

1. Consider the hemispherical closed surface in Fig. 1. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through (i) the flat surface S_1 and (ii) the hemispherical surface S_2 .

Solution (i) $\Phi_B|_{\text{flat}} = \vec{B} \cdot \vec{A} = B\pi R^2 \cos(\pi - \theta) = -B\pi R^2 \cos \theta$. (ii) The net flux out of the closed surface is zero: $\Phi_B|_{\text{flat}} + \Phi_B|_{\text{curved}} = 0$, hence $\Phi_B|_{\text{curved}} = B\pi R^2 \cos \theta$.

2. A cube of edge length $\ell = 2.50$ cm is positioned as shown in Fig. 2. A uniform magnetic field given by $\vec{B} = (5\hat{i} + 4\hat{j} + 3\hat{k})$ T exists throughout the region. (i) Calculate the flux through the shaded face. (ii) What is the total flux through the six faces?

Solution (i) $\Phi_B = \sum \vec{B} \cdot \Delta\vec{A} = \vec{B} \cdot \vec{A} = (5\hat{i} + 4\hat{j} + 3\hat{k}) \text{ T} \cdot (2.50 \times 10^{-2} \text{ m})^2 \hat{i} = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = 3.12 \text{ mWb}$. (ii) $\Phi_B^{\text{total}} = \sum_{\text{surface}}^{\text{closed}} \vec{B} \cdot \Delta\vec{A} = 0$ for any closed surface (Gauss' law for magnetism).

3. A solenoid 2.50 cm in diameter and 30.0 cm long has 300 turns and carries 12.0 A. (i) Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as shown in Fig. 3 (a). (ii) Figure 3 (b) shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is defined by an annulus that has an inner radius of 0.400 cm and outer radius of 0.800 cm.

Solution $\Phi_B = \sum \vec{B} \cdot \Delta\vec{A} = BA_s$, where A_s is the cross-sectional area of the solenoid, $\Phi_B = \frac{\mu_0 NI}{L} \pi r^2 = 7.40 \mu\text{Wb}$. (ii) $\Phi_B = \sum \vec{B} \cdot \Delta\vec{A} = BA_a = \frac{\mu_0 NI}{L} \pi (r_2^2 - r_1^2) = 2.27 \mu\text{Wb}$.

4. Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00- Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails (Fig. 4). The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00- Ω resistor.

Solution Name the currents as shown in Fig. 4: left loop, $+Bdv_2 - I_2 R_2 - I_1 R_1 = 0$; right loop, $+Bdv_3 - I_3 R_3 + I_1 R_1 = 0$; and at the junction, $I_2 = I_1 + I_3$, where d is the length of the bar. Then, $Bdv_2 - I_1 R_2 - I_3 R_2 - I_1 R_1 = 0$, with $I_3 = \frac{Bdv_3}{R_3} + \frac{I_1 R_1}{R_3}$. Hence $Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3 R_2}{R_3} - \frac{I_1 R_1 R_2}{R_3} = 0$, yielding $I_1 = Bd \frac{v_2 R_3 - v_3 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} = 145 \mu\text{A}$ upward.

5. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Fig. 5. The magnitude of B inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?

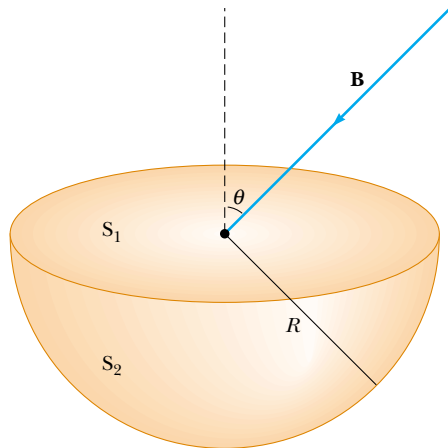


Figure 1: Problem 1.

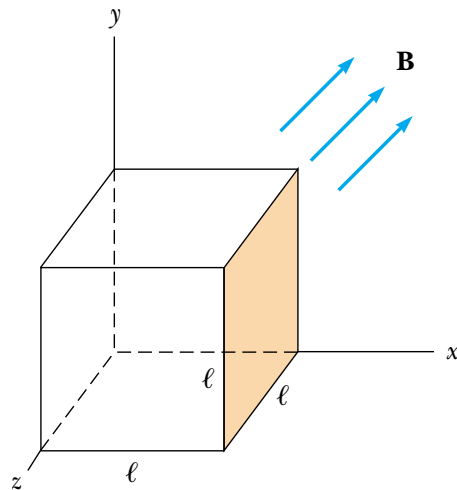


Figure 2: Problem 2.

Solution In the loop on the left, the induced emf is $|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = \pi(0.10 \text{ m})^2 100 \text{ T/s} = \pi \text{ V}$ and it attempts to produce a counterclockwise current in this loop. In the loop on the right, the induced emf is $|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = \pi(0.15 \text{ m})^2 100 \text{ T/s} = 2.25\pi \text{ V}$ and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00Ω resistor, I_2 flows down through the 5.00Ω resistor, and that I_3 flows up through the 3.00Ω resistor. From Kirchhoff's junction rule: $I_3 = I_1 + I_2$. Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$. Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ Solving these three equations simultaneously, $I_1 = 0.062 \text{ A}$, $I_2 = 0.860 \text{ A}$, and $I_3 = 0.923 \text{ A}$.

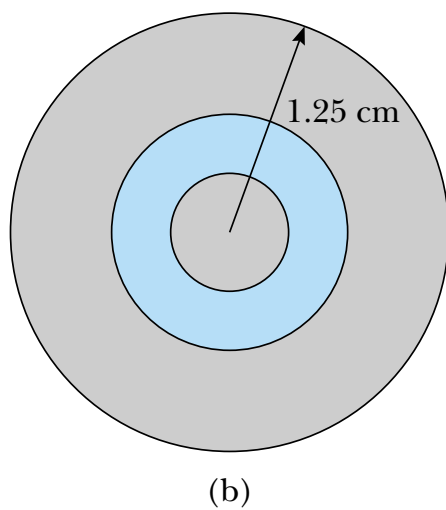
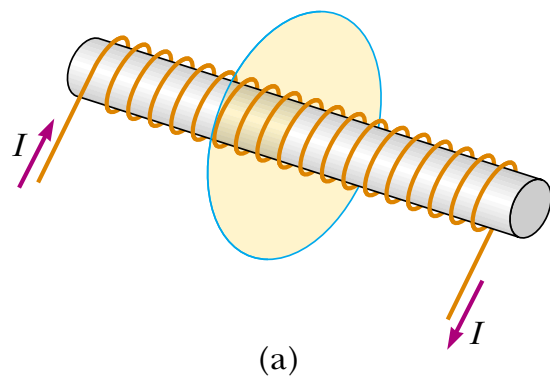


Figure 3: Problem 3.

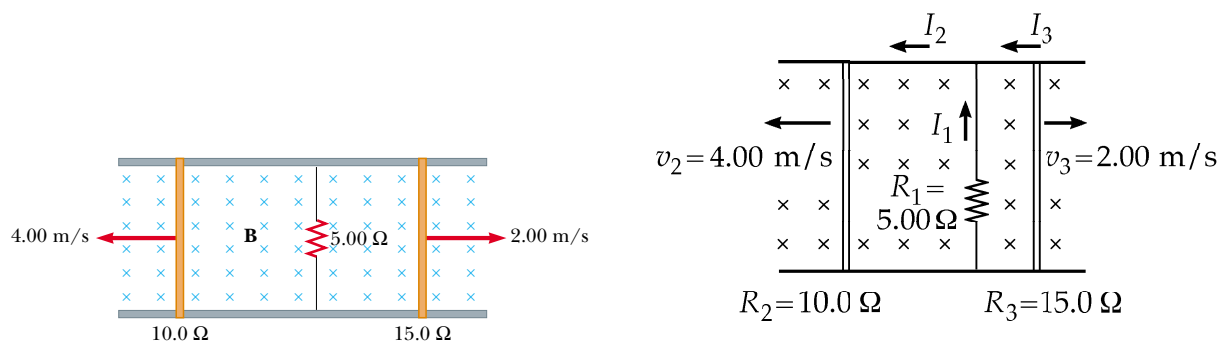


Figure 4: Problem 4.

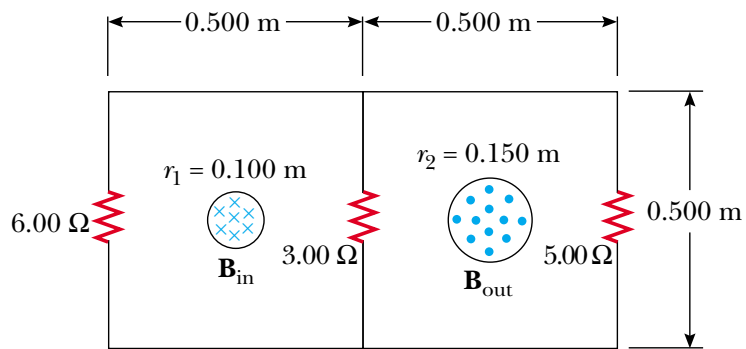


Figure 5: Problem 5.