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Problems set # 10 **Physics 167** **Solutions**

1. Calculate the wavelength (i) of a 60-Hz EM wave, *(ii)* of a 93.3-MHz FM radio wave, and *(iii)* of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

Solution All of these waves are electromagnetic waves, so their speed is $c = 3.00 \times 10^8$ m/s. *(i)* $\lambda = c/f = 5.0 \times 10^6$ m, or 5000 km. 60 Hz is the frequency of AC current in the United States, and, as we see here, one wavelength stretches all the way across the continental USA. *(ii)* $\lambda = c/f = 3.22$ m. The length of an FM radio antenna is often about half this ($\lambda/2$), or 1.6 m. *(iii)* $\lambda = c/f = 6.33 \times 10^{-7}$ m $= 633$ nm.

2. Radiation from the Sun that reaches the Earth's surface (after passing through the atmosphere) transports energy at a rate of about 1000 W/m^2. Estimate the pressure and force exerted by the Sun on your outstretched hand.

Solution The radiation is partially reflected and partially absorbed, so we estimate simply $P = \overline{I}/c \approx 3 \times 10^{-6}$ N/m^2. An estimate of the area of your outstretched hand might be about 10 cm by 20 cm, so $A \approx 0.02$ m^2. Then the force is $F = PA \approx 6 \times 10^{-8}$ N. These numbers are tiny. The force of gravity on your hand, say of $m \approx 0.2$ kg, is $F_g = mg \approx 0.2~\text{kg} \times 9.8~\text{m/s}^2 \approx 2$ N. The radiation pressure on your hand is imperceptible compared to gravity.

3. Some science fiction writers have described solar sails that could propel interstellar spaceships. Imagine a giant sail on a spacecraft subjected to radiation pressure from our Sun. *(i)* Explain why this arrangement works better if the sail is highly reflective rather than highly absorptive. *(ii)* If the sail is assumed highly reflective, show that the force exerted by the sunlight on the spacecraft's sail is given by $F_{\rm rad} = \frac{P_\odot A}{2\pi r^2 c}$, where $P_\odot$ is the power output of the Sun (3.8×10^{26} W), A is the surface area of the sail, r is the distance from the Sun, and c is the speed of light. (Assume that the area of the sail is much larger than the area of the spacecraft so that all the force is due to radiation pressure on the sail, only. *(iii)* Using a reasonable value for A, compute the force on the spacecraft due to the radiation pressure and the force on the spacecraft due to the gravitational force of the Sun on the spacecraft. Does this result imply that such a system will work? Explain your answer.

Solution *(i)* If the sail is reflective then it gets twice as much of a momentum kick from the light as it does if it was absorptive. This is because the reflective sail has to reflect the light back, pushing the sail back harder. This accelerates the sail better than simply absorbing the light. *(ii)* The radiation force can be expressed in terms of the radiation pressure, $F_{\rm rad} = P_{\rm rad} A$, where A is the area of the sail. The radiation pressure is $2I/c$, where I is the intensity, and the factor of 2 comes from the fact that the sail is reflective. Now, the intensity comes from the sun, and can be written as $I = \frac{P_\odot}{4\pi r^2}$ where r is the distance to the sail. Thus, we finally find that the force is $F_{\rm rad} = \frac{P_\odot A}{2\pi r^2 c}$. *(iii)* The ratio of the radiation force to the Newtonian gravitational force is $\frac{F_{\text{rad}}}{F_G} = \frac{\frac{P_\odot A}{2\pi r^2 c}}{\frac{G_N m M_\odot}{r^2}} = \frac{P_\odot A}{2\pi G_N m M_\odot c}$. All of these are constants, except for the area, A, and mass, m, of the ship. So, plugging in the numbers for everything except m and A, $\frac{F_{\text{rad}}}{F_G} = \frac{P_\odot A}{2\pi G_N m M_\odot c} = \frac{3.8\times10^{26}}{2\pi\cdot6.672\times10^{-11}\cdot2.00\times10^{30}\cdot3\times10^{8}}\frac{A}{m} = 0.0015\ \frac{A}{m}$. In order for this to be an effective means of propulsion we need $F_{\text{rad}}/F_G > 1$, which requires that $0.0015\frac{A}{m} > 1 \to m/A < 0.0015$. So, we would need a tremendously huge sail, and a very light ship. For example, for a 1000 kg ship we would need an area bigger of at least 670,000 square meters, would be a circle of more than 460 meters! It seems like this would be a practically difficult method of space travel, at least if powered by the Sun. However, perhaps by firing lasers from the surface of the Earth to the sail and pushing it with extra light we could build up a good speed.

4. A pulsed laser fires a 1000 MW pulse that has a 200 ns duration at a small object that has a mass equal to 10.0 mg and is suspended by a fine fiber that is 4.00 cm long. If the radiation is completely absorbed by the object, what is the maximum angle of deflection of this pendulum? [*Hint*: Think of the system as a ballistic pendulum and assume the small object was hanging vertically before the radiation hit it.]

Solution Consider the pendulum in Fig. 1. Initially the object has zero energy, but it is then hit with the pulse which gives it a kick, lifting it up to a height h, which can be expressed in terms of the angle as $h = L - L\cos\theta = L(1-\cos\theta)$. When it is pushed up to the height h, the object has a potental energy $E_P = mgh = mgL(1-\cos\theta)$. Equating this to the initial kinetic energy of the pulse $E_K = mgL(1-\cos\theta)$. Solving this expression for the angle gives $\theta = \cos^{-1}\left[1 - \frac{E_K}{mgL}\right]$. Now, we just need to figure out the kinetic energy of the pulse. The pulse carries momentum, which transfers to the object. Hence, $p_{\text{pulse}} = p_{\text{object}}$, which gives it kinetic energy equal to the kinetic energy of the pulse. Hence, $E_{K,\text{pulse}} = E_{K,\text{object}}$. Now, $E_{K,\text{object}} = \frac{p^2_{\text{object}}}{2m} = \frac{p^2_{\text{pulse}}}{2m}$. Hence, $\theta = \cos^{-1}\left[1 - \frac{p^2_{\text{pulse}}}{2m^2 gL}\right]$. To finish we just need to find the momentum of the pulse. This can be found by looking at the energy of the wave, which is related to the momentum by $E = pc$, and the energy can be related to the power, P , by $E = P\Delta t$, which, finally, gives $\theta = \cos^{-1}\left[1 - \frac{P^2\Delta t^2}{2m^2c^2 gL}\right]$. Thus, we can plug in the numbers to find $\theta = \cos^{-1}\left[1 - \frac{P^2\Delta t^2}{2m^2c^2gL}\right] = \cos^{-1}\left[1 - \frac{(10^9)^2(2\times10^{-7})^2}{2(0.01)^2(3\times10^8)^2\cdot9.8\cdot0.04}\right] = 0.0061°$.

5. A dish antenna having a diameter of 20 m receives (at normal incidence) a radio signal from a distant source as shown in Fig. 2. The radio signal is a continuous sinusoidal wave with amplitude $E_{\text{m}} = 0.2\mu\text{V/m}$. Assume the antenna absorbs all the radiation that falls on the dish. *(i)* What is the amplitude of the magnetic field in this wave? *(ii)* What is the intensity of the radiation received by the antenna? *(iii)* What is the power received by the antenna? *(iv)* What force is exerted by the radio waves on the antenna?

Solution *(i)* The magnitude of the electric field vector and the magnitude of the magnetic field vector are proportional to each other $B_{\text{m}} = E_{\text{m}}/c = \frac{0.2\times10^{-6}\ \text{V/m}}{3\times10^{8}\text{m/s}} = 6.7\times10^{-16}$ T. *(ii)* By definition, the intensity of electromagnetic wave is equal to the average value of the magnitude of the Poynting vector. It can also be expressed in terms of the magnitude of the electric field vector or the magnitude of the magnetic field vector. The intensity is $I = \langle S\rangle = \frac{E^2_{\text{m}}}{2\mu_0 c} = \frac{cB^2_{\text{m}}}{2\mu_0} = \frac{3\times10^8\ \text{m/s}(6.7\times10^{-16}\ \text{T})^2}{2\cdot4\pi\times10^{-7}\ \text{Tm/A}} =$

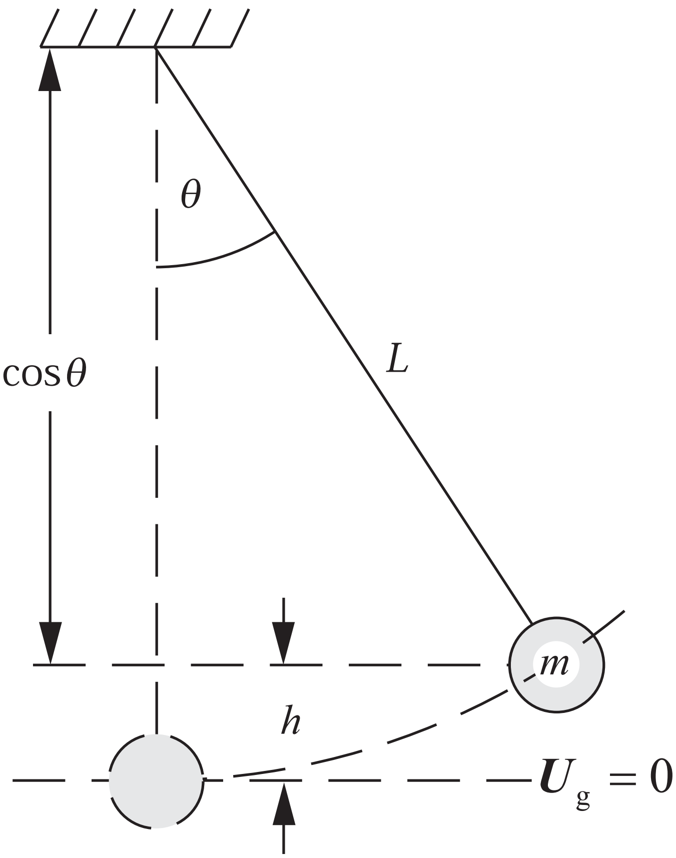

Figure 1: Problem 4.

5.31×10^{-17} W/m^2. *(iii)* The power received by the antenna is related to the size of the antenna and the intensity of the approaching wave $\langle P\rangle = I\frac{\pi D^2}{4} = 5.31 \times 10^{-17}\ \frac{\mathrm{W}}{\mathrm{m}^2}\frac{\pi(20\mathrm{m})^2}{4} = 1.67 \times 10^{-14}$ W. *(iv)* The force exerted on the antenna is equal to product of the antena's area and the wave pressure, related to the magnitude of the Poyting vector $F = PA = \frac{\langle S\rangle}{c}\frac{\pi D^2}{4} = \frac{5.31\times10^{-17}\ \mathrm{W/m^2}}{3\times10^8\ \mathrm{m/s}} \cdot \frac{\pi(20\ \mathrm{m})^2}{4} = 5.56 \times 10^{-23}$ N.

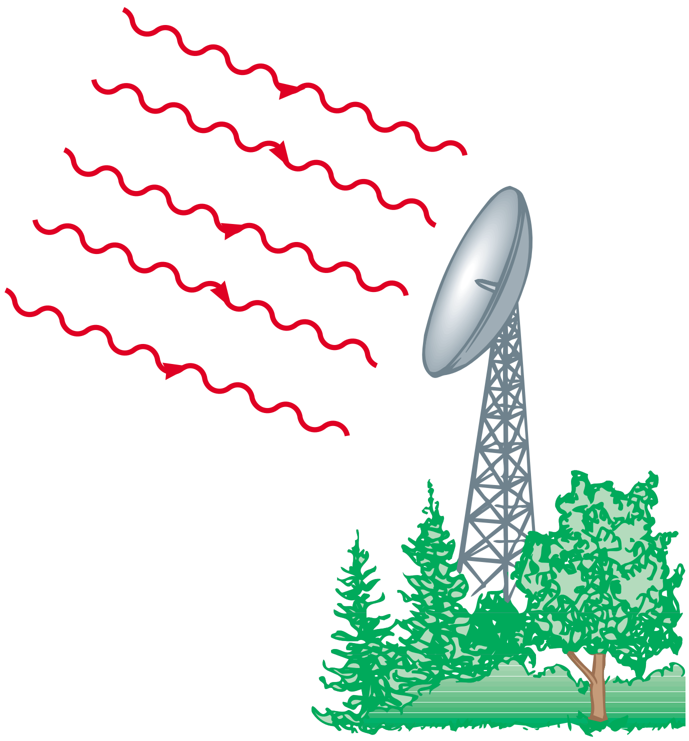

Figure 2: Problem 5.