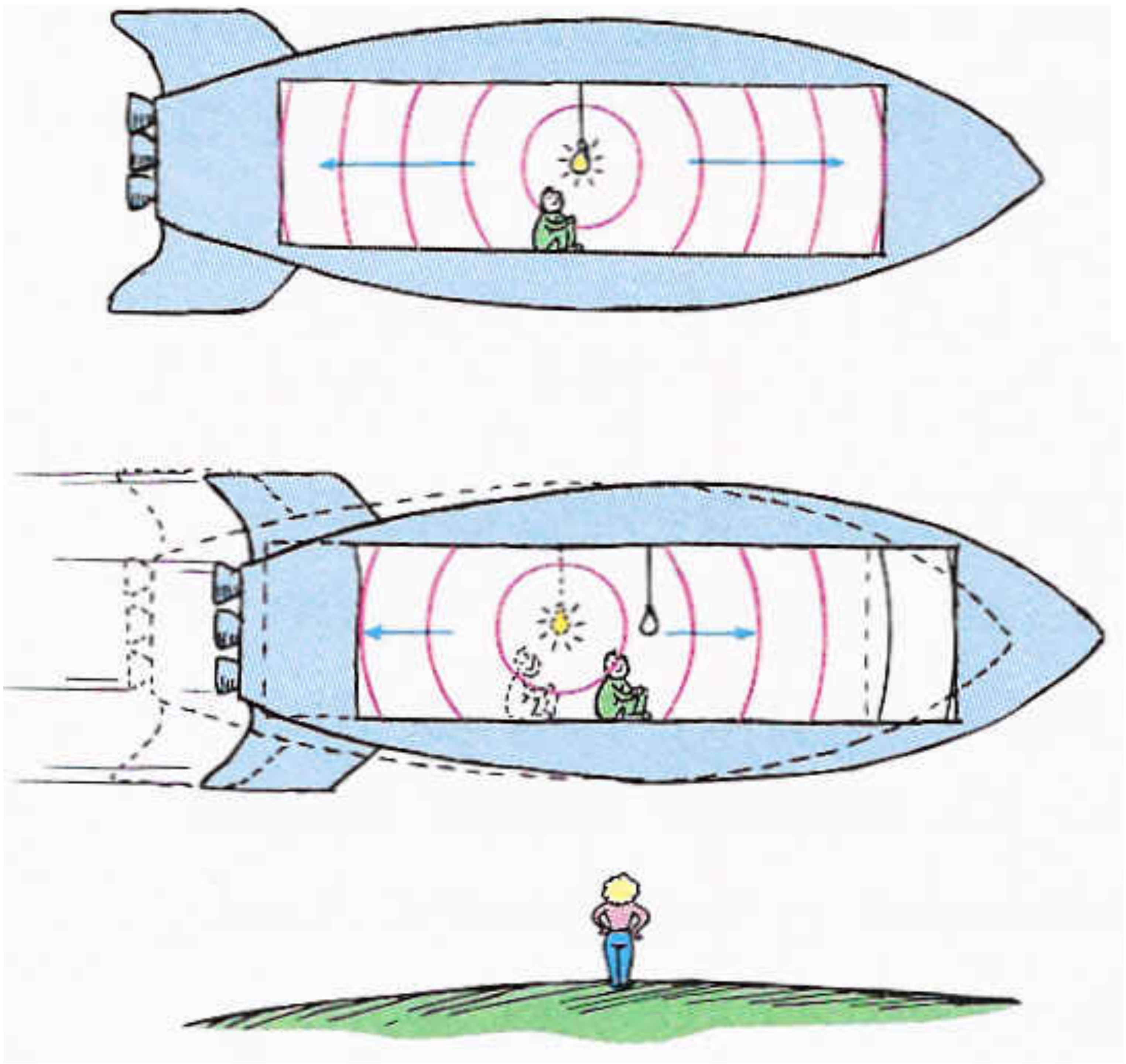
1. *All laws of nature are the same*
   in all uniformly moving reference frames
2. *Speed of light in free space has the same value for all observers regardless of the motion of source or motion of observer*
   ☞ *speed of light (in free space) is a constant*

Galileo and Science of Motion

- Galilean transformation relates coordinates of 2 reference frames which differ only by constant relative motion within constructs of Newtonian physics
- Two balls are launched simultaneously at same speed and opposite direction from center of wagon
- Balls are observed to hit simultaneously the ends of wagon in two reference frames



Relativity of simultaneity

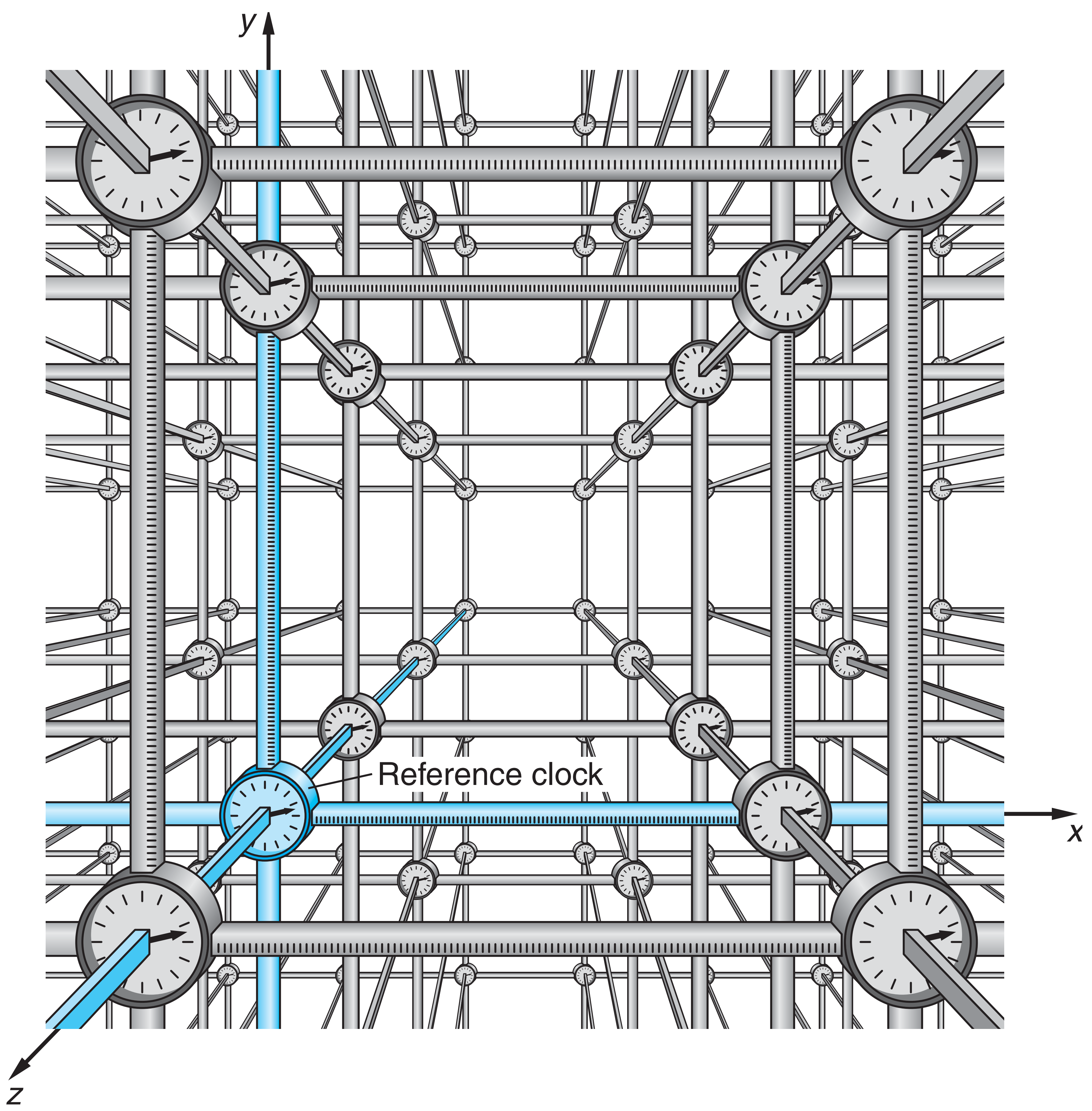


- From Harry's viewpoint light from travels equals distances to both ends of rocket ☞ striking both ends simultaneously
- Events of striking front and the end of spacecraft are not simultaneous in Sally's reference frame
- Because of rocket's motion light strikes back end sooner than front end

How Does Observer in Uniformly Moving Frame Descibe Event?

- Event ☞ an occurrence characterized by: three space coordinates and one time coordinate
- Events are described by observers who do belong to particular uniformly moving frames of reference
- Different observers in different uniformly moving (u.m.) frames would describe same event with different spacetime coordinates
- Observer's rest frame is also known as proper frame
- Up until now it was enough for us to have a measuring stick for each reference frame a rigid body that defined units of a coordinate system
- But we could all depend on just one clock a master timepiece that was used by all observers
- Now what we need is a measuring stick with clocks all along it so that when something happens we can record both time and place

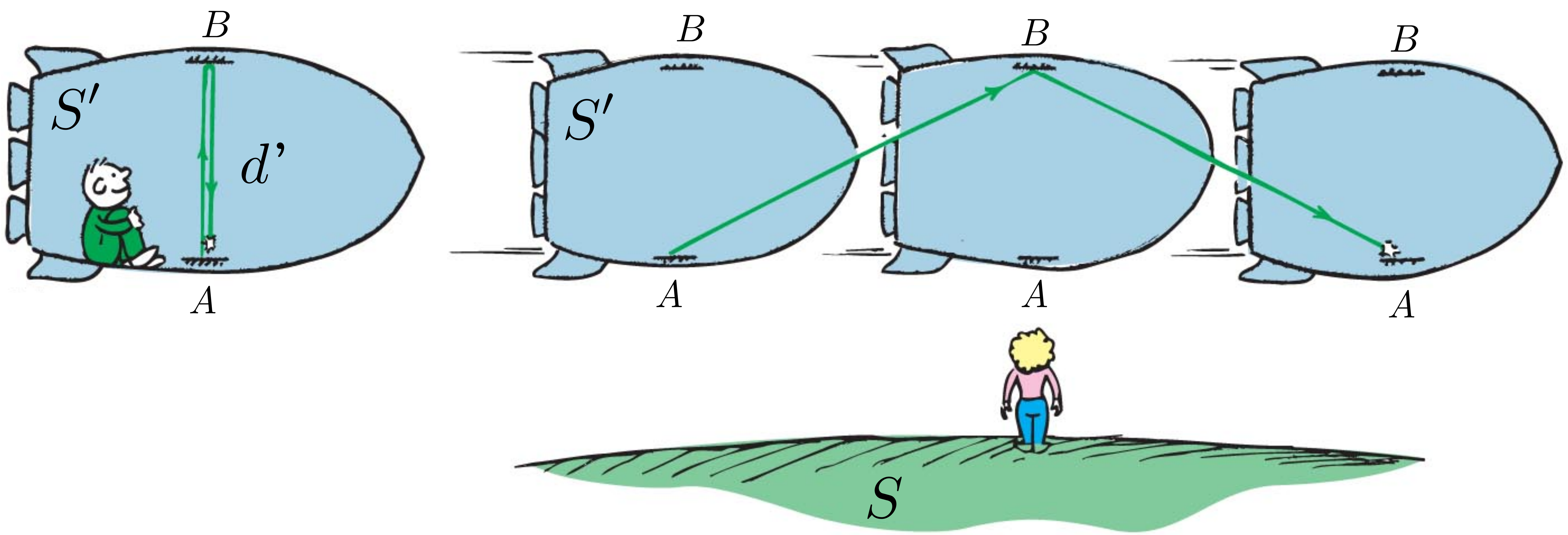
Confederate Scheme for Coordinatizing Any Event

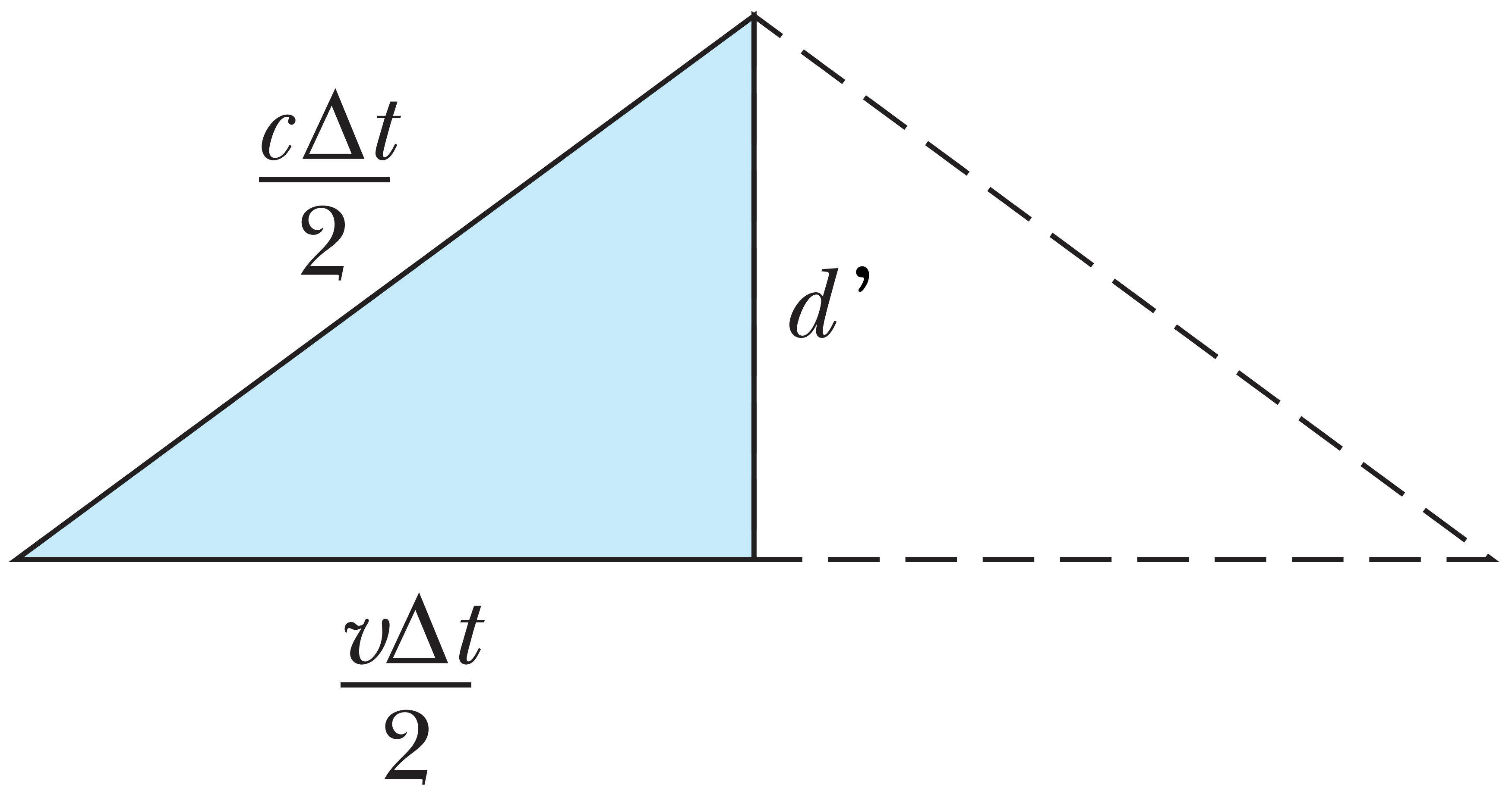


- Observer establishes lattice of confederates with identical synchronized clocks
- Label of any event in spacetime is reading of clock and location of nearest confederate to event

Einstein's Thought Experiment

- Idealized clock
  light wave is bouncing back and forth between two mirrors
- Clock "ticks" when light wave makes a round trip
  from mirror A to mirror B and back
- Assume mirrors A and B are separated a distance d' in rest frame
- Llight wave will take $\Delta t' = 2d'/c$ for round trip $A \to B \to A$





Time Dilation

Since light has velocity c in all directions

$$d'^2 + \left(v\,\frac{\Delta t}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$\Delta t = \frac{2d'}{\sqrt{c^2 - v^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

Ticking of clock in Hary's frame
which moves @ v wrt Sally in direction $\perp$ to separation of mirrors
is slower by $\gamma = (1 - v^2/c^2)^{-1/2}$

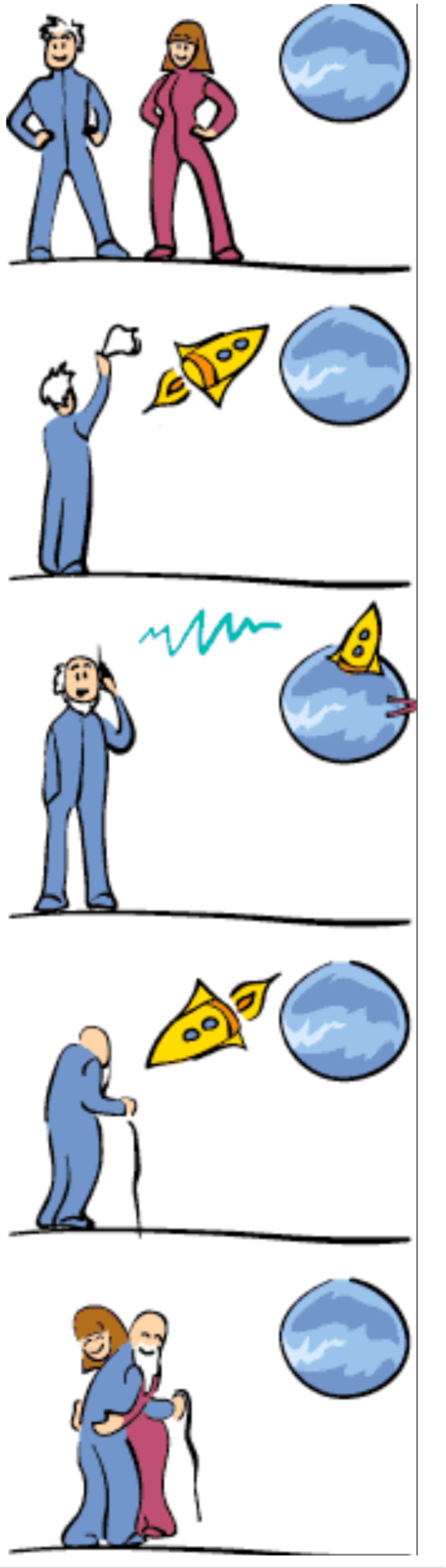
Twin Paradox

- Consider two synchronized standard clocks A and B at rest at point P of uniformly moving frame S
- Let A remain @ P while B is briefly accelerated to some velocity v with which it travels to distant point Q
- There it is decelerated and made to return with velocity v to P
- If one of two twins travels with B while other remains with A ☞ B twin will be younger than A twin when meet again

Can't B claim with equal right it was her who remained where she was while A went on round-trip ☞ A should be younger when meet again?

Answer is NO ☛ This solves paradox

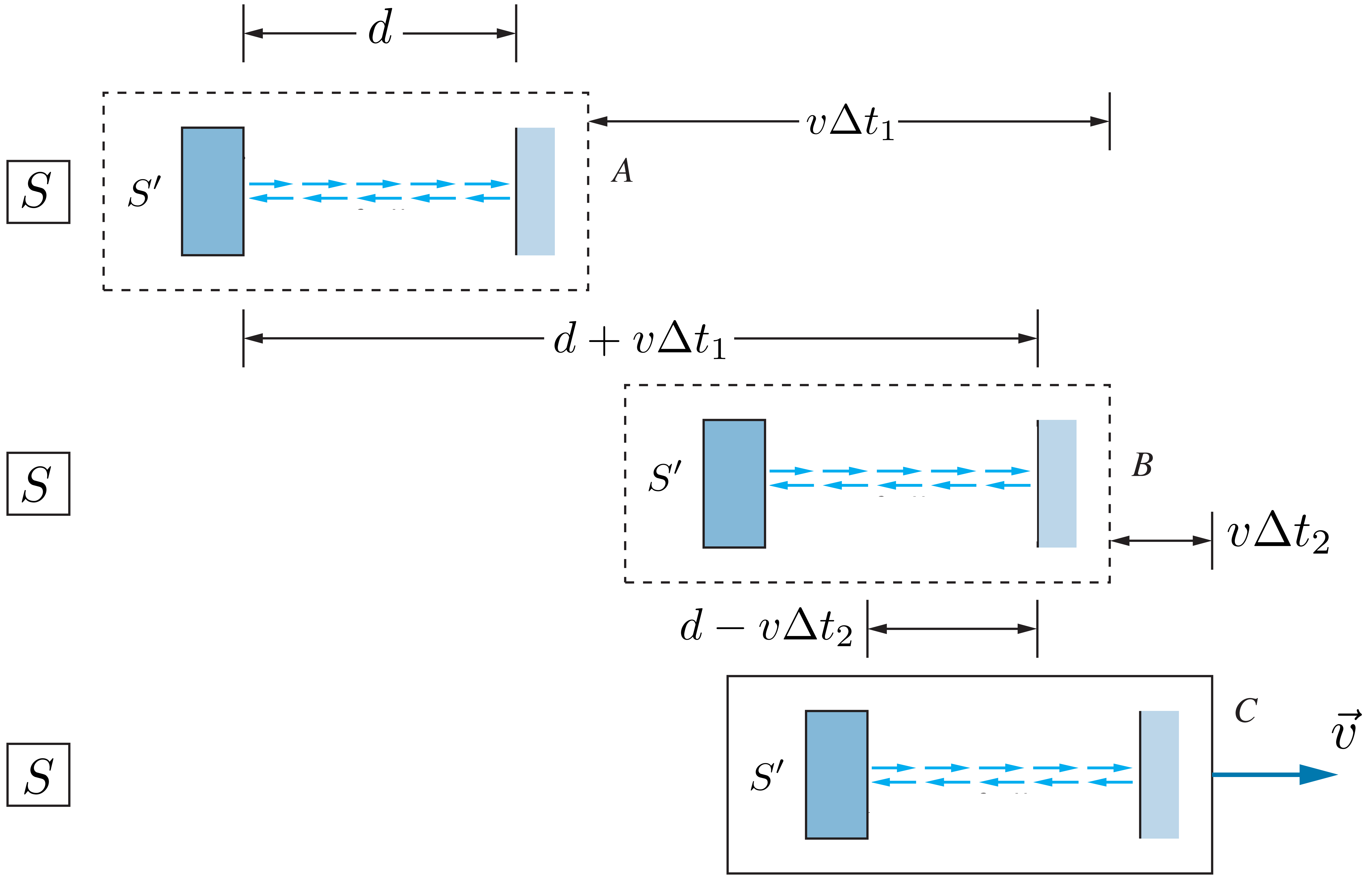
- A remained at rest in single u.m. frame while B accelerated out of his rest frame: @P, @Q, and once again @P
- Accelerations recorded on B's accelerometer she can be under no illusion that it was her who remain at rest
- Two accelerations at P are not essential (age comparison could be made in passing) but acceleration in Q is vital



Answer is NO ☛ This solves paradox

- Eperimet involves 3 u.m. frames:
  1. earth-bound frame S
  2. S' of outbound rocket
  3. S'' of returning rocket
- Experiment not symmetrical between twins:
  - A stays at rest in single uniformly moving frame S
  - but B occupies at least two different frames
- This allows result to be unsymmetrical

Rotate clock by 90° before setting it in motion



A-B

$$d + v\,\Delta t_1 = c\,\Delta t_1$$

$$\Delta t_1 = \frac{d}{c - v}$$

B-A

$$d - v\Delta t_2 = c\Delta t_2$$

$$\Delta t_2 = \frac{d}{c + v}$$

Length Contraction

1. Interval between two consecutive ticks in the moving frame is

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2d}{c(1 - v^2/c^2)}$$
$$= \left(\frac{d}{d'}\right) \frac{\Delta t'}{1 - v^2/c^2}$$

2. Because of time dilation

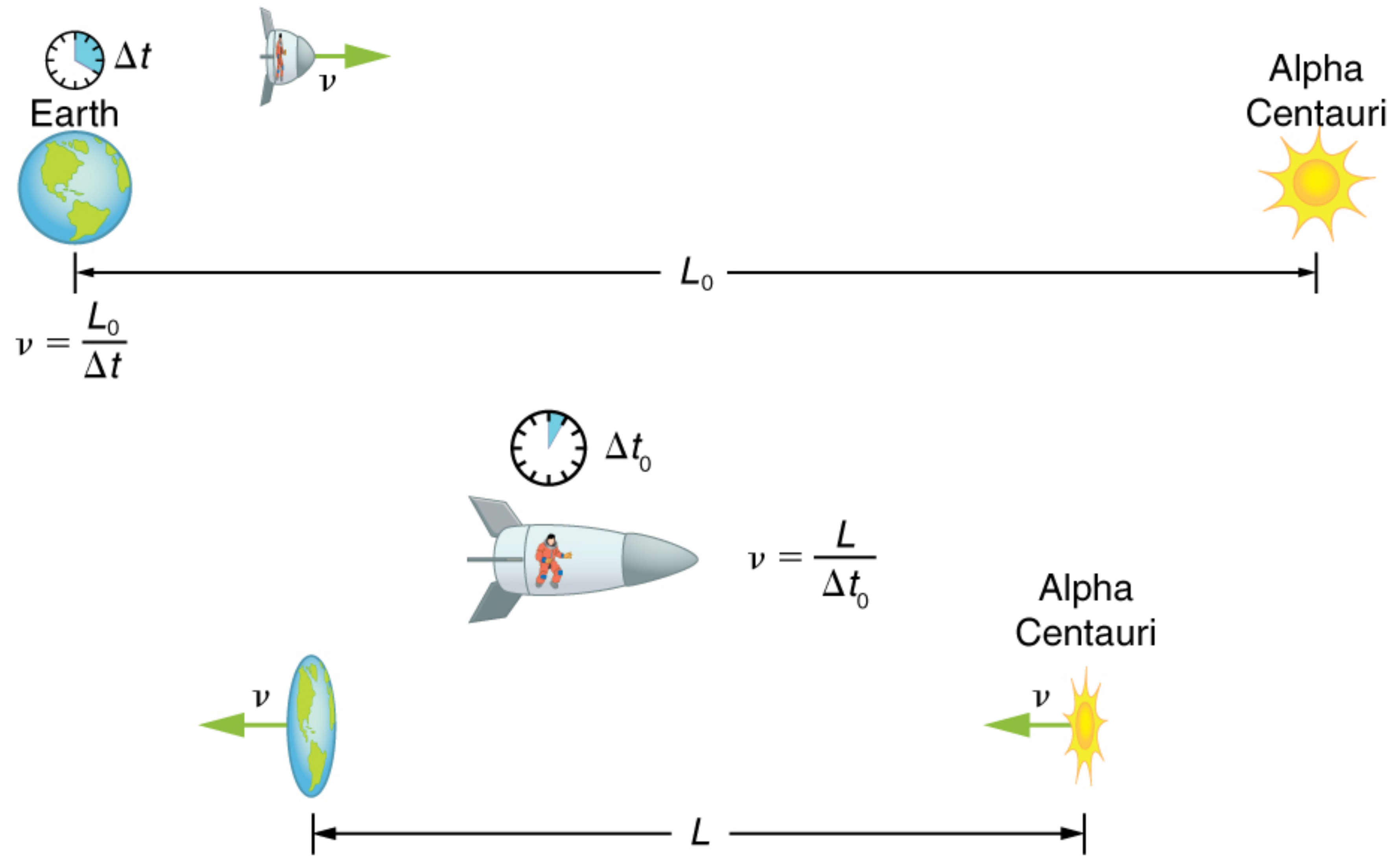
$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2}$$

we get

$$d = \left(1 - \frac{v^2}{c^2}\right)^{1/2} d'$$

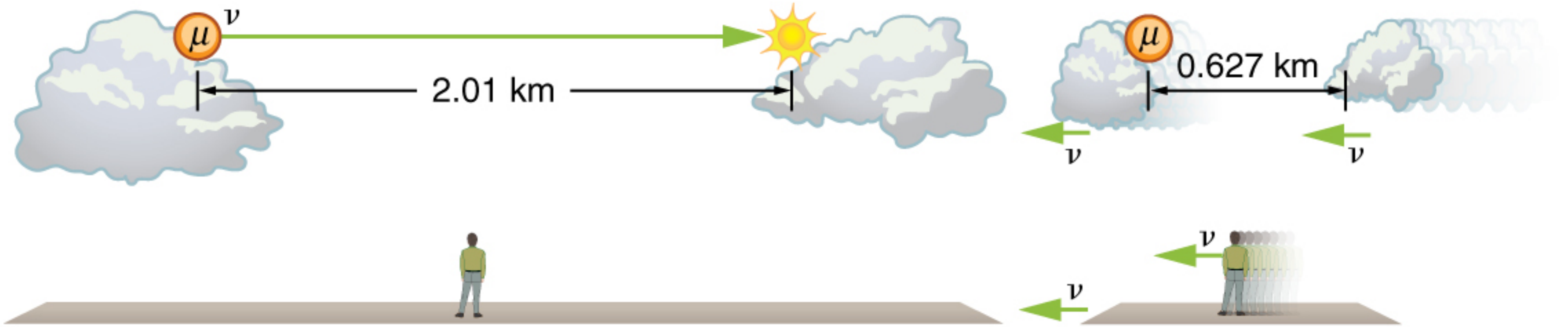
A Trip to Alpha Centauri

- One thing all observers agree upon is relative speed
- Even though clocks measure different elapsed times for same process ☞ they still agree that relative speed is the same
- Distance too depends on observer's relative motion!
- If two observers see different times ☞ they must also see different distances for relative speed to be same to each of them



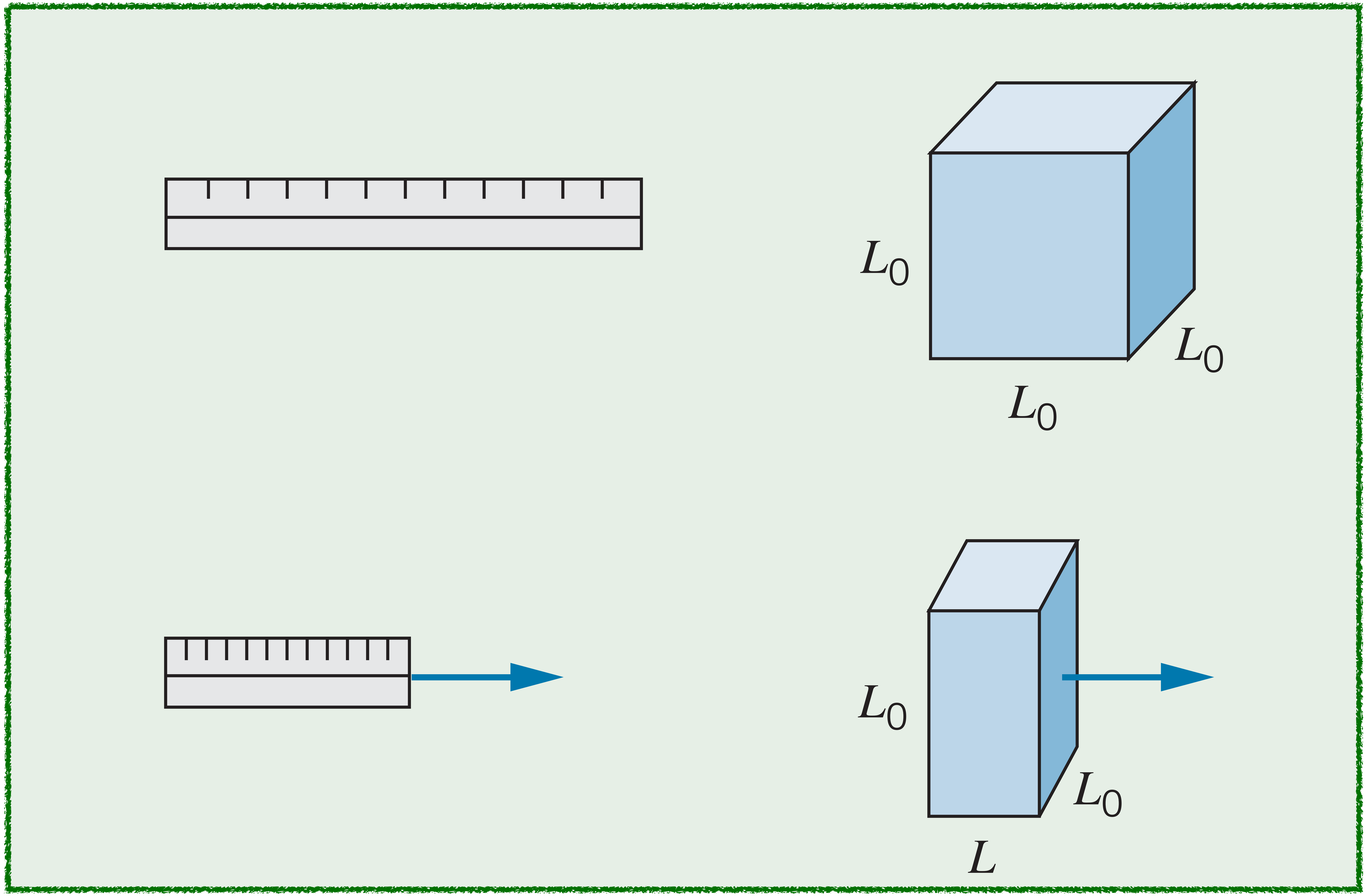
Life of a Muon

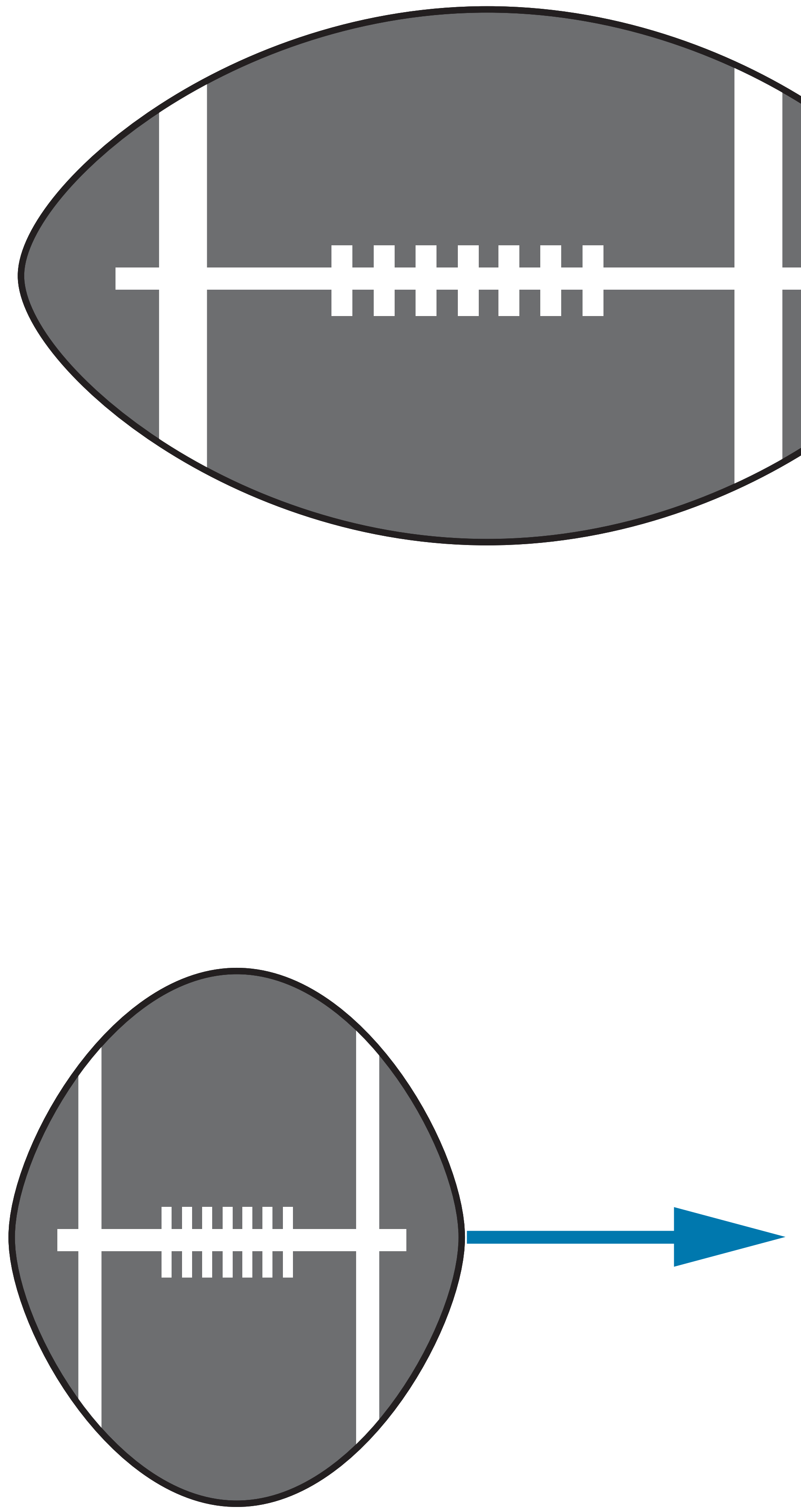
- Earth-bound observer sees muon travels at $0.95c$ for $7.05\ \mu s$ from time it is produced until it decays
- It travels distance ☞ $L_0 = v\,\Delta t = 2.1$ km relative to Earth
- In muon's rest frame ☞ its lifetime is only $2.20\ \mu s \Rightarrow$ it has enough time to travel only $L = v\Delta t_0 = 0.627$ km
- Distance between same two events (muon production and decay) depends on who measures it + how they are moving relative to it



Einstein time dilation factor agrees with experiment with fractional error of 2×10^{-3} at 95% confidence!

Example





12

Giving a Quick Rundown of $\vec{E} \leftrightharpoons \vec{B}$ dilema

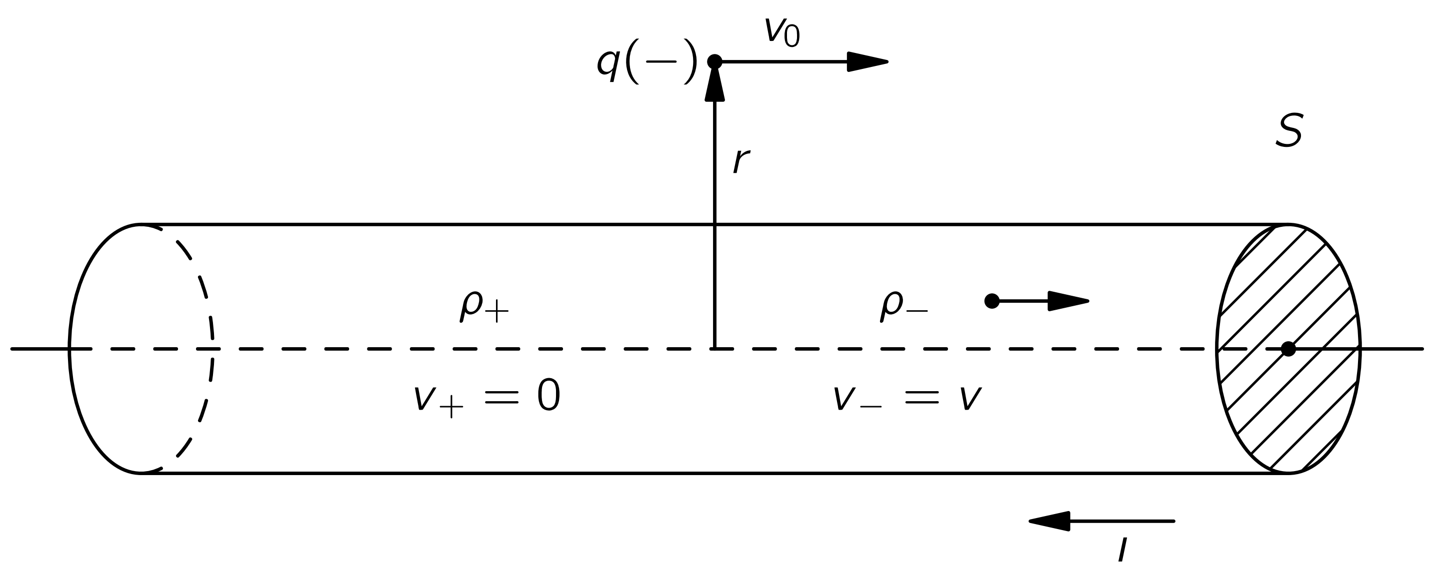
- When we said that magnetic force on charge was proportional to its velocity ☞ you may have wondered:
  1. What velocity?
  2. With respect to which reference frame?
- From definition of $\vec{B}$ ☞ what this vector is depends on our choice of reference frame for specification of velocity of charges
- But we have said nothing about which is the proper frame for specifying the magnetic field
- It turns out that any inertial frame will do
- Although static Maxwell's equations separate into $\vec{E}$ and $\vec{B}$ with no apparent connection between the two fields ☞ in nature there is intimate relation between them that arises from relativity principle
- Let's see what our knowledge of relativity would tell us about magnetic forces if we assume that relativity principle is applicable – as it is – to electromagnetism

Feynman's Example

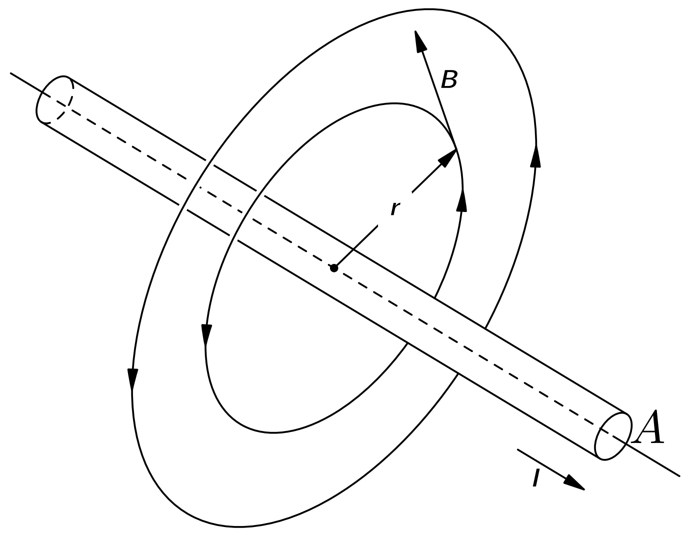
- Think about what happens when negative charge moves with velocity v_0 parallel to current-carrying wire
- Try to understand what goes on in two reference frames: one fixed wrt wire (S) and one fixed wrt particle (S')
- In S-frame ☞ there is magnetic force on particle
- Force is directed toward wire ☞ if charge were moving freely we would see it curve in toward wire
- But in S'-frame there can be no magnetic force on particle ☞ because its velocity is zero
- Does it then stay where it is?
- Would we see different things happening in the two systems?
- Principle of relativity would say that in S' we should also see particle move closer to wire
- We must try to understand why that would happen

Atomic Description Of Current-Carrying Wire in S-Frame

- In conductor electric currents come from motion of negative conduction electrons while positive nuclear charges and remainder of electrons stay fixed in body of material
- ρ_- ☞ charge density of conduction electrons of velocity v
- ρ_+ ☞ density of charges at rest $= \rho_-$ ☞ wire is uncharged
- There is no $\vec{E}$ field outside wire
- Force on moving particle ☞ $\vec{F} = q\vec{v}_0 \times \vec{B}$



- Recall Ampère's law ☞ $\sum_{\substack{\text{closed}\\ \text{path}}} B_{\|}\,\Delta\ell = \mu_0 I_{\text{encl}}$
- $\vec{B}$ field at distance r from axis of wire: $B = \frac{1}{4\pi\epsilon_0 c^2}\frac{2I}{r}$
- $c = 1/\sqrt{\mu_0\epsilon_0}$

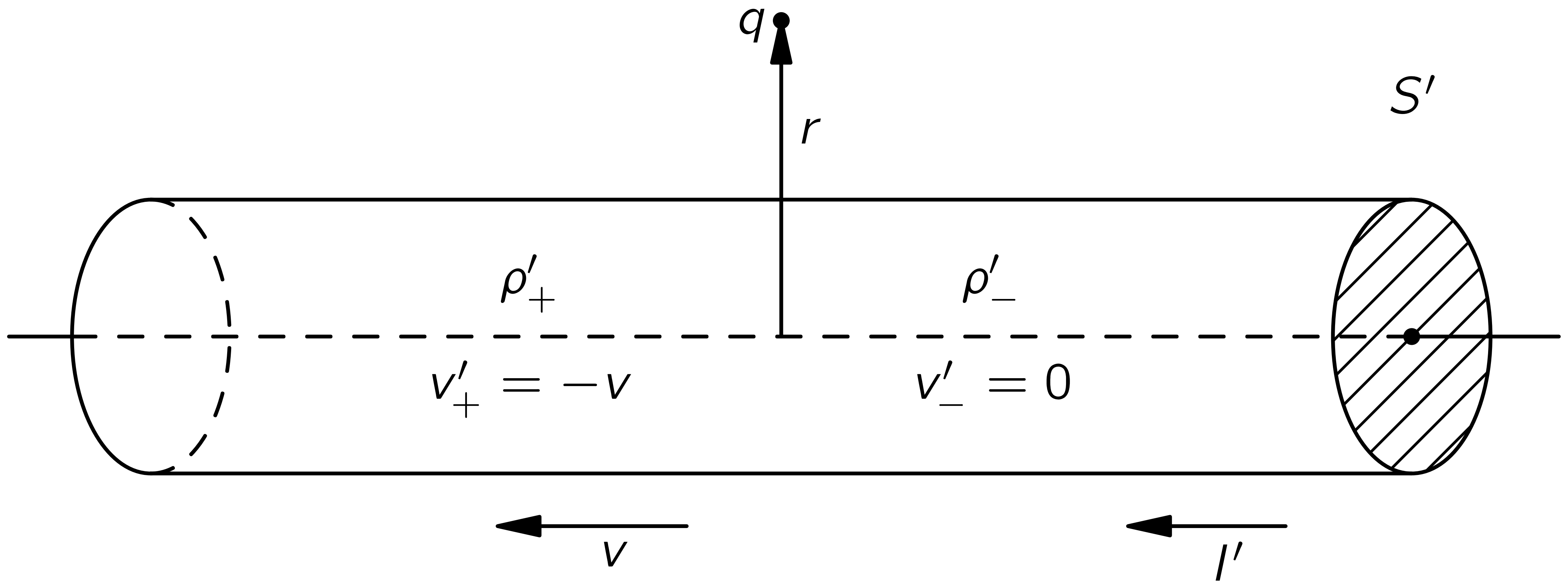


Force in S-Frame

- Conclude that:
  1. Force on particle is directed toward wire
  2. Force has magnitude ☞ $F = \frac{1}{4\pi\epsilon_0 c^2}\frac{2Iqv_0}{r}$
- Since $I = \rho_- v A$ ☞ $F = \frac{1}{4\pi\epsilon_0 c^2}\frac{2q\rho_- A v v_0}{r}$
- We could continue to treat general case of arbitrary velocities but it will be just as good to look at special case $v_0 = v$
- Taking $v_0 = v$ ☞ $F = \frac{q}{2\pi\epsilon_0}\frac{\rho_- A}{r}\frac{v^2}{c^2}$

What Happens in *S'* ?

- Particle is at rest and wire is running past with speed v
- Positive charges moving with wire will make some B' at particle
- But particle is now at rest ☞ there is no magnetic force on it!
- If there is any force on particle it must come from $\vec{E}$
- It must be that moving wire has produced an $\vec{E}$
- But it can do that only if it appears charged ☞ it must be that neutral wire with current appears to be charged when set in motion



Charge Density in *S and S'*

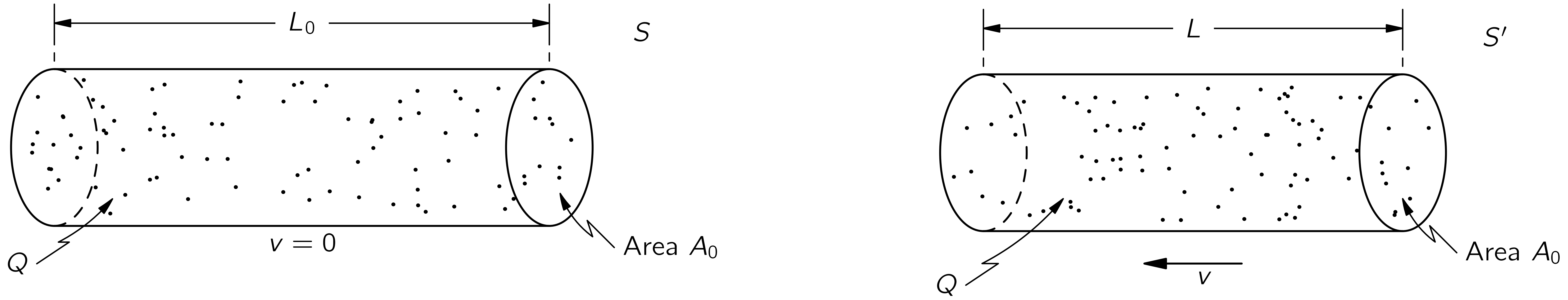
- Compute ρ of wire in S' from what's known about it in S
- Aren't ρ and ρ' the same?
- Charge q on particle is invariant scalar quantity
  ☞ independent of reference frame
- In any frame ☞ charge density of distribution of electrons is proportional to number of electrons per unit volume
- BUT we know that lengths are changed between S and S'
  ☞ so volumes will change also
- Since charge densities depend on volume occupied by charges
  ☞ densities will change too
- Must calculate:
  volume changes because of relativistic contraction of distances

Length Contraction of Current-Carrying Wire

- Take length L_0 of wire with charge density ρ_0 of stationary charges
- Total charge $Q = \rho_0 L_0 A_0$
- If same charges are observed in different frame moving v ☞ they will all be found in piece of material with shorter length

$$L = L_0\sqrt{1 - v^2/c^2}$$

but same area



Current and Charge Distribution Within Wire

- ρ ☞ density of charges in S
- Charge conservation implies:
  $Q = \rho L A_0 = \rho_0 L_0 A_0$ ☞ $\rho L = \rho_0 L_0 \Rightarrow \rho = \rho_0 / \sqrt{1 - v^2/c^2}$
- ρ_+ charges are at rest in S ☞ BUT move with spped v in S'
  $$\rho'_+ = \rho_+ / \sqrt{1 - v^2/c^2} \equiv \gamma \rho_+$$
- Negative charges are at rest in S' ☞ rest density $\equiv \rho_0 = \rho'_-$
  because they have density ρ_- when wire is at rest in S
  where speed of negative charges is v
- For conductor electrons ☞ $\rho_- = \gamma \rho'_- \Rightarrow \rho'_- = \rho_- \sqrt{1 - v^2/c^2}$
- In S' we have a net charge ☞ $\rho' = \rho'_+ + \rho'_- \neq 0$

$$\rho' = \rho_+ \frac{1}{\sqrt{1 - v^2/c^2}} + \rho_- \sqrt{1 - v^2/c^2}$$

- Since stationary wire is neutral ☞ $\rho_- = -\rho_+ \Rightarrow \rho' = \rho_+ \frac{v^2/c^2}{\sqrt{1 - v^2/c^2}}$

- Recall Gauss' law ☞ $\sum_{\substack{\text{closed} \\ \text{surface}}} E_\perp \, \Delta A = Q/\epsilon_0$
- Take $Q = \rho A L$ and $A = 2\pi r L$
- $\vec{E}$ field at distance r from axix of wire ☞ $E' = \frac{\rho' A}{2\pi\epsilon_0 r} = \frac{\rho_+ A v^2/c^2}{2\pi\epsilon_0 r\sqrt{1-v^2/c^2}}$

Force in *S'*-Frame

- Force on negatively charged particle in is also towards wire
- Magnitude of force in S' ☞ $F' = \frac{q}{2\epsilon_0}\frac{\rho_+ A}{r}\frac{v^2/c^2}{\sqrt{1-v^2/c^2}}$
- Comparing F with F' ☞ $F' = \frac{F}{\sqrt{1-v^2/c^2}}$
- For small velocities we've been considering ☞ $F = F'$!
- Conclude that ☞ for low velocities electricity and magnetism are just "two ways of looking at the same stuff"
- But wait ☞ things are even better than that!!!

No Contraction in Orthogonal Directions

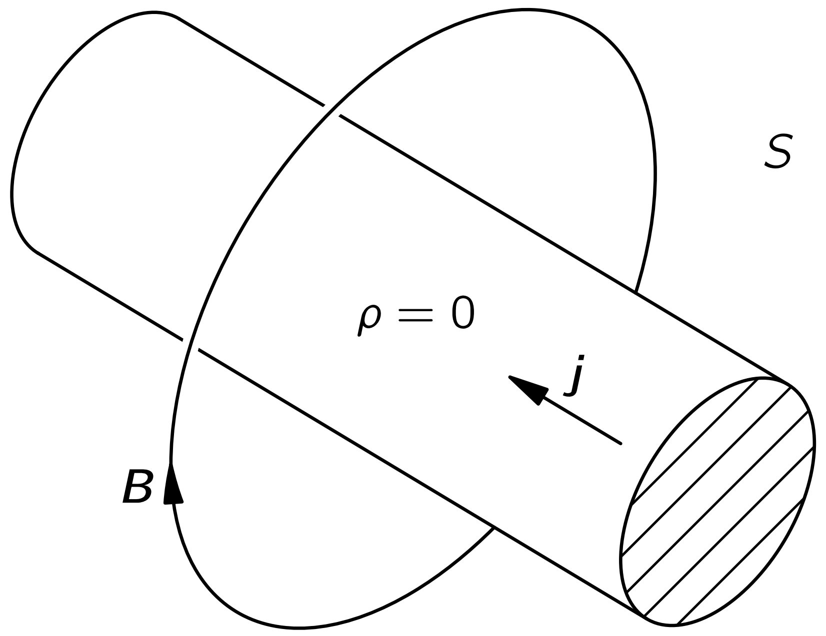
- What transverse momentum will particle have after force has acted for little while?
- Transverse momentum of particle should be the same in both S- and S'-frames
- Calling transverse coordinate y ☞ $\Delta p_y = F \Delta t$ and $\Delta p'_y = F' \Delta t'$
- We must compare Δp_y and $\Delta p'_y$ for time intervals Δt and $\Delta t'$
- Since particle is initially at rest in S' ☞ for small time interval

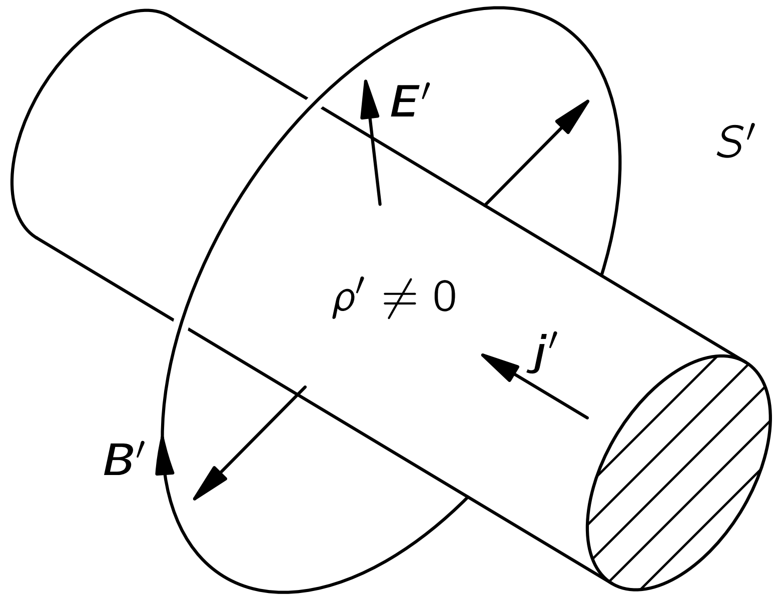
$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

- We conclude that

$$\frac{\Delta p'_y}{\Delta p_y} = \frac{F' \, \Delta t'}{F \, \Delta t} = 1!!!$$

Relativity of Electric and Magnetic Fields





- In S frame
  1. Charge density is zero and current density is J
  2. There is only $\vec{B}$ field
- In S' frame
  1. There is charge density $\rho' \neq 0$ and different current density J'
  2. $\vec{B}'$ field is different and there is $\vec{E}'$ field
- We must not attach too much reality to $\vec{E}$ and $\vec{B}$ "lines" ☞ they may disappear if weobserve them from different coordinate system
- Conclude that ☞ **electricity and magnetism are just "two ways of looking at the same stuff"**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