Prof. Anchordoqui

Problems set # 3 **Physics 167** **Solutions**

1. A point charge q is located at the center of a uniform ring having linear charge density λ and radius a, as shown in Fig. 1. Determine the total electric flux through a sphere centered at the point charge and having radius R, where $R < a$.

Solution Only the charge inside radius R contributes to the total flux, hence $\Phi_E = q/\epsilon_0$.

2. An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$. *(i)* Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. *(ii)* Find an expression for the electric flux for $r > a$. *(iii)* Plot the flux versus r. [Part *(iii)* is optional. You have to figure out how the flux behaves at small and large radii.]

Solution The charge density is determined by $Q = \frac{4}{3}\pi a^3 \Rightarrow \rho = \frac{3Q}{4\pi a^3}$. *(i)* The flux is that created by the enclosed charge within radius r: $\Phi_E = \frac{q_{\rm in}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{Qr^3}{\epsilon_0 a^3}$. *(ii)* $\Phi_E = \frac{Q}{\epsilon_0}$. Note that the answers to parts *(i)* and *(ii)* agree at $r = a$. *(iii)* This is shown in Fig. 2.

3. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Fig. 3. *(i)* Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface. *(ii)* What is the direction of the electric field at $r > c$? *(iii)* Find the electric field at $r \geq c$. *(iv)* Find the electric field in the region with radius r where $b < r < c$ *(v)* Construct a spherical gaussian surface of radius r, where $b < r < c$, and find the net charge enclosed by this surface. *(vi)* Construct a spherical gaussian surface of radius r, where $a < r < b$, and find the net charge enclosed by this surface. *(vii)* Find the electric field in the region $a < r < b$. *(viii)* Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r. Note that the charge inside this surface is less than $3Q$. *(ix)* Find the electric field in the region $r < a$. *(x)* Determine the charge on the inner surface of the conducting shell. *(xi)* Determine the charge on the outer surface of the conducting shell. *(xii)* Make a plot of the magnitude of the electric field versus r.

Solution *(i)* $q_{\rm in} = 3Q - Q = 2Q$. *(ii)* The charge distribution is spherically symmetric and $q_{\rm in} > 0$. Thus, the field is directed radially outward. *(iii)* For $r \geq c$, $E = \frac{1}{4\pi\epsilon_0}\frac{q_{\rm in}}{r^2} = \frac{Q}{2\pi\epsilon_0 r^2}$. *(iv)* Since all points within this region are located inside conducting material, $E = 0$, for $b < r < c$. *(v)* $\Phi_E = \sum E_\perp \, \Delta A = 0 \Rightarrow q_{\rm in} = \epsilon_0 \Phi_E = 0$. *(vi)* $q_{\rm in} = 3Q$. *(vii)* For $a \leq r < b$, $E = \frac{1}{4\pi\epsilon_0}\frac{q_{\rm in}}{r^2} = \frac{3Q}{4\pi\epsilon_0 r^2}$ (radially outward). *(viii)* $q_{\rm in} = \rho V = \frac{3Q}{\frac{4}{3}\pi a^3}\frac{4}{3}\pi r^3 = 3Q\frac{r^3}{a^3}$. *(ix)* For $0 \leq r \leq a$, $E = \frac{q_{\rm in}}{4\pi\epsilon_0 r^2} = \frac{3Qr}{4\pi\epsilon_0 a^3}$

(radially outward). *(x)* From part *(iv)*, for $b < r < c$, $E = 0$.Thus, for a spherical gaussian surface with $b < r < c$, $q_{\rm in} = 3Q + q_{\rm inner} = 0$ where $q_{\rm inner}$ is the charge on the inner surface of the conducting shell. This yields $q_{\rm inner} = -3Q$. *(xi)* Since the total charge on the conducting shell is $q_{\rm net} = q_{\rm outer} + q_{\rm inner} = -Q$, we have $q_{\rm outer} = -Q - q_{\rm inner} = -Q - (-3Q) = 2Q$. *(xii)* This is shown in Fig. 3.

4. Consider an infinite cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

<u>Solution</u> If is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$, see Fig. 4. Because the charge distribution is infinite (L is very large), no electric flux passes through the circular end caps. Gauss' law, $\sum E_\perp \, \Delta A = \frac{q}{\epsilon_0}$. becomes $\sum E_\perp \, \Delta A = \frac{\rho \pi r^2 L}{\epsilon_0}$. Now the lateral surface area of the cylinder is $2\pi r L$, yielding $E 2\pi r L = \rho \pi r^2 L/\epsilon_0$. Thus, $\vec{E} = \frac{\rho r}{2\epsilon_0}$ radially away from the cylinder axis.

5. Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Fig. 5. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points *(i)* to the left of, *(ii)* in between, and *(iii)* to the right of the two sheets. *(iv)* Repeat the calculations when both sheets have positive uniform surface charge densities of value σ.

<u>Solution</u> Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets, see Fig. 5. The field at any distance from each sheet has a magnitude given by $|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$. *(i)* To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{E} = 0$. *(ii)* In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is $\vec{E} = \sigma/\epsilon_0$ to the right. *(iii)* To the right of the negative sheet, E_- and E_+ are again oppositely directed and $\vec{E} = 0$. *(iv)* If both charges are positive (see Fig. 5), in the region to the left of the pair of sheets, both fields are directed toward the left and the net field is $\vec{E} = \sigma/\epsilon_0$ to the left; in the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is $\vec{E} = 0$; in the region to the right of the pair of sheets, both are fields are directed toward the right and the net field is $\vec{E} = \sigma/\epsilon_0$ to the right.

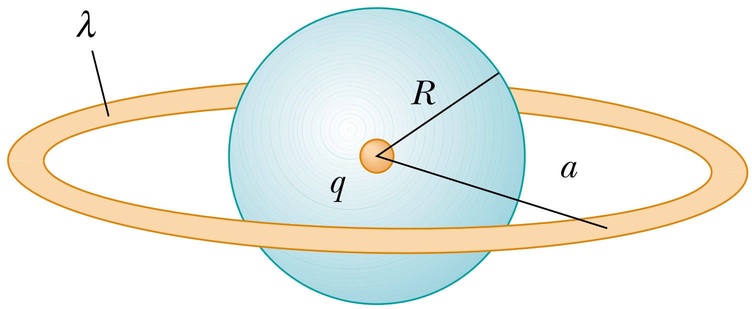

Figure 1: Problem 1.

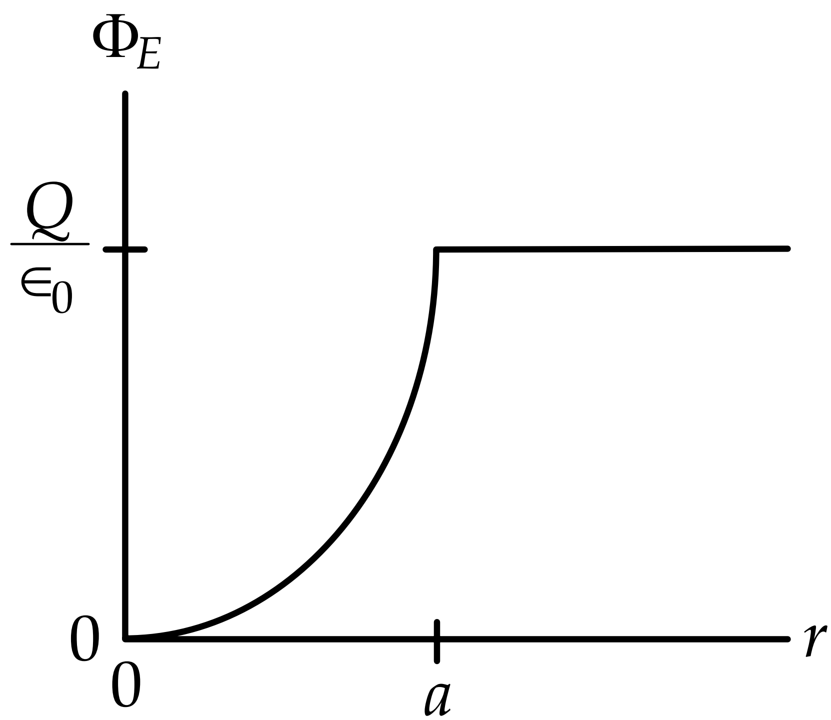

Figure 2: Problem 2.

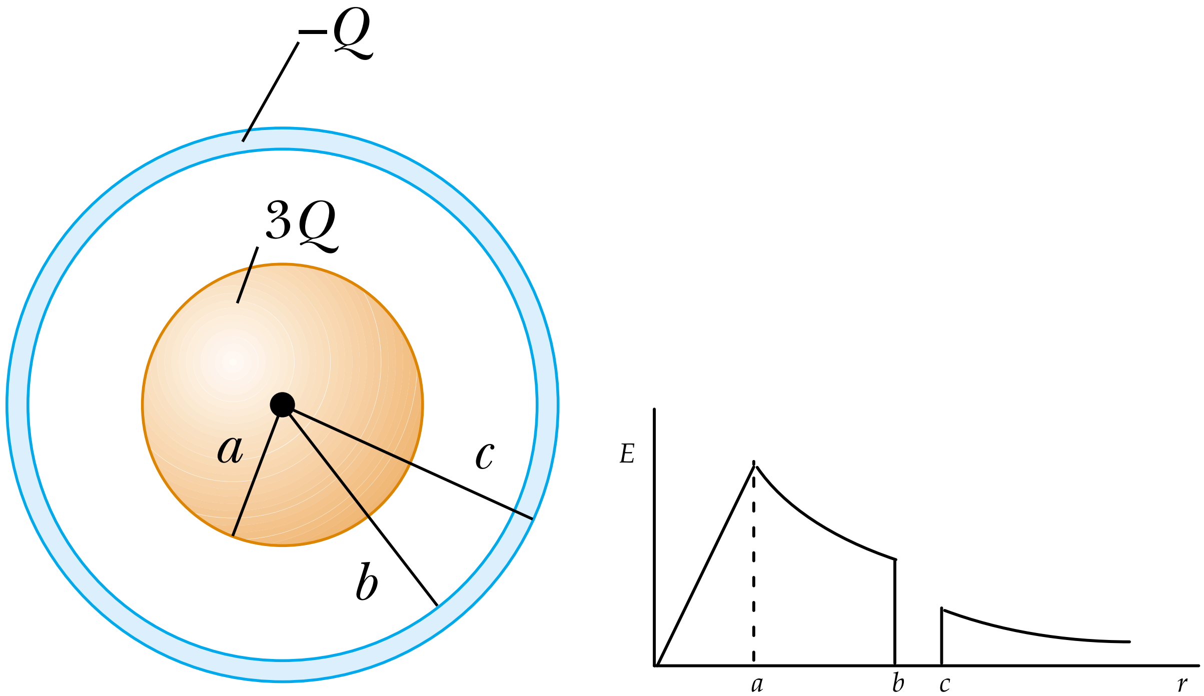

Figure 3: Problem 3.

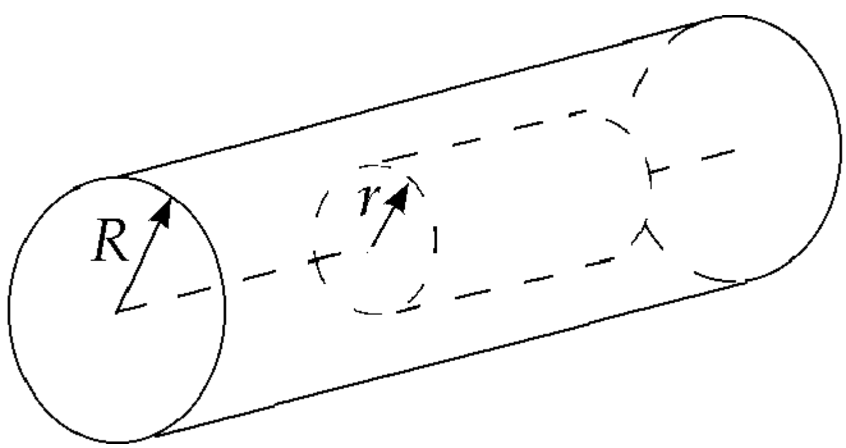

Figure 4: Problem 4.

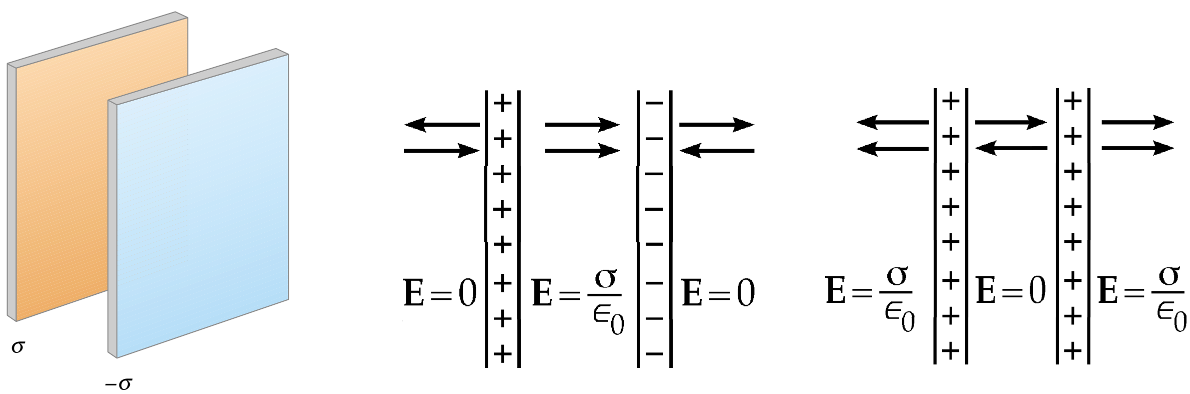

Figure 5: Problem 5.