Prof. Anchordoqui

Problems set # 1 **Physics 167** **Solutions**

1. *(i)* Compare the electric force holding the electron in orbit ($r = 0.53 \times 10^{-10}$ m) around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton. What is the ratio of these two forces. *(ii)* Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions? Explain.

Solution *(i)* Take the ratio of the electric force divided by the gravitational force, that is $\frac{F_E}{F_G} = \left(\frac{q_1 q_2}{4\pi\epsilon_0\, r^2}\right) / \left(\frac{G\, m_1 m_2}{r^2}\right) = \frac{8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\ (1.602\times 10^{-19}\ \mathrm{C})^2}{6.67\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2}\ 9.11\times 10^{-31}\ \mathrm{kg}\, 1.67\times 10^{-27}\ \mathrm{kg}} \simeq 2.3 \times 10^{39}$. The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario. *(ii)* No. Life would be no different if electrons were positively charged and protons were negatively charged. Opposite charges would still attract, and like charges would still repel. The designation of charges as positive and negative is merely a definition.

2. The nucleus of ^{8}Be, which consists of 4 protons and 4 neutrons, is very unstable and spontaneously breaks into two alpha particles (helium nuclei, each consisting of 2 protons and 2 neutrons). *(i)* What is the force between the two alpha particles when they are 5.00×10^{-15} m apart, and *(ii)* what will be the magnitude of the acceleration of the alpha particles due to this force? Note that the mass of an alpha particle is 4.0026 u.

Solution *(i)* Since the charges of the particles are positive, the force is repulsive. The magnitude of the force is given by Coulomb's law, $F = \frac{1}{4\pi\epsilon_0}\frac{4e^2}{r^2} = 8.99 \times 10^9\ \frac{\mathrm{Nm^2}}{\mathrm{C^2}}\frac{4(1.6\times 10^{-19}\ \mathrm{C})^2}{(5.00\times 10^{-15}\ \mathrm{m})^2} = 36.8$ N. *(ii)* The mass of an alpha particle is $m = 4.0026\ u$, where $u = 1.66 \times 10^{-27}$ kg is the unified mass unit. Applying Newton's 2nd law, the acceleration of either alpha particle is then $a = \frac{F}{m} = \frac{36.8\ \mathrm{N}}{4.0026\cdot 1.66\times 10^{-27}\ \mathrm{kg}} = 5.54 \times 10^{27}\ \mathrm{m/s^2}$. Of course from Newton's 3rd law, both alpha particles experience the same force, and hence undergo the same acceleration.

3. A charge of 6.00×10^{-9} C and a charge of -3.00×10^{-9} are at a distance of 60.0 cm. Find the position at which a third charge of 12.0×10^{-9} C can be placed so that the net electrostatic force on it is zero.

Solution The required position is shown in Fig. 1. Note that this places q closer to the smaller charge, which will allow the two forces to cancel. Applying Coulmob's law, and requiring that $F_6 = F_3$ gives $\frac{1}{4\pi\epsilon_0}\ \frac{6.00\ \mathrm{nC}\ q}{(x+0.600\ \mathrm{m})^2} = \frac{1}{4\pi\epsilon_0}\ \frac{3\ \mathrm{nC}\, q}{x^2}$, or $2x^2 = (x + 0.600\ \mathrm{m})^2$. Solving for x gives the equilibrium position as $x = \frac{0.600\ \mathrm{m}}{\sqrt{2}-1} = 1.45$ m beyond the -3.00 nC charge.

4. An electron is released a short distance above Earth's surface. A second electron (directly below it) exerts an electrostatic force on the first electron just great enough to cancel the gravitational force on it. How far below the first electron is the second?

Solution The magnitude of the repulsive force between electrons must equal the weight of an electron. Thus by using Coulomb's law and Newton's 2nd law (applied to gravity), we have $\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = m_e g$, so rearranging this expression, and then making the substitutions, you obtain $r = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e g}} = \sqrt{\frac{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\,(1.60\times 10^{-19}\text{ C})^2}{9.11\times 10^{-31}\text{ kg } 9.80\text{ m/s}^2}} = 5.08$ m.

5. Eight point charges, each of magnitude q, are located on the corners of a cube of edge s, as shown in Fig. 2 *(i)* Determine the x, y, and z components of the resultant force exerted by the other charges on the charge located at point A. *(ii)* What are the magnitude and direction of this resultant force? *(iii)* Show that the magnitude of the force on a charge q at the center of any face of the cube has a value of $2.18\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$. *(iv)* What is the direction of the force at the center of the top face of the cube?

Solution *(i)* There are 7 terms that contribute. There are 3 charges a distance s away (along sides), 3 a distance $\sqrt{2}s$ away (face diagonals), and one charge a distance $\sqrt{3}s$ away (body diagonal). By symmetry, the x, y, and z components of the electric force must be equal. Thus, we only need to calculate one component of the total force on the charge of interest. We will choose the coordinate system as indicated in Fig. 2, and calculate the y component of the force. We can already see that several charges will not give a y component of the force at all, just from symmetry - charges 3, 4 and 7. This leaves only charges 1, 2, 5, and 6 to deal with. Charge 6 will give a force purely in the y direction: $F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$. Charges 5 and 2 are both a distance $s\sqrt{2}$ away, and a line connecting these charges with the charge of interest makes an angle $\theta = 45°$ with the y-axis in both cases. Hence, noting that $\cos\theta = 1/\sqrt{2}$, we obtain $F_{2,y} = F_{5,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{2})^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2s^2}\frac{1}{\sqrt{2}}$. Finally, we have charge 1 to deal with. It is a distance $s\sqrt{3}$ away (see the Fig. 3). What is the y component of the force from charge 1? First, we can find the component of the force in the $x-y$ plane (see Fig. 3): $F_{1,x-y} = F_1\cos\phi = F_1\frac{\sqrt{2}}{\sqrt{3}}$. Now, we can find the component of the force along the y direction: $F_{1,y} = F_{1,x-y}\cos\theta = F_{1,x-y}\frac{1}{\sqrt{2}} = F_1\frac{\sqrt{2}}{\sqrt{3}}\frac{1}{\sqrt{2}} = F_1\frac{1}{\sqrt{3}}$. Since we know charge 1 is a distance $s\sqrt{3}$ away, we can calculate the full force F_1 easily, and complete the expression for $F_{1,y}$, that is $F_{1,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{3})^2}\frac{1}{\sqrt{3}} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\frac{1}{3\sqrt{3}}$. Now we have the y component for the force from every charge; the net force in the y direction is just the sum of all those: $F_{y,\text{net}} = F_{1,y}+F_{2,y}+F_{5,y}+F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right] = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$. Since the problem is symmetric in the x, y, and z directions, all three components must be equivalent. The force is then $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right](\hat{\imath}+\hat{\jmath}+\hat{k}) = 1.90\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}(\hat{\imath}+\hat{\jmath}+\hat{k})$. *(ii)* $F = \sqrt{F_x^2+F_y^2+F_z^2} = 3.29\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$ away from the origin. *(iii)* There is zero contribution from the same face due to symmetry. The opposite face contributes $\frac{q^2\sin\phi}{\pi\epsilon_0 r^2}$ where $r = \sqrt{\frac{(\sqrt{2}s)^2}{4}+s^2} = \sqrt{1.5}\,s = 1.22s$ and $\sin\phi = s/r$, see Fig. 3. All in all, $F = \frac{q^2 s}{\pi\epsilon_0 r^3} = 2.18\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$. *(iv)* The direction is $\hat{k}$.

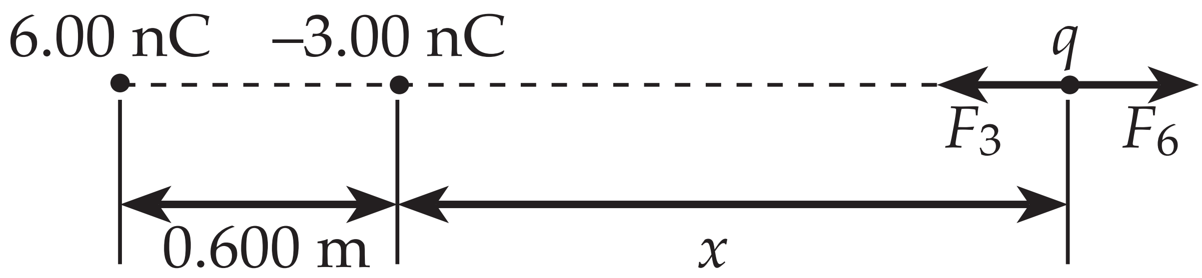

Figure 1: Problem 4.

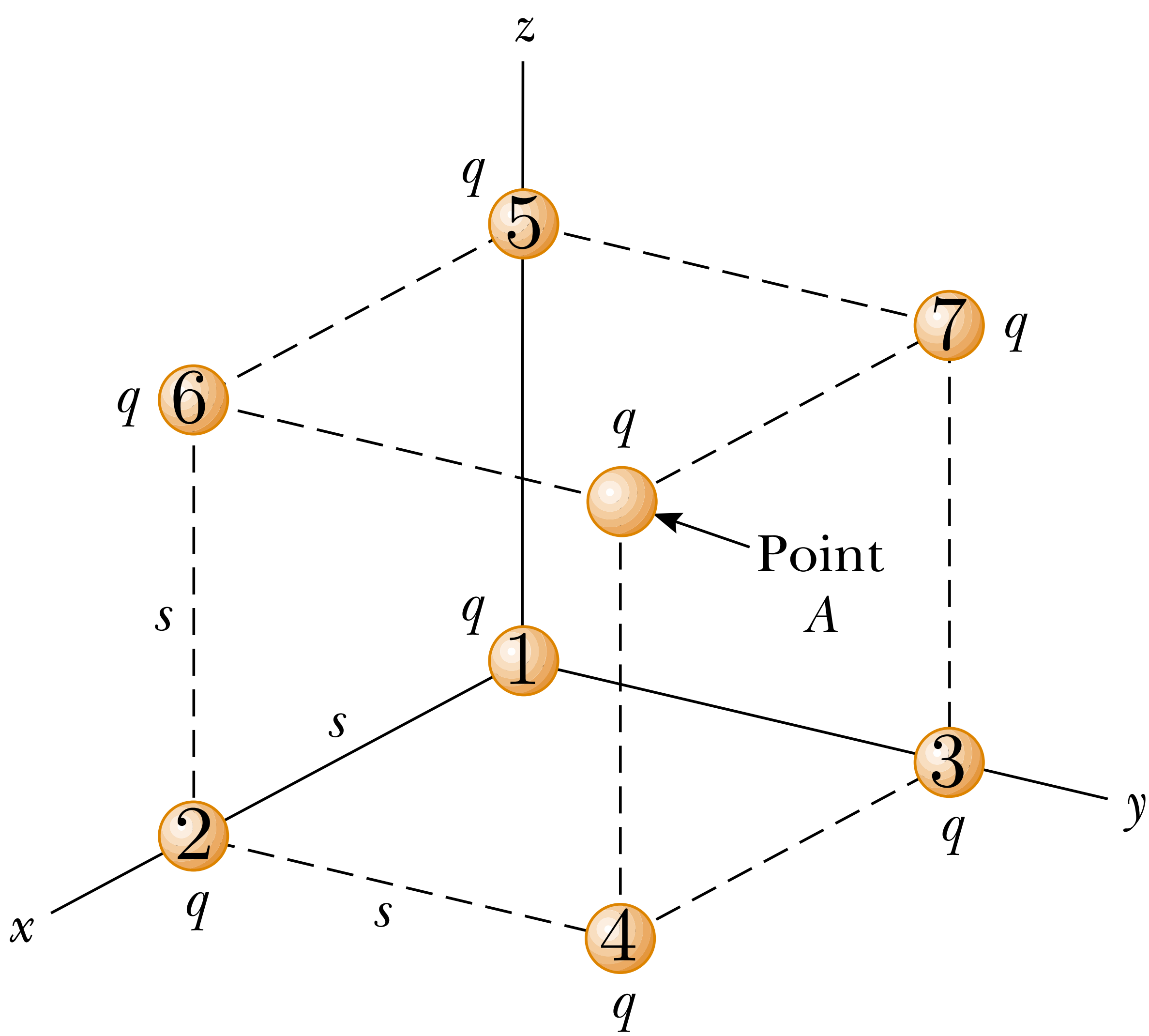

Figure 2: Problem 5.

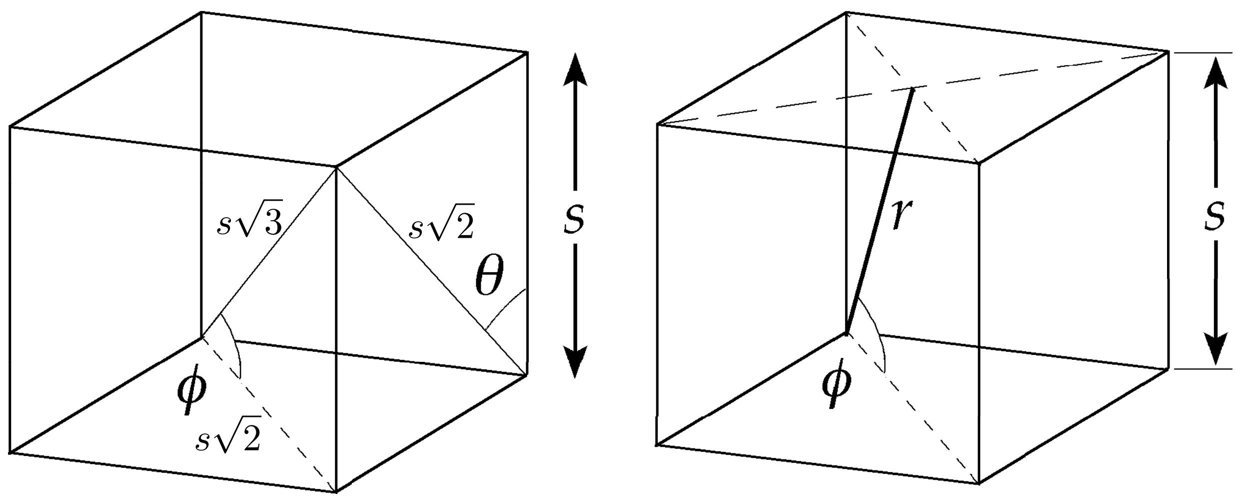

Figure 3: Problem 5, solution.