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| **Problems set # 11** | **Physics 167** | **Solutions** |
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1. A spherical mirror has a focal length of 10.0 cm. *(i)* Locate and describe the image for an object distance of 25.0 cm. *(ii)* Locate and describe the image for an object distance of 10.0 cm. *(iii)* Locate and describe the image for an object distance of 5.00 cm.

Solution Because the focal length of the mirror is positive, it is a concave mirror. We expect the possibilities of both real and virtual images. *(i)* Because the object distance is larger than the focal length, we expect the image to be real. The image is at a distance $q$ is found from the relation $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, where $p$ is the position. We have $q = 16.7$ cm. The magnification of the image is $M = -\frac{q}{p} = -0.667$. The absolute value of $M$ is less than unity, so the image is smaller than the object, and the negative sign for $M$ tells us that the image is inverted. Because $q$ is positive, the image is located on the front side of the mirror and is real. Look into the bowl of a shiny spoon or stand far away from a shaving mirror to see this image. *(ii)* Because the object is at the focal point, we expect the image to be infinitely far away. The distance $q$ of the image is found by the relation $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ and so as $p \to f$ the image distance $q \to \infty$. This result means that rays originating from an object positioned at the focal point of a mirror are reflected so that the image is formed at an infinite distance from the mirror; that is, the rays travel parallel to one another after reflection. Such is the situation in a flashlight or an automobile headlight, where the bulb filament is placed at the focal point of a reflector, producing a parallel beam of light. *(iii)* Because the object distance is smaller than the focal length, we expect the image to be virtual. We have $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, yielding $q = -10.0$ cm. The magnification is $M = -\frac{q}{p} = 2.00$. The image is twice as large as the object, and the positive sign for $M$ indicates that the image is upright. The negative value of the image distance tells us that the image is virtual, as expected. Put your face close to a shaving mirror to see this type of image.

2. An automobile rearview mirror shows an image of a truck located 10.0 m from the mirror. The focal length of the mirror is $-0.60$ m. *(i)* Find the position of the image of the truck. *(ii)* Find the magnification of the image.

Solution Because the mirror is convex, we expect it to form an upright, reduced, virtual image for any object position. *(i)* The image distance is $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, so $q = -0.57$ m. *(ii)* The magnificastion is $M = -\frac{q}{p} = 0.057$. The negative value of $q$ in part *(i)* indicates that the image is virtual, or behind the mirror. The magnification in part *(ii)* indicates that the image is much smaller than the truck and is upright because $M$ is positive. The image is reduced in size, so the truck appears to be farther away than it actually is. Because of the image's small size, these mirrors carry the inscription, "Objects in this mirror are closer than they appear." Look into your rearview mirror or the back side of a shiny spoon to see an image of this type.

3. A converging lens has a focal length of 10.0 cm. *(i)* An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(ii)* An object is

placed 10.0 cm from the lens. Find the image distance and describe the image. *(iii)* An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

Solution Because the lens is converging, the focal length is positive. We expect the possibilities of both real and virtual images. *(i)* Because the object distance is larger than the focal length, we expect the image to be real. The ray diagram for this situation is shown in the left panel of Fig. 1. The image distance is found through the relation $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, yielding $q = 15.0$ cm. The magnification is $M = -\frac{q}{p} = -0.5$. The positive sign for the image distance tells us that the image is indeed real and on the back side of the lens. The magnification of the image tells us that the image is reduced in height by one half, and the negative sign for $M$ tells us that the image is inverted. *(ii)* Because the object is at the focal point, we expect the image to be infinitely far away. Indeed, we have $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, yielding $q \to \infty$. This result means that rays originating from an object positioned at the focal point of a lens are refracted so that the image is formed at an infinite distance from the lens; that is, the rays travel parallel to one another after refraction. *(iii)* Because the object distance is smaller than the focal length, we expect the image to be virtual. The ray diagram for this situation is shown in the right panel of Fig. 1. The image distance follows from the relation $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ and so $q = -10$ cm. The magnification is $M = \frac{-q}{p} = 2.00$. The negative image distance tells us that the image is virtual and formed on the side of the lens from which the light is incident, the front side. The image is enlarged, and the positive sign for $M$ tells us that the image is upright.

4. A diverging lens has a focal length of 10.0 cm. *(i)* An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(ii)* *(ii)* An object is placed 10.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image. *(iii)* An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

Solution *(i)* Because the lens is diverging, the focal length is negative. The ray diagram is shown in the left panel of Fig. 2. Because the lens is diverging, we expect it to form an upright, reduced, virtual image for any object position. The image distance is obtain from the relation $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$. We have, $q = -7.50$ cm. The magnification is $M = -\frac{q}{p} = 0.250$. This result confirms that the image is virtual, smaller than the object and upright. *(ii)* The ray diagram is shown in the middel panel of Fig. 2. The image is located at $q = -5.00$ cm and the magnification is $M = 0.5$. Note the difference between this situation and that for a converging lens. For a diverging lens, an object at the focal point does not produce an image infinitely far away. *(iii)* The ray diagram is shown in the right panel of Fig. 2. The image distance is $q = -3.33$ cm and the magnification is $M = 0.667$. For all three object positions, the image position is negative and the magnification is a positive number smaller than 1, which confirms that the image is virtual, smaller than the object, and upright.

5. Two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm. An object is placed 30.0 cm to the left of lens 1. Find the position and the magnification of the final image.

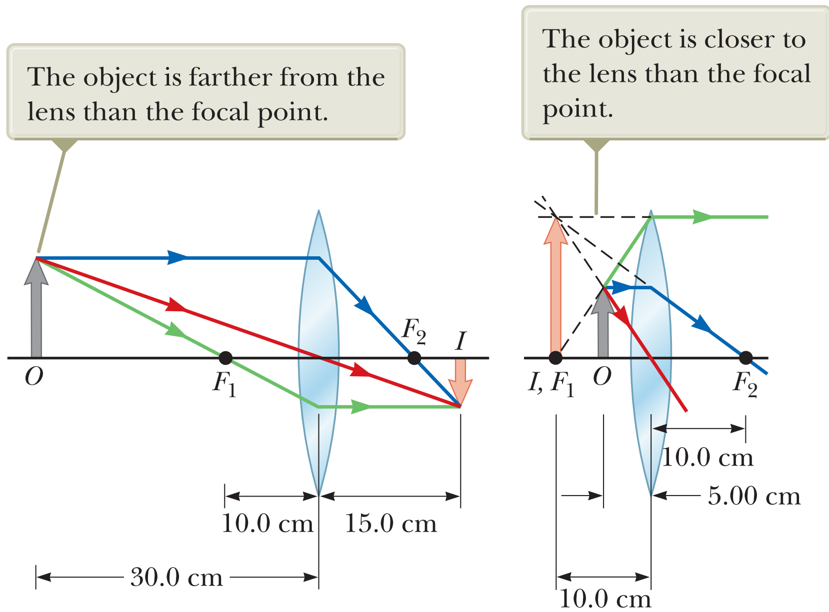


Figure 1: The situations in problem 3.

Solution Imagine light rays passing through the first lens and forming a real image (because $p > f$) in the absence of a second lens. Fig. 3 shows these light rays forming the inverted image $I_1$. Once the light rays converge to the image point, they do not stop. They continue through the image point and interact with the second lens. The rays leaving the image point behave in the same way as the rays leaving an object. Therefore, the image of the first lens serves as the object of the second lens. Using the thin lens equation, $\frac{1}{q_1} = \frac{1}{f} - \frac{1}{p_1}$, we find the location of the image formed by lens 1, $q_1 = 15.0$ cm. The magnification of the image is$M_1 = -\frac{q_1}{p_1} = -0.5$. The image formed by this lens acts as the object for the second lens. Therefore, the object distance for the second lens is $p_2 = 20.0$ cm $-$ 15.0 cm $= 5.00$ cm. We find the location of the image formed by lens 2 from the thin lens equation, $q_2 = -6.67$ cm, and so the magnification is $M_2 = -\frac{q_2}{p_2} = 1.33$. The overall magnification of the system is $M = M_1 M_2 = -0.667$. The negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. Because the absolute value of the magnification is less than 1, the final image is smaller than the object. Because $q_2$ is negative, the final image is on the front, or left, side of lens 2. These conclusions are consistent with the ray diagram shown in Fig. 3.

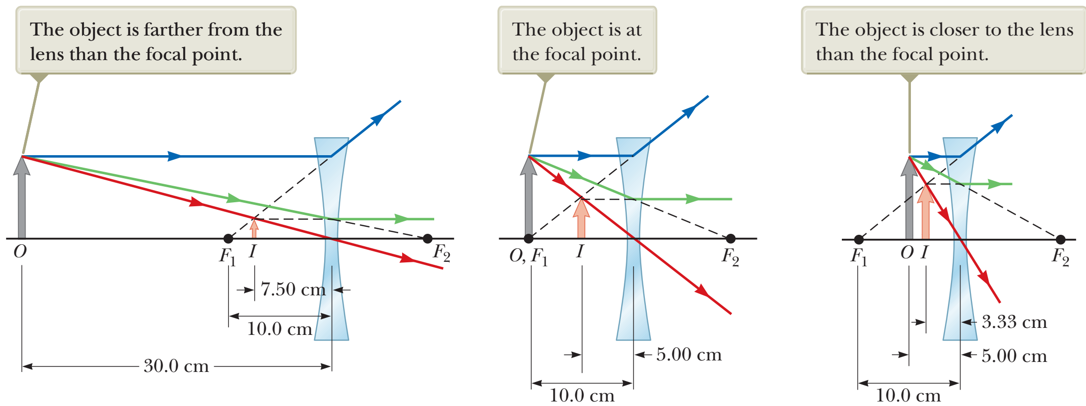


Figure 2: The situations in problem 4.

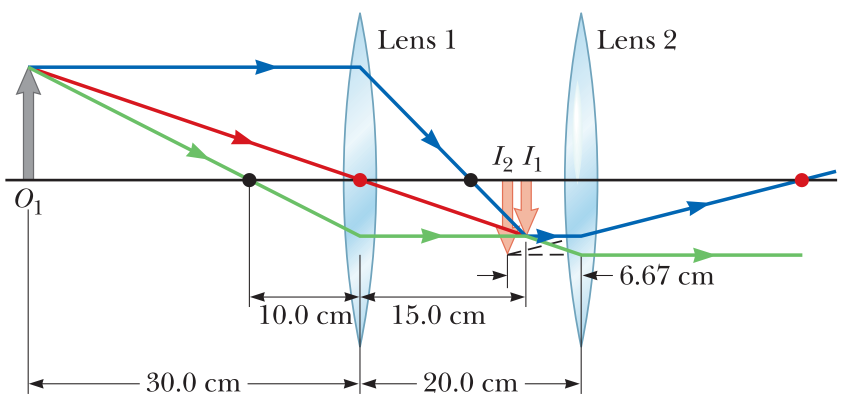


Figure 3: Problem 5. A combination of two converging lenses. The ray diagram shows the location of the final image ($I_2$) due to the combination of lenses. The black dots are the focal points of lens 1, and the red dots are the focal points of lens 2.