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Problems set # 8 **Physics 167** **Solutions**

1. A particle of charge $-e$ is moving with an initial velocity v when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in Fig. 1. You may ignore effects of the gravitational force. *(i)* Is the trajectory of the particle deflected upward or downward? *(ii)* What is the magnitude of the velocity of the particle if it just strikes the end of the plate?

Solution: *(i)* Choose unit vectors as shown in Fig. 1. The force on the particle is given by $\vec{F} = -e(v\hat{\imath} \times B\hat{\jmath}) = -evB\hat{k}$. The direction of the force is downward. *(ii)* Remember that when a charged particle moves through a uniform magnetic field, the magnetic force on the charged particle only changes the direction of the velocity hence leaves the speed unchanged so the particle undergoes circular motion. Therefore we can use Newton's second law in the form $evB = m\frac{v^2}{R}$. The speed of the particle is then $v = eBR/m$. In order to determine the radius of the orbit we note that the particle just hits the end of the plate. From the figure above, by the Pythagorean theorem, we have that $R^2 = (R - d/2)^2 + l^2$ Expanding this equation yields $R^2 = R^2 - Rd + d^2/4 + l^2$. We can now solve for the radius of the circular orbit: $R = \frac{d}{4} + \frac{l^2}{d}$. We can now substitute this value in the equation for the velocity and find the speed necessary for the particle to just hit the end of the plate: $v = \frac{eB}{m}\left(\frac{d}{4} + \frac{l^2}{d}\right)$.

2. The entire $x - y$ plane to the right of the origin O is filled with a uniform magnetic field of magnitude B pointing out of the page, as shown in Fig. 2. Two charged particles travel along the negative x axis in the positive x direction, each with velocity $\vec{v}$, and enter the magnetic field at the origin O. The two particles have the same mass m, but have different charges, q_1 and q_2. When propagate thorugh the magnetic field, their trajectories both curve in the same direction (see sketch in Fig. 2), but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly twice as big as the radius of the semi-circle traced out by particle 1. *(i)* Are the charges of these particles positive or negative? Explain your reasoning. *(ii)* What is the ratio q_2/q_1?

Solution: *(i)* Because $\vec{F}_B = q\vec{v} \times \vec{B}$, the charges of these particles are positive. *(ii)* We first find an expression for the radius R of the semi-circle traced out by a particle with charge q in terms of q, $v \equiv |\vec{v}|$, B, and m. The magnitude of the force on the charged particle is qvB and the magnitude of the acceleration for the circular orbit is v^2/R. Therefore applying Newton's second law yields $qvB = \frac{mv^2}{R}$. We can solve this for the radius of the circular orbit $R = \frac{mv}{qB}$. Therefore the charged ratio $\frac{q_2}{q_1} = \frac{mv/(R_2B)}{mv(R_1B)} = \frac{R_1}{R_2}$.

3. Shown in Fig. 3 are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass m and charge $q = +e$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, S_1, in which there is an adjustable magnetic field $\vec{B}_1$, pointing

out of the page and a deflecting electric field $\vec{E}$, pointing from positive to negative plate. Only particles of a uniform velocity $\vec{v}$ leave the selector. The emerging particles at S_2, enter a second magnetic field B_2, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance x from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{E}|$, e, x, m, $B_2 \equiv |\vec{B}_2|$, and ΔV. *(i)* What magnetic field B_1 in the selector chamber is needed to insure that the particle travels straight through? *(ii)* Find an expression for the mass of the particle after it has hit the electronic sensor at a distance x from the entry slit.

Solution: *(i)* We first find an expression for the speed of the particle after it is accelerated by the potential difference ΔV, in terms of m, e, and ΔV. The change in kinetic energy is $\Delta K = \frac{1}{2}mv^2$. The change in potential energy is $\Delta U = -e\Delta V$. From conservation of energy, $\Delta K = -\Delta U$, we have that $\frac{1}{2}mv^2 = e\Delta V$. So the speed is $v = \sqrt{\frac{2e\Delta V}{m}}$ Inside the selector the force on the charge is given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B}_1)$. If the particle travels straight through the selector then force on the charge is zero, therefore $\vec{E} = -\vec{v} \times \vec{B}_1$. Because the velocity is to the right in Fig. 3 (define this as the $+\hat{\imath}$ direction), the electric field points up (define this as the $+\hat{\jmath}$ direction) from the positive plate to the negative plate, and the magnetic field is pointing out of the page (define this as the $+\hat{k}$ direction). Then $E\hat{\jmath} = -v\hat{\imath} \times B_1\hat{k} = vB_1\hat{\jmath}$. Thus, $\vec{B}_1 = \frac{E}{v}\hat{k} = \sqrt{\frac{m}{2e\Delta V}}E\hat{k}$. *(ii)* The force on the charge when it enters the magnetic field $\vec{B}_2$ is given by $\vec{F} = ev\hat{\imath} \times B_2\hat{k} = -evB_2\hat{\jmath}$. This force points downward and forces the charge to start circular motion. You can verify this because the magnetic field only changes the direction of the velocity of the particle and not its magnitude which is the condition for circular motion. Recall that in circular motion the acceleration is towards the center. In particular when the particle just enters the field $\vec{B}_2$ the acceleration is downward $\vec{a} = -\frac{v^2}{x/2}\hat{\jmath}$. Newton's Second Law becomes $-evB_2 = -m\frac{v^2}{x/2}$. Thus, the particle hits the electronic sensor at a distance $x = \frac{2mv}{eB_2} = \frac{2}{eB_2}\sqrt{2em\Delta V}$ from the entry slit. The mass of the particle is then $m = \frac{eB_2^2x^2}{8\Delta V}$.

4. The unit of magnetic flux is named for Wilhelm Weber. The practical-size unit of magnetic field is named for Johann Karl Friedrich Gauss. Both were scientists at Göttingen, Germany. Along with their individual accomplishments, together they built a telegraph in 1833. It consisted of a battery and switch, at one end of a transmission line 3 km long, operating an electromagnet at the other end. (André Ampérè suggested electrical signaling in 1821; Samuel Morse built a telegraph line between Baltimore and Washington in 1844.) Suppose that Weber and Gauss's transmission line was as diagrammed in Fig. 4. Two long, parallel wires, each having a mass per unit length of 40.0 g/m, are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current I, the wires repel each other so that the angle θ between the supporting strings is 16.0°. *(i)* Are the currents in the same direction or in opposite directions? *(ii)* Find the magnitude of the current.

Solution The separation between the wires is a $a = 2 \cdot 6.00cm \cdot \sin 8.00° = 1.67$ cm. *(i)* Because the wires repel, the currents are in opposite directions. *(ii)* Because the magnetic force acts horizontally, $\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a m g} = \tan 8.00°$, yielding $I^2 = \frac{mg2\pi a}{\mu_0 \ell}\tan 8.00°$ and so $I = 67.8$ A.

5. Figure 5 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by

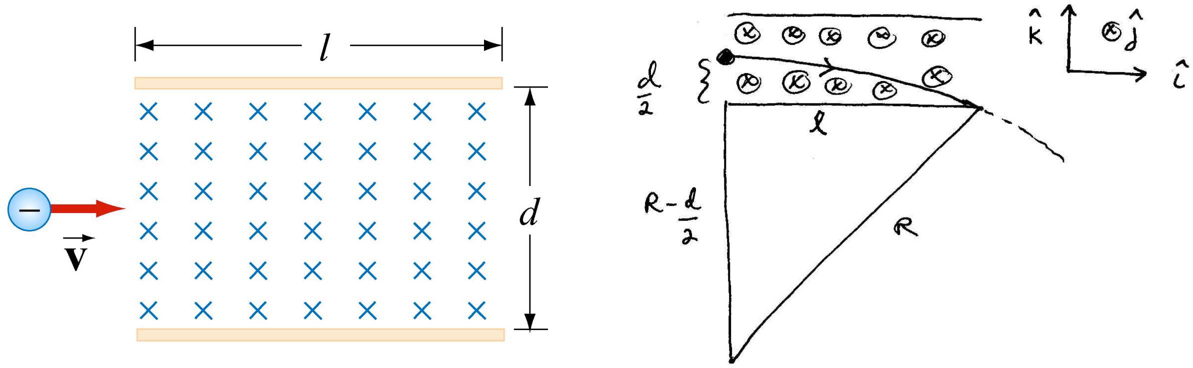

Figure 1: Problem 1.

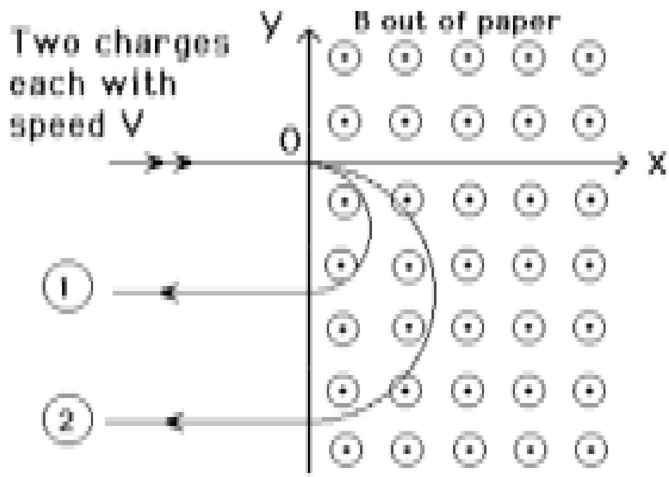

Figure 2: Problem 2.

a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1.00 A out of the page and the current in the outer conductor is 3.00 A into the page. Determine the magnitude and direction of the magnetic field at points a and b.

Solution From Ampere's law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so $B_a = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\cdot 1\ \text{A}}{2\pi\cdot1.00\times10^{-3}\ \text{m}} = 200\ \mu\text{T}$ toward top of the page. Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00$ A, or $I_b = 2.00$ A into the page. Therefore, $B_b = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\cdot 2\ \text{A}}{2\pi\cdot3.00\times10^{-3}\ \text{m}} = 133\ \mu\text{T}$ toward the bottom of the page.

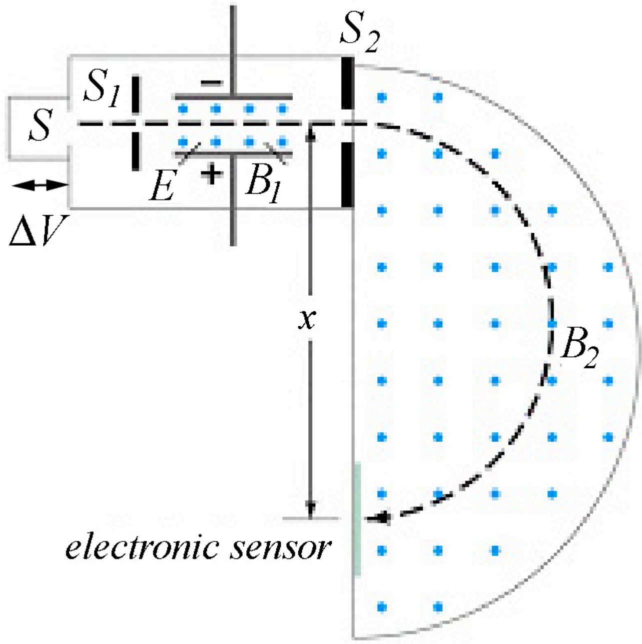

Figure 3: Problem 3.

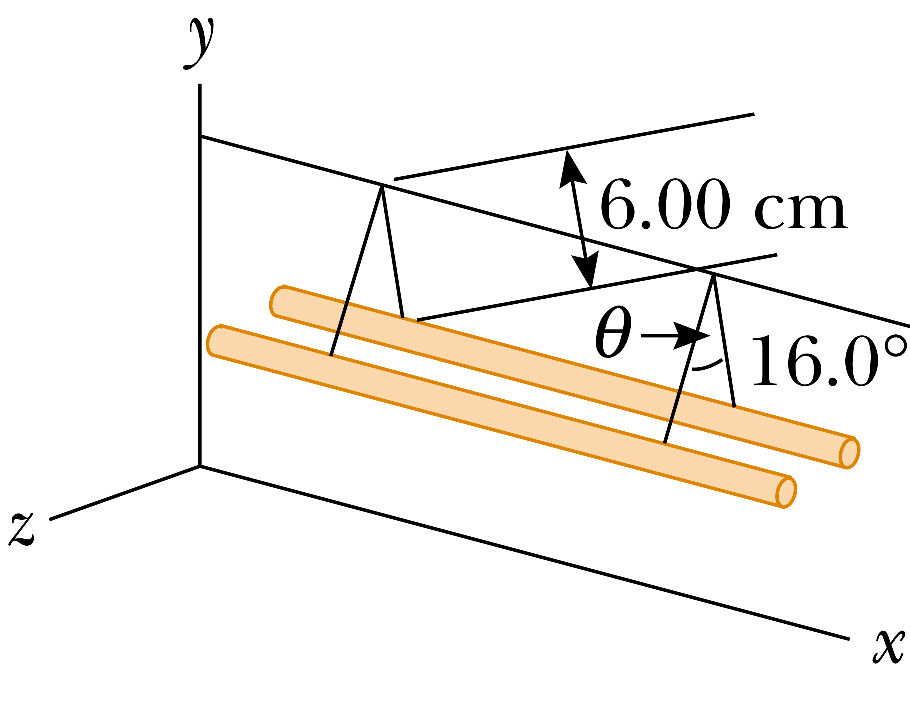

Figure 4: Problem 4.

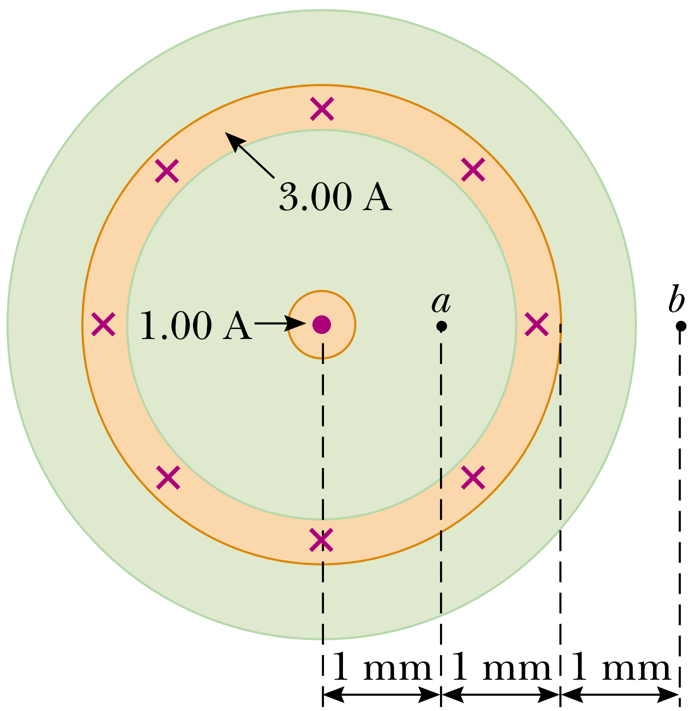

Figure 5: Problem 5.