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I. ASTRONOMY BEFORE THE COMMON ERA

The alteration between day and night, the succession of the seasons, and the observation of the celestial bodies and their movements in the sky introduced the notion of time. The first effort humans made to define time was to combine the notion of time and the movements of celestial bodies. We now know that we experience day and night because of Earth's rotation around itself and we experience seasons because of the tilt of Earth's axis of rotation as the Earth moves around the Sun in a year. However, the precise understanding of these phenomena came about through careful observations and gradual application of the scientific method across the world and over thousands of years.

Astronomy has its roots in the work done by the Babylonian and Egyptian civilisations. Over a thousand years before the common era (BCE), the Babylonians already had extensive astronomical records, with good measurements of time and of the positions of the Moon, as well as stars and planets [1] in the sky (from which we inherit both our systems of angular and time measurement: the 360° circle and the time units of 24 hrs, 60 minutes, and 60 seconds).[1]

The Babylonian calendar was a lunisolar calendar based on the lunar phases which was used in Babylon and surrounding regions for administrative, commercial and ritualistic purposes. The Babylonian year consisted of twelve lunar months, each beginning on the evening (i.e. after sunset) of the first observed (or computed) lunar crescent after the astronomical new moon. The year began around the spring equinox and in order to keep the calendar in step with the seasons, an intercalary month was inserted at (semi-)regular intervals. At first the intercalary months were inserted at irregular intervals, based on the observed discrepancies between the calendar and the seasons, but after about 500 BCE a regular intercalation scheme consisting of seven intercalary months in a 19-year cycle was adopted.

Just like Earth, the Moon rotates on its own axis and experiences daylight and dark cycles. The Moon's day and night cycles are a little longer than Earth's: the Moon spins on its axis once every 27.3 days. The Moon's period of rotation matches the time of revolution around Earth. This implies that it takes the Moon the same length of time to turn once on its axis as it takes it to go once completely around the Earth. This means that Earth observers always see the same side of the Moon (called the "nearside"). The side we do not see from Earth, called the "farside," has been mapped during lunar missions.

The Moon looks different during its revolution around the Earth, because at each position it is getting a different amount of sunlight on its surface; see Fig. 1. When the Moon is positioned between the Earth and the Sun, we face the dark side, so cannot see the Moon at all. This is called a new moon. As each day passes and the Moon moves at an angle out from between the Earth and the Sun, we begin to see a sliver of the Moon getting sunlight. By day 4 this is called a waxing crescent. When the moon has revolved to a 90° angle from the Earth and Sun, on about day 7, it has reached its first quarter. We can now see half the Moon, while the other half sits in invisible shadows. The next phase, at about day 10, we can see roughly three quarters of the Moon. This is called the waxing gibbous phase. After roughly 2 weeks, the Moon is now in position with the Earth sitting between it and the Sun, so we see its fully lit side as a full moon. It is not an exact alignment though, or the Earth would block the Sun from the moon causing a lunar eclipse. As it continues on in its revolution around the Earth, the moon begins to move into shadow as the waning gibbous Moon by day 18, then the third quarter half Moon at day 22, then waning crescent at day 26 and finally the invisible new moon again on day 29.5. Like the full moon, the new moon does not block the sun from reaching the Earth, because it is not an exact alignment. On the months where the alignment is exact, we experience a solar eclipse.

A careful reader should have noticed already the difference between the sideral month and the synodic month. The Moon's sidereal orbital period (the sidereal month) is roughly 27.3 days; this is the time interval that the Moon takes to orbit 360° around the Earth relative to the "fixed" stars. The period of the lunar phases (the synodic month), e.g. the full moon to full moon period, is longer at about 29.5 days. This is because while the Moon is orbiting the Earth, the Earth is progressing in its orbit around the Sun. After completing a sidereal month, the Moon must move a little further to reach the new position having the same angular distance from the Sun, appearing to move with respect to the stars since the previous month. Therefore, the synodic month takes 2.2 Earth's days longer than the sidereal month.

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[1] BCE and BC (before Christ) mean the same thing, i.e. previous to year 1 of the common era. This is the same as the year AD 1 (Anno Domini); the latter means "in the year of the lord," often translated as "in the year of our lord."

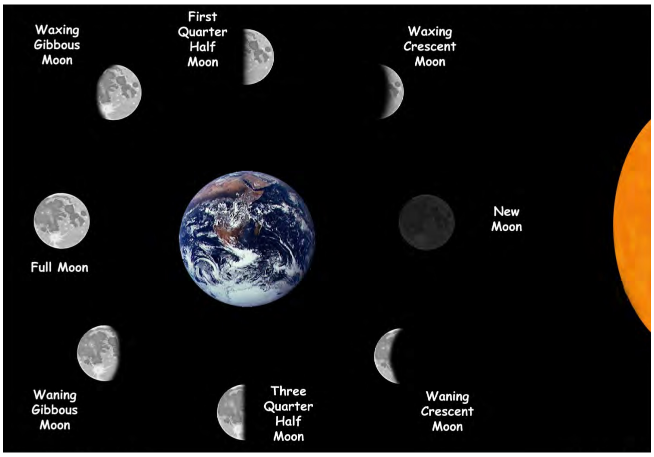

FIG. 1: Lunar phases.

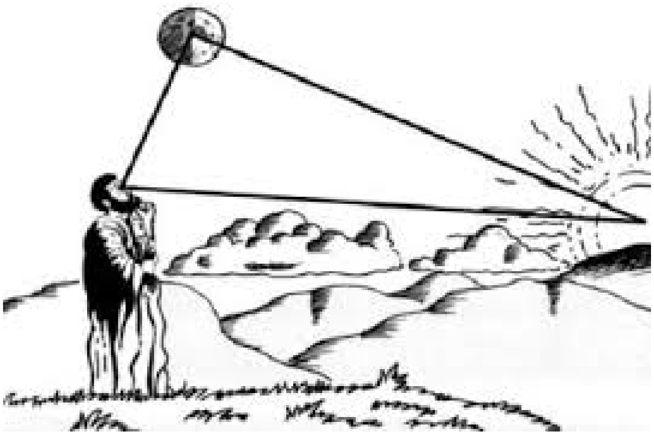

FIG. 2: Aristarchus observation.

The first precise astronomical measurements were carried out in the middle of the 2nd century BCE by Aristarchus of Samos, and Eratosthenes of Cyrene. The first distance to be measured with any accuracy was the ratio of the Earth distances to the Sun and the Moon. Aristarchus realized that when the Moon was exactly half illuminated, it formed a right triangle with the Earth and the Sun [2]. By observing the angle between the Sun and Moon, ϕ, the ratio of the distances to the Sun and Moon could be deduced using a form of trigonometry. From the diagram in Fig. 2 and trigonometry, we can calculate that

$$\frac{S}{L} = \frac{1}{\cos\phi} = \sec\phi\,. \tag{1}$$

The diagram is greatly exaggerated, because in reality, $S = 390\,L$. Aristarchus determined ϕ to be a thirtieth of a quadrant (in modern terms, 3°) less than a right angle: in current terminology, $\phi = 87°$. Trigonometric functions had not yet been invented, but using geometrical analysis in the style of Euclid, Aristarchus determined that

$$18 < \frac{S}{L} < 20\,. \tag{2}$$

In other words, the distance to the Sun was somewhere between 18 and 20 times greater than the distance to the Moon. It was brilliant reasoning undermined by insufficient observations. With nothing but his eyes to go on, Aristarchus estimate was not terribly far from the true value of 89.853°. But when the distances involved are enormous, small errors can be quickly magnified. His result was off by a factor of about 20.

We are told by Cleomedes, in a story often retold, that Eratosthenes determined the size of the Earth by observing known phenomena and applying basic arithmetic and geometry to them [3]. Here is how he did it. Eratosthenes was the head librarian of the famous Library of Alexandria. While working at the library, he learned that on the first day of summer the Egyptian town of Syene cast no shadows; see Fig. 3. This happens because at noon on the day of the summer solstice the Sun is positioned directly above the town of Syene, near the modern city of Aswan, Egypt. On the same date at noon in Alexandria, a rod perpendicular to the ground cast a shadow that is 7° 12′ from perpendicular.[2] Eratosthenes then divided 360° by 7°12′ and determined that 7° 12′ was 1/50th of a circle. Since the distance between and Syene and Alexandria was measured to be 5,000 stades and these two places lie on the same meridian, by means of a simple geometric argument Eratosthenes calculated the circumference of the Earth to be 250,000 stades. The best modern guess is that 1 stadia = 185 m. Putting Eratosthenes result into modern units, his estimate of the circumference of the Earth is 46,250 km. The modern measurement is 40,070 km. The largest uncertainty in Eratosthenes measurement comes from the definition of the stadia, there are also some minor erros in Eratosthenes calculations. Syene is not on the Tropic of Cancer, where the Sun's rays are directly overhead during the summer solstice. It is actually 37 km (23 miles) north of the Tropic of Cancer. Alexandria is not due north of Syene and the distance between them is not 925 km. The actual distance corresponds to an angular measurement not of 7°12′, but rather of 7°30′. All in all, Eratosthenes' estimate is only about 15% too large!

In closing, we note that Aristarchus also presented the first known heliocentric model. Though the original text has been lost, a reference in Archimedes' book [4] describes a work by Aristarchus in which he advanced the heliocentric model as an alternative hypothesis to geocentrism. Archimedes wrote: "Now you [*you* being King Gelon] are aware the *universe* is the name given by most astronomers to the sphere the center of which is

[2] Note that noon is determined when the shadow is shortest, and therefore one does not need any clock for this.

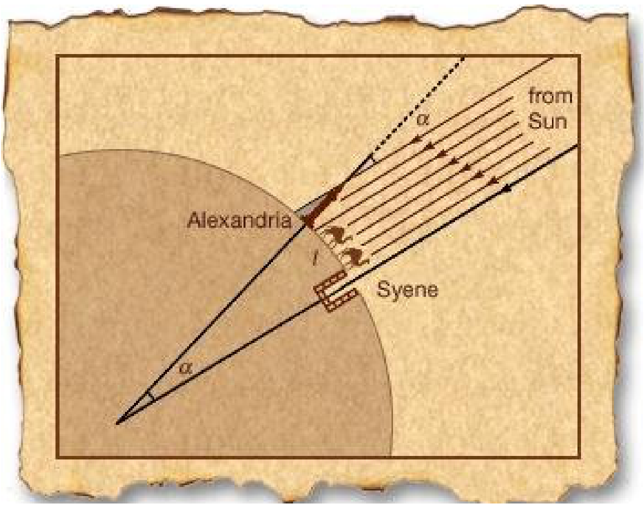

FIG. 3: Eratosthenes observation.

the center of the Earth, while its radius is equal to the straight line between the center of the Sun and the center of the Earth. This is the common account as you have heard from astronomers. But Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the *universe* just mentioned. His hypotheses are that the fixed stars and the Sun remain unmoved, that the Earth revolves about the Sun on the circumference of a circle, the Sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same center as the Sun, is so great that the circle in which he supposes the Earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface." Aristarchus' astronomical ideas were rejected in favor of the geocentric theories of Aristotle and Ptolemy until they were successfully revived nearly 1,800 years later by Copernicus and extensively developed and built upon by Kepler and Newton. Copernicus attributed the heliocentric theory to Aristarchus [2].

II. COPERNICUS REVOLUTION, KEPLER'S LAWS OF PLANETARY MOTION, AND GALILEO'S VISION

A scientific theory should be: *(i)* simple; *(ii)* without fewest unproven assumptions; *(iii)* verifiable. A scientific theory begins with a hypothesis, which tries to describe and predict the natural world (i.e. explain observations/experiments), possibly contributing to or encompassing physical laws. However, theories may break down if they are not able to explain new observations/experiments. A new hypothesis is then proposed to modify or replace the current explanations. The new hypothesis must also be under observational/experimental scrutiny. All results must be repeatable/verifiable or else a new hypothesis must be proposed as an explanation. According to Occam's razor, when there exist two competing theories that make exactly the same prediction, then the simpler one is the better.[3]

One of the earliest scientific questions is whether the Sun or the Earth is at the center of our Universe? The answer is neither. However, we can ask ourselves which view best explains the motions of the stars, planets, and Sun in our sky? How this question was tackled over the years gives insight as to how science is performed. It also gives a historical context to astronomy.

On the one hand, Aristotle argued for a geocentric cosmology: the Earth is a sphere that is positioned at the center of the universe. On the other hand, Aristarchus proposed that: *(i)* the Sun is at the center of the universe; *(ii)* the Sun and stars are stationary; *(iii)* the Earth and planets revolve around the sun. In contemporary Greece, Aristotle was far more influential than Aristarchus and therefore the Earth-centered universe became the accepted norm. The observational evidence indicated that the earth is not felt to move beneath ones feet, so it must be stationary. The stars, planets, and Sun seem to revolve around the Earth.

Claudius Ptolemaeus (Ptolemy) devised a Geocentric model to describe the motion of the heavenly bodies. Based on the teachings of Aristotle and other Greek scholars he argued that the motion of celestial objects must have perfect uniform circular motion. To explain the observed retrograde motion of the planets Ptolemaic model incorporates epicycles. The epicycle orbits on a circle called the deferent, see Fig. 4. The planet moves along the epicycle. Even though Ptolemaic model reasonably explains the retrograde motion, further predictions of planetary positions using the Ptolemaic model did not match observations.

Aristarchus ideas were rekindled by Nicolaus Copernicus. He devised a heliocentric model of the universe in which the Sun is at the center, and planets revolved around it in perfect circles [5]. This model correctly placed the position of the known planets of the time (Mercury, Venus, Earth, Mars, Jupiter, and Saturn). The Moon orbits the Earth and the stars are fixed. Copernicus model elegantly describes the retrograde motion of the planets, see Fig. 5.

The Copernican revolution was a major event during the European Renaissance, a time when scientific thoughts began to flourish. In this climate Tycho Brahe performed and recorded detailed observations of the stars, planets, and the Sun. Brahe observations encompassed the best astronomical data yet collected at the time.

[3] Occam's razor is a principle (atributed to William of Ockham) that underlies all scientific modelling and theory building. The principle states that one should not make more assumptions than the minimum needed.

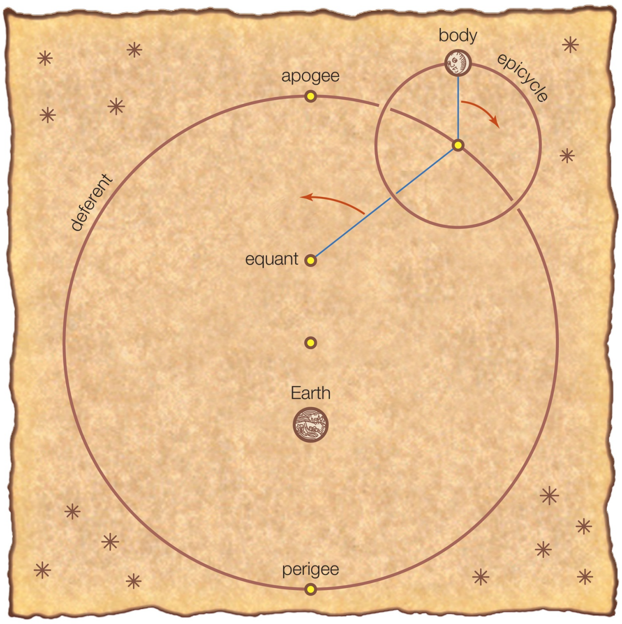

FIG. 4: Ptolemy's geocentric model of the universe. The Sun, the Moon, and each planet orbit a stationary Earth. To explain the erratic apparent paths of the planets, Ptolemy shifted the center of each planet orbit (deferent) from Earth – accounting from the planet's apogee and perigee – and added a second orbital motion (epicycle) to explain retrogade motion. The equant is the point from which each body sweeps out equal angles along the deferent in equal times. The center of the deferent is midway between the equant and the Earth.

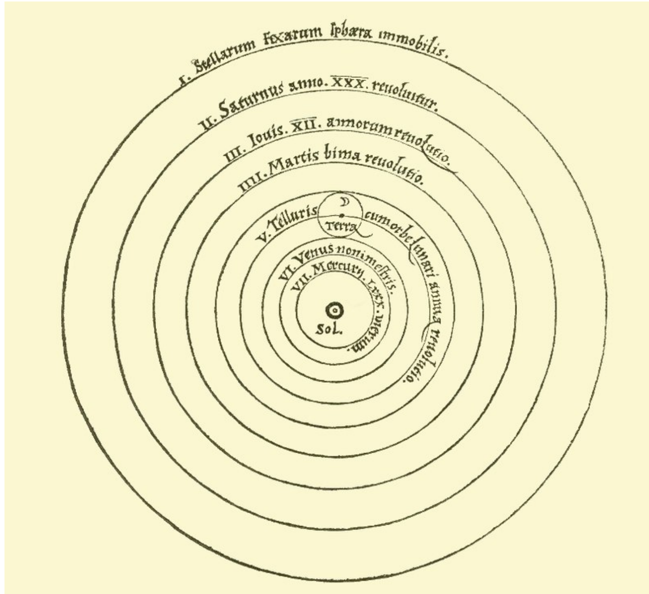

FIG. 5: Copernicus heliocentric model showing: *(i)* the stationary sphere of immobile stars; *(ii)* Saturn is returned in the 30th year; *(iii)* the revolution of Jupiter of 12 years; *(iv)* the two-year revolution of Mars; *(v)* the year revolution of the Earth with its Moon's orbit; *(vi)* nine-month Venus revolution; *(vii)* of the 80 days of Mercury revolution; the Sun.

Through meticulous observation of the planets and other objects the Copernican model was corroborated much further, but the stage was set for further refinements of the model as well. Copernicus thought that the planets travel in perfectly circular orbits around the sun, but this change when Brahe's young assistant Johannes Kepler got his hand on this new data [6]. He used this to determine that planets do not trace circles as they go around the sun, but they trace ellipses. Unlike a circle with its one center an ellipse has two foci and the sum of the distances from the foci to a particular point on the ellipse is a constant, see Fig. 6. When these foci get very close together it starts to resemble a circle. This is bacause a circle is just an ellipse with foci that coincide. This is also why the orbits of the planets seem circular and why we operated under that assumption for so long.

The brilliance of Kepler was to realize that the planets actually follow elliptical paths, with the Sun acting as one focus of the elipse. The point in a planet's orbit that is closest to the Sun is called perihelion and the point furthest away is called aphelion. The distances to these points are very close for most of the planets because the ellipses have very low *eccentricity*. This fact is the first of three laws known as Kepler's laws of planetary motion, which essentially mark the birth of Celestial Mechanics. The second of Kepler's laws says that the orbital speed of a planet will vary according to its distance from the Sun. It slows down when further away and speeds up when closer in. A little more formally, it says that a planet will sweep out equal areas of the ellipse over equal time intervals. So if we take one of the areas shown in Fig. 7 involving a part of the orbit that is closer to the Sun and any other equal area of the ellipse involving a part of the orbit that is further away from the Sun, we see that the distances traveled by the planet in order to sweep out these areas, say from January to February and July to August, are of different magnitudes. However, the planet moves faster when close and slower when far away, in a manner that results in precisely equivalent time intervals. Kepler's third law states that the amount of time a planet takes to orbit the Sun, (a.k.a. the period τ) is related to the length of the semi-major axis of the orbit a. More concretely,

$$\tau^2 \propto a^3 \,, \tag{3}$$

when τ is measured in years and a in astronomical units (AU), and the precise relation will depend on the mass of the planet.

Kepler's laws were derived from observation and used to calculate the relative distances of all the planets from the Sun. These laws had other practical applications as well, but the impact of these laws was much deeper than the merely practical. Kepler's laws represented the first time that a handful of extremely simple mathematical formulae were able to describe and predict the motions of the heavens to the upmost precision.

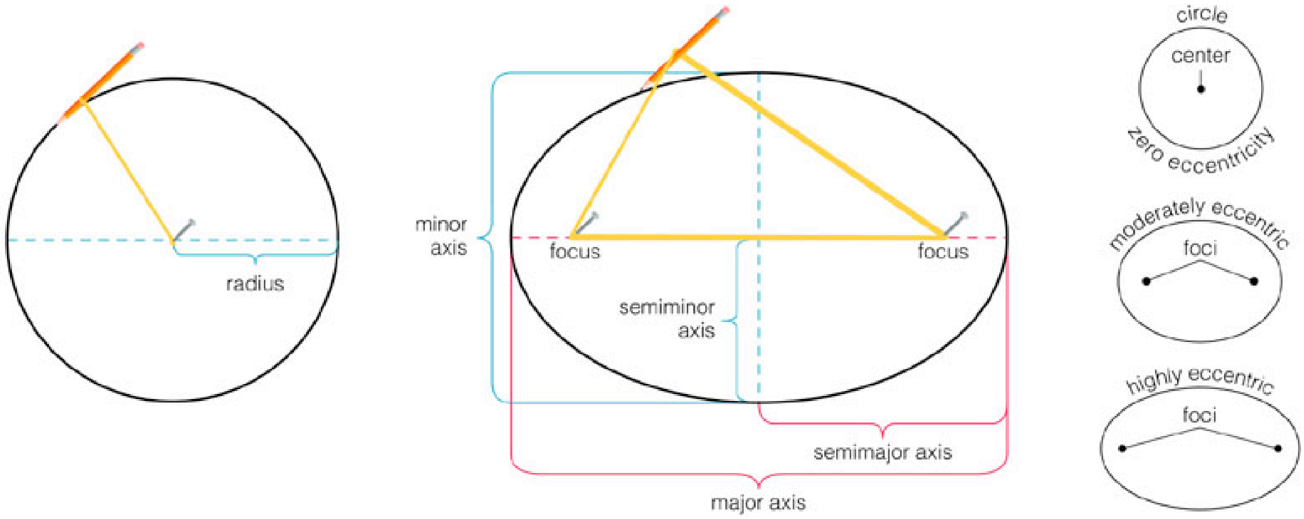

FIG. 6: Geometry of ellipses.

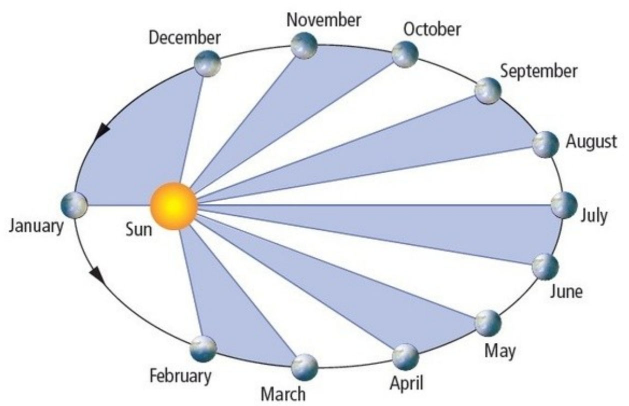

FIG. 7: Second law of planetary motion.

It was undeniable proof that the Universe obeys mathematical principles that are decipherable by mankind, which is the basis of the revolution that produced modern scientific thought.

Roughly simultaneous to Kepler's model of the solar system, Galileo Galilei was mapping the same night sky, but focusing on entirely different observations [7, 8]. He used the best telescopes of the time to see that the Moon has distinct terrestrial features, like mountains and craters. This transformed the Moon into a world unto itself, rather than just a mysterious glowing disk. Galileo also noticed the dark sunspots on the Sun, which changed position over time, thereby deducing the rotation of the Sun. He observed Jupiter's moons, and as they orbit Jupiter, this proved that not everything in the solar system orbits around the Sun or the Earth, no matter which of the two you put at the center. He saw what we would later understand to be Saturn's rings. He observed the phases of Venus, demonstrating that it must be in orbit around the Sun just as the Moon is in orbit around the Earth, the match in the powder barrel for the geocentric model. Galileo also observed far more stars than were previously visible to the naked eye, indicating that the Universe is much bigger than was thought at the time.

III. NEWTONIAN CELESTIAL MECHANICS

The newtonian idea of *force* is based on experimental observation [9]. Experiment tells us that everything in the universe seems to have a preferred configuration: e.g., *(i)* masses attract each other; *(ii)* magnets can repel or attract one another. The concept of a force is introduced to quantify the tendency of objects to move towards their preferred configuration. If objects accelerate very quickly towards their preferred configuration, then we say that there is a big force acting on them. If they don't move (or move at constant velocity), then we say there is no force.

We cannot see a force; we can only deduce its existence by observing its effect. More specifically, forces are defined through Newton's laws of motion:

0. A *particle* is a small mass at some position in space.

1. When the sum of the forces acting on a particle is zero, its velocity is constant.

2. The sum of forces acting on a particle of constant mass is equal to the product of the mass of the particle and its acceleration:

$$\text{force} = \text{mass} \times \text{acceleration}\,. \tag{4}$$

3. For every thing that moves there is an equal and opposite reaction that happens because of the action. In other words, the forces exerted by two

particles on each other are equal in magnitude and opposite in direction.

The standard unit of force is the newton, given the symbol N. The newton is a derived unit, defined through Newton's second law of motion: one newton is the force needed to accelerate one kilogram (kg = 2.2 lb) of mass at the rate of one meter per second squared in direction of the applied force.

Now, a point worth noting at this juncture is that forces are vectors, which evidently have both magnitude and direction. For example, the *gravitational force* is a force that attracts any two objects with mass [9]. The magnitude of this force is directly dependent upon the masses of both objects m and M and inversely proportional to the square of the distance r that separates their centers,

$$\text{gravitational force} = F_g = \frac{G\,M\,m}{r^2}\,, \qquad (5)$$

where $G = 6.673 \times 10^{-11}\ \mathrm{N\,m^2/kg}$ is the proportionality constant.[4] The direction of the force is along the line joining the two objects. Near the Earth's surface, the acceleration due to gravity is approximately constant,

$$\text{gravitational acceleration} = g = \frac{G\,M_\oplus}{R_\oplus^2} \approx 9.8\ \mathrm{m/s^2}\,, \qquad (6)$$

where $M_\oplus = 1.3 \times 10^{25}$ lb is the mass of the Earth and $R_\oplus = 3,959$ miles its radius.

So, the Earth pulls on the Moon because of gravity? Why doesn't the moon get pulled into the Earth and crash? To answer this provocative question we first note that an object can move around in a circle with a constant speed yet still be accelerating because its direction is constantly changing. This acceleration, which is always directed in toward the center of the circle, is called centripetal acceleration. The magnitude of this acceleration is written as

$$\text{centripetal acceleration} = \frac{v^2}{r}\,, \qquad (7)$$

where v is the speed of the object and r the radius of the circle. If a mass is being accelerated toward the center of a circle, it must be acted upon by an unbalanced force that gives it this acceleration. The *centripetal force* is the net force required to keep an object moving on a circular path. It is always directed inward toward the center of the circle. So, we can say that the Moon continuously *falls* towards Earth due to gravity, but does not get any closer to Earth because its motion is an orbit. In other words, the Moon *is* constantly trying to fall upon the Earth, due to the force of gravity; but it is *constantly missing*, due to its *tangential velocity*.

[4] For a quick overview of scientific notation, see Appendix A.

We now turn to study the implications of Newton's laws for the planetary motion. We have seen that the elliptical orbits of the planets have such small eccentricities that, to a very good approximation, we can think of them as circular orbits. (Only very precise measurements, like those available to Kepler, are able to detect the difference.) This means that we can use the idea of uniform circular motion to analyze planetary motion. We have also seen that a body in uniform circular motion is constantly accelerating towards the center of its circular track. Thus, according to Newton's first law of motion, there must be a force acting on the planet that is always directed toward the center of the orbit – that is toward the Sun.

Newton's second law of motion allows us to state what the magnitude of that force must be. The required force is just the mass of the planet times its acceleration. We know that the acceleration of an object moving in uniform circular motion is v^2/r. Thus, we can calculate the force that is required to keep the Earth on its circular path.

Before proceeding though, we apply Newton's third law to the bound Earth-Sun system. If there is a force that attracts the Earth toward the Sun, then there must be an equal and opposite force attracting the Sun towards the Earth. Why, then, does not the Sun move? The answer is that it does move, but by a very small amount since the mass of the Sun is about half a million times that of the Earth. Thus, when subjected to the equal and opposite force required by the third law, it accelerates about half a million times less than the Earth as well. For this reason, to a very good approximation, we can treat the Sun as stationary in our studies of planetary motion.

We can now combine Newton's gravitation and circular acceleration to obtain a nice relation between the period, the orbital distance, and the mass of the central body. We being by equating the centripetal force due to the circular motion to the gravitational force,

$$\frac{GM_\oplus M_\odot}{a^2} = M_\oplus \frac{v^2}{a}\,, \qquad (8)$$

where $M_\oplus$ is the mass of the Earth, $M_\oplus$ the mass of the Sun, a the Earth-Sun distance and v the orbiting velocity. Note that the $M_\oplus$ cancels, so that circular orbital motion is independent of the mass of the orbiting body.

The orbital velocity of the Earth can be described as distance/time, or circumference of the circular orbit divided by the orbital period, $v = 2\pi a/\tau$. Substituting v into (8) and rearranging to place all the a-terms on the right and all the τ-terms on the left we have

$$a^3 = \frac{GM_\odot}{4\pi^2}\tau^2\,, \qquad (9)$$

which looks startlingly like Kepler's third law. To use a and τ to solve for the mass, we manipulate once more (9) so that

$$M_\odot = \frac{4\pi^2 a^3}{G\tau^2}\,. \qquad (10)$$

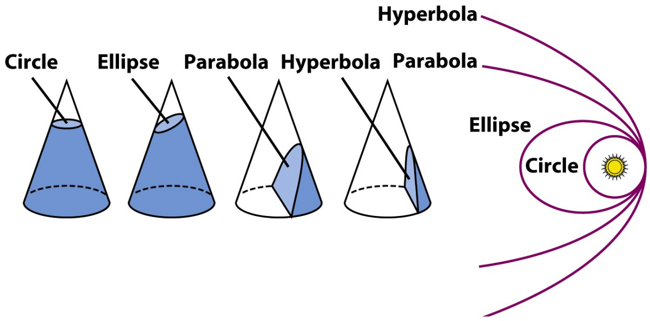

FIG. 8: Classification of cellestial orbits.

All in all, Newton was able to re-derive Kepler's laws based upon physical laws. He gave a concrete physical insight as to the motion of the planets.

Newton's laws also predicted (parabolic and hyperbolic) unbound orbits. Comets typically have such orbits, in which the object comes very close to the Sun and then travels out of the solar system. The four types of orbit of celestial objects are described by the conic sections; see Fig. 8.

The planet Uranus was discovered in 1781 by Herschel, as a byproduct of a systematic sky-survey of the brighter stars, and once he had reported his observation (of what he thought be a comet), it then took about half a year before it gradually became evident it was a new planet due to its nearly circular orbit [10]. Uranus, being the 7th planet in the solar system, was the first to be discovered with the aid of a telescope. Newtons laws were then used to study the gravitational perturbations of the orbit of Uranus. Discrepancies found between the observed and theoretical orbit of Uranus seemed to indicate that an undiscovered planet beyond Uranus was the cause. The planet's actual position was predicted by Adams and Le Verrier (independently of each other's work) [11]. The planet, Neptune, was subsequently discovered at the Berlin Observatory by Galle near that position [12]. The discovery of Neptune represents a great testament to the power of Newton's laws and to science.

IV. STARS AND GALAXIES

A look at the night sky provides a strong impression of a changeless universe. We know that clouds drift across the Moon, the sky rotates around the polar star, and on longer times, the Moon itself grows and shrinks and the Moon and planets move against the background of stars. Of course we know that these are merely local phenomena caused by motions within our solar system. Far beyond the planets, the stars appear motionless. Indeed we have seen that according to the ancient cosmological belief, the celestial objects, except for a few that appeared to move (the planets), where fixed on a sphere beyond the last planet. In today's class we are going to see that this impression of changelessness is illusory.

The astronomical distances are so large that we specify them in terms of the time it takes the light to travel a given distance. For example,

$$\text{one light second} = 3 \times 10^8 \text{m} = 300,000 \text{ km}\,, \tag{11}$$

$$\text{one light minute} = 1.8 \times 10^7 \text{ km}\,, \tag{12}$$

and

$$\text{one light year} = 1 \text{ ly} = 9.46 \times 10^{15} \text{ m} \approx 10^{13} \text{ km}. \tag{13}$$

For specifying distances to the Sun and the Moon, we usually use meters or kilometers, but we could specify them in terms of light. The Earth-Moon distance is 384,000 km, which is 1.28 ls. The Earth-Sun distance is $150,000,000$ km; this is equal to 8.3 lm. Far out in the solar system, Pluto is about 6×10^9 km from the Sun, or 6×10^{-4} ly. The nearest star to us, Proxima Centauri, is about 4.2 ly away. Therefore, the nearest star is 10,000 times farther from us that the outer reach of the solar system.

On clear moonless nights, thousands of stars with varying degrees of brightness can be seen, as well as the long cloudy strip known as the Milky Way. Galileo first observed with his telescope that the Milky Way is comprised of countless numbers of individual stars. A half century later Wright suggested that the Milky Way was a flat disc of stars extending to great distances in a plane, which we call the Galaxy [13].

Our Galaxy has a diameter of 100,000 ly and a thickness of roughly 2,000 ly. It has a bulging central *nucleus* and spiral arms. Our Sun, which seems to be just another star, is located half way from the Galactic center to the edge, some $26,000$ ly from the center. The Sun orbits the Galactic center approximately once every 250 million years or so, so its speed is

$$v = \frac{2\pi \quad 26,000 \times 10^{13} \text{ km}}{2.5 \times 10^8 \text{ yr } 3.156 \times 10^7 \text{ s/yr}} = 200 \text{ km/s}\,. \tag{14}$$

The total mass of all the stars in the Galaxy can be estimated using the orbital data of the Sun about the center of the Galaxy. To do so, assume that most of the mass is concentrated near the center of the Galaxy and that the Sun and the solar system (of total mass m) move in a circular orbit around the center of the Galaxy (of total mass M),

$$\frac{GMm}{r^2} = m\frac{v^2}{r}\,, \tag{15}$$

where we recall that $a = v^2/r$ is the centripetal acceleration. All in all,

$$M = \frac{r\,v^2}{G} \approx 2 \times 10^{41} \text{ kg}\,. \tag{16}$$

Assuming all the stars in the Galaxy are similar to our Sun ($M_\odot \approx 2 \times 10^{30}$ kg), we conclude that there are roughly 10^{11} stars in the Galaxy.

In addition to stars both within and outside the Milky Way, we can see with a telescope many faint cloudy patches in the sky which were once all referred to as *nebulae* (Latin for clouds). A few of these, such as those in the constellations of Andromeda and Orion, can actually be discerned with the naked eye on a clear night. In the XVII and XVIII centuries, astronomers found that these objects were getting in the way of the search for comets. In 1781, in order to provide a convenient list of objects not to look at while hunting for comets, Messier published a celebrated catalogue [14]. Nowadays astronomers still refer to the 103 objects in this catalog by their Messier numbers, e.g., the Andromeda Nebula is M31.

Even in Messier's time it was clear that these extended objects are not all the same. Some are star clusters, groups of stars which are so numerous that they appeared to be a cloud. Others are glowing clouds of gas or dust and it is for these that we now mainly reserve the word nebula. Most fascinating are those that belong to a third category: they often have fairly regular elliptical shapes and seem to be a great distance beyond the Galaxy. Kant seems to have been the first to suggest that these latter might be circular discs, but appear elliptical because we see them at an angle, and are faint because they are so distant [15]. At first it was not universally accepted that these objects were extragalactic (i.e. outside our Galaxy). In 1920, Sir Hubble's observations revealed that individual stars could be resolved within these extragalactic objects and that many contain spiral arms [16]. The distance to our nearest spiral galaxy, Andromeda, is over 2 million ly, a distance 20 times greater than the diameter of our Galaxy. It seemed logical that these nebulae must be galaxies similar to ours. Today it is thought that there are roughly 4×10^{10} galaxies in the observable universe – that is, as many galaxies as there are stars in the Galaxy.

V. ASTRONOMICALLY FAR AWAY

Last class we have been talking about the vast distance of the objects in the universe. Today, we will discuss different methods to estimate these distances.

How can we measure the distance to an object we cannot reach? The answer to this question is simple: use triangles. The small triangle in Fig. 9 has the same shape as the large one. Then, by measuring the two sides of the small triangle and the short side of the big triangle, we can calculate the length of the long side of the big triangle, i.e.,

$$\frac{B}{A} = \frac{b}{a} \Rightarrow B = \frac{A \times b}{a} . \tag{17}$$

So, how can we measure the distance to stars? Take two telescopes some distance apart and observe the same star. Measure the tilt between the two telescopes, which sets all the angles for the triangles; see Fig. 10. Then we can find the distance to the star from the distance

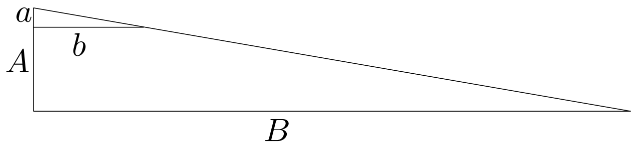

FIG. 9: Triangle proportionality.

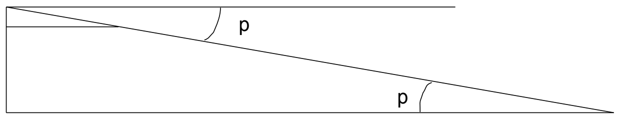

FIG. 10: Observing a star with two telescopes.

between the telescopes and the angle of the tilt. To this end, we want to use the largest distance we can for the short side of the big triangle. Now, what is the largest distance we can get between the two telescopes (if both of them have to be on Earth - no spacecraft)? The largest distance is not obtained by placing the two telescopes at opposite ends of the Earth. Instead, we can use one telescope and just let the earth move.

This basic method to measure distances to nearby stars employs simple geometry and stellar parallax. Parallax is the apparent displacement of an object because of a change in the observer's point of view. One way to see how this effect works is to hold your hand out in front of you and look at it with your left eye closed, then your right eye closed. Your hand will appear to move against the background. By stellar parallax we mean the apparent motion of a star against the background of more distant stars, due to Earth's motion around the Sun; see Fig. 11. The sighting angle of a star relative to the plane of Earth's orbit can be determined at two different times of the year separated by six months. Since we know the distance d from the Earth to the Sun, we can determine the distance D to the star. For example, if the parallax angle of a star is measured to be $p = 0.00006°$, using trigonometry $\tan p = d/D$, and since the distance to the Sun is $d = 1.5 \times 10^8$ km = 1 AU the distance to the star is

$$D = \frac{d}{\tan p} \approx \frac{d}{p} = \frac{1.5 \times 10^8 \text{ km}}{1 \times 10^{-6}} = 1.5 \times 10^{14} \text{ km} , \tag{18}$$

or about 15 ly.

Distances to stars are often specified in terms of parallax angles given in seconds of arc: 1 second (1″) is 1/60 of a minute (1′) of arc, which is 1/60 of a degree, so 1″ = 1/3600 of a degree. The distance is then specified in parsecs (meaning *par*allax angle in *sec*onds of arc), where the parsec is defined as $1/p$ with p in seconds. For example, if $p = 6 \times 10^{-5\,\circ}$, we would say that the star is at a distance $D = 4.5$ pc. We can also use the parallax to determine distance to the bright star Vega. Vega has a measured parallax of 0.1 arcsec ($p = 0.1''$). This

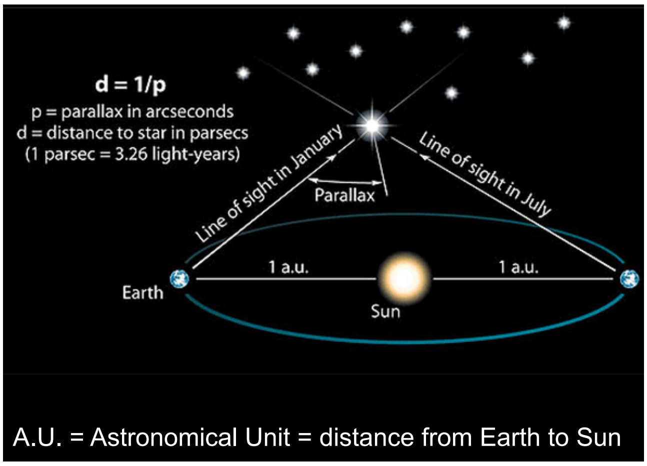

FIG. 11: The parallax method of measuring a star's distance.

means that Vega appears to move from +0.1″ to −0.1″ with respect to distant stars over a year of observation: $D(\mathrm{pc}) = 1/p('') = 1/0.1 = 10$ pc. Vega is 10 pc (parsec) from Earth (about 32.6 light years).

To understand the other method used for estimating long distances in space, we first need to introduce a few general concepts adopted to describe the starlight observed on Earth.

A star produces light: the total amount of energy that a star puts out as light each second is called its Luminosity. If we have a light detector (eye, camera, telescope) we can measure the light produced by the star: the total amount of energy intercepted by the detector divided by the area of the detector is called the Flux. To find the Luminosity, we take a shell which completely encloses the star and measure all the light passing through the shell. To find the Flux, we take our detector at some particular distance from the star and measure the light passing only through the detector. How bright a star looks to us is determined by its Flux, not its Luminosity, i.e. Brightness = Flux.

The Flux decreases as we get farther from the star like $1/\mathrm{Distance}^2$, see Fig. 12. Mathematically, if we have two stars A and B

$$\frac{\mathrm{Flux}_A}{\mathrm{Flux}_B} = \frac{\mathrm{Luminosity}_A}{\mathrm{Luminosity}_B} \times \left(\frac{\mathrm{Distance}_B}{\mathrm{Distance}_A}\right)^2 . \quad (19)$$

Now we can look more careful at the Distance-Luminosity relation. Assume the Luminosity of A is twice the Luminosity of B and the Distance to A is twice the Distance to B. Which star appears brighter to the observer? We have that

$$\frac{\mathrm{Luminosity}_A}{\mathrm{Luminosity}_B} = 2 \quad (20)$$

and

$$\frac{\mathrm{Distance}_B}{\mathrm{Distance}_A} = \frac{1}{2}, \quad (21)$$

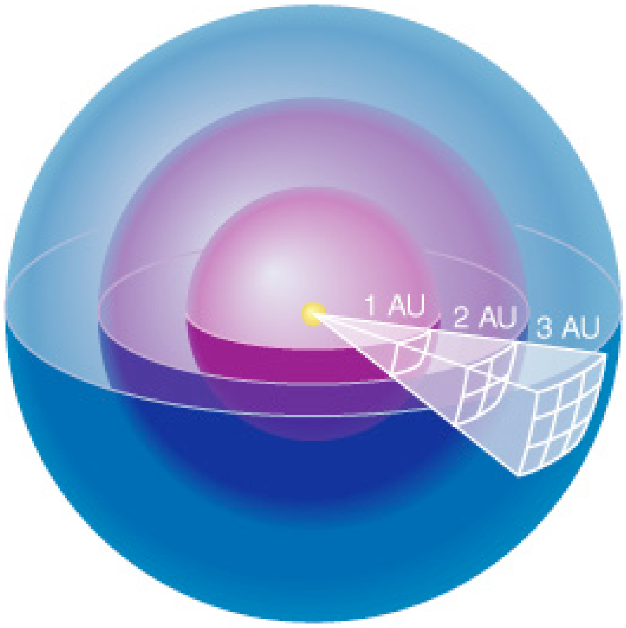

FIG. 12: The inverse square law.

hence

$$\begin{aligned}\frac{\mathrm{Flux}_A}{\mathrm{Flux}_B} &= \frac{\mathrm{Luminosity}_A}{\mathrm{Luminosity}_B} \left(\frac{\mathrm{Distance}_B}{\mathrm{Distance}_A}\right)^2 \\ &= 2 \times \left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}. \end{aligned} \quad (22)$$

This means that the nearest star is the brightest on Earth.

How do we measure the brightness of stars? Ptolemy (150 B.C.E.) grouped stars into 6 *magnitude* groups according to how bright they looked to his eye. In the 1800s, Herschel first measured the brightness of stars quantitatively and matched his measurements onto Ptolemy's magnitude groups and assigned a number for the magnitude of each star. In Herschel's system, if a star is 1/100 as bright as another then the dimmer star has a magnitude 5 higher than the brighter one. Note that dimmer objects have higher magnitudes.

The magnitude of a star gives it brightness or flux when observed from Earth. To talk about the properties of star, independent of how far they happen to be from Earth, we use *absolute magnitude*. Absolute magnitude is the magnitude that a star would have viewed from a distance of 10 parsecs. The absolute magnitude is directly related to the luminosity of the star.

VI. CLASSIFYING STARS: THE HERTZSPRUNG-RUSSELL DIAGRAM

Stars are so small compared to their distance to us that we almost never have the resolution to see their sizes and details directly. They are usually described as "point sources." We deduce everything by measuring the amount of light (brightness) which is emitted at different wavelengths (color, spectra).

The rate at which an object radiates energy has been found to be proportional to the fourth power of the Kelvin temperature T and to the area A of the emitting object, i.e.,

$$\text{emitted radiation} = \sigma A T^4\,, \tag{23}$$

where $\sigma = 5.670 \times 10^{-5}\ \text{erg}\,\text{cm}^{-2}\,\text{K}^{-4}\,\text{s}^{-1}$ is the Stefan-Boltzmann constant [17, 18]. At normal temperatures (≈ 300 K) we are not aware of this electromagnetic radiation because of its low intensity. At higher temperatures, there is sufficient infrared radiation that we can feel heat if we are close to the object. At still higher temperatures (on the order of 1000 K), objects actually glow, such as a red-hot electric stove burner. At temperatures above 2000 K, objects glow with a yellow or whitish color, such as the filament of a lightbulb.

Last class we have seen that a useful parameter for a star or galaxy is its luminosity (hereafter L), by which we mean the total power radiated in watts, and is given by the product of its surface area and the radiation emitted per unit area,

$$L = 4\pi R^2 \sigma T^4\,, \tag{24}$$

where R is the radius of the star (or galaxy). In addition, we have seen that also important is the flux of energy (hereafter $\mathcal{F}$), defined as the power crossing unit area at the Earth perpendicular to the path of light. Given that energy is conserved and ignoring any absorption in space, the total emitted power L when it reaches a distance D from the star will be spread over a sphere of surface area $4\pi D^2$. If D is the distance from the star to Earth, then

$$\mathcal{F} = \frac{L}{4\pi D^2}\,. \tag{25}$$

Careful analyses of nearby stars have shown that the absolute luminosity for most of the stars depends on the mass: *the more massive the star, the greater the luminosity*.

Another important parameter of a star is its surface temperature, which can be determined from the spectrum of electromagnetic frequencies it emits.[5] The wavelength at the peak of the spectrum, $\lambda_{\rm max}$, is related to the temperature by Wien's displacement law [19]

$$\lambda_{\rm max} T = 2.9 \times 10^{-3}\ \text{m K}\,. \tag{26}$$

We can now use Wien's law and the Steffan-Boltzmann equation (power output or luminosity $\propto AT^4$) to determine the temperature and the relative size of a star. Suppose that the distance from Earth to two nearby stars

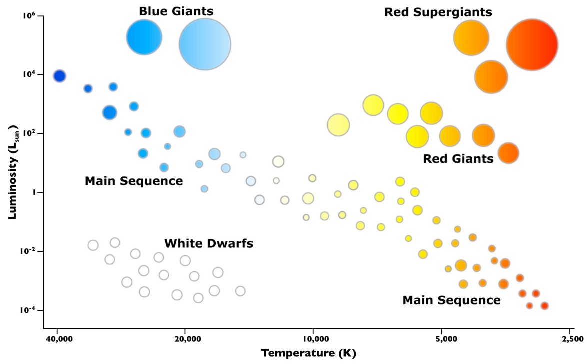

FIG. 13: HR diagram. The vertical axis depicts the inherent brightness of a star, and the horizontal axis the surface temperature increasing from right to left.

can be reasonably estimated, and that their apparent luminosities suggest the two stars have about the same absolute luminosity, L. The spectrum of one of the stars peaks at about 700 nm (so it is reddish). The spectrum of the other peaks at about 350 nm (bluish). Using Wien's law, the temperature of the reddish star is $T_{\rm red} \simeq 4140$ K. The temperature of the bluish star will be double because its peak wavelength is half, $T_{\rm blue} \simeq 8280$ K. The power radiated per unit of area from a star is proportional to the fourth power of the Kelvin temperature (24). Now the temperature of the bluish star is double that of the redish star, so the bluish must radiate 16 times as much energy per unit area. But we are given that they have the same luminosity, so the surface area of the blue star must be 1/16 that of the red one. Since the surface area is $4\pi R^2$, we conclude that the radius of the redish star is 4 times larger than the radius of the bluish star (and its volume 64 times larger).

An important astronomical discovery, made around 1900, was that for most of the stars, the color is related to the absolute luminosity and therefore to the mass. A useful way to present this relationship is by the so-called Hertzsprung-Russell (HR) diagram [20, 21]. On the HR diagram, the horizontal axis shows the temperature T, whereas the vertical axis the luminosity L, each star is represented by a point on the diagram shown in Fig. 13. Most of the stars fall along the diagonal band termed the main sequence. Starting at the lowest right, we find the coolest stars, redish in color; they are the least luminous and therefore low in mass. Further up towards the left we find hotter and more luminous stars that are whitish like our Sun. Still farther up we find more massive and more luminous stars, bluish in color. There are also stars that fall outside the main sequence. Above and to the right we find extremely large stars, with high luminosity but with low (redish) color temperature: these are called red giants. At the lower left, there are a few stars of low luminosity but with high temperature: these are white dwarfs.

Suppose that a detailed study of a certain star suggests

[5] An electromagnetic wave can be characterized by its frequency ν and its wavelength λ, they are related by $\lambda = c/\nu$, where c is the speed of light.

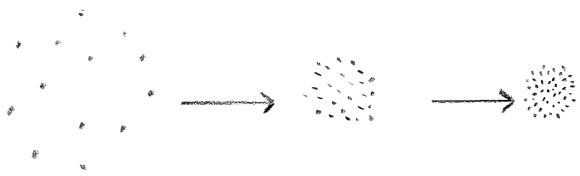

FIG. 14: Contraction of a gas cloud. As the collapse due to gravitational attraction proceeds it speeds up. The gas cloud heats up, in the center most of all. Nuclear fusion starts. The contraction stops and a balance is established between pressure in the gas cloud and gravity.

that it most likely fits on the main sequence of the HR diagram. The observed flux is $\mathcal{F} = 1\times10^{-12}$ W m^{-2}, and the peak wavelength of its spectrum is $\lambda_{\rm max} \approx 600$ nm. We can first find the temperature using Wien's law and then estimate the absolute luminosity using the HR diagram; namely, $T \approx 4800$ K. A star on the main sequence of the HR diagram at this temperature has absolute luminosity of about $L \approx 10^{26}$ W. Then, using (25) we can estimate its distance from us, $D = 3 \times 10^{18}$ m or equivalently 300 ly.

VII. STELLAR EVOLUTION

§ In the beginning $\cdots$! there is a huge cloud of hydrogen and helium, about a trillion miles in diameter (this is a million times the sun's diameter). If the cloud has a mass greater than about 10^{30} lb, the attractive forces due to gravitation will be sufficient to overcome the dispersive effects of the random motion of the atoms in the cloud, and the cloud will begin to contract; see Fig. 14.

As the cloud contracts, one can think of all the atoms falling in toward the center, and just like any other kind of falling, they pick up speed. Their speed gets randomized through collisions, so that the net effect is a large increase in the temperature (like the rise in temperature of a falling brick when it strikes the ground).

§ Formation of the plasma. After about 10 million (10^7) years of contraction, the hydrogen cloud shrunk to a diameter of 100 million miles ($100D_\odot$, or 100 times the diameter of the sun). This is like shrinking the entire Lehman campus down to an inch. The temperature of the cloud has gone from about 100°K (representing an average speed of 1 mile per second for the hydrogen atoms) to about 50,000°K (representing an average speed of 20 miles per second). The density of the cloud is about $\frac{1}{1000}$ of the density of air. At this speed and density the collisions are frequent enough (each atom collides about a billion times/second) and violent enough to ionize all the atoms in the cloud (that is, remove the electrons from the atoms). So at this stage, the star-to-be (called a protostar) consists of gas of positively charged protons and helium nuclei, and negatively charged electrons in equal balance. This kind of hot gas is called a *plasma*.

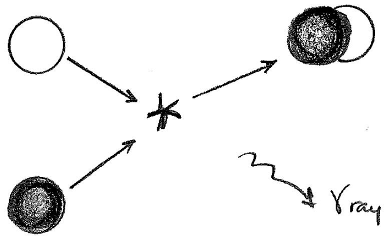

FIG. 15: $p + n \to D + \gamma$.

§ Further contraction. The plasma continues to contract under the influence of the gravitational force, getting hotter and denser. The temperature rises because the surface area of the star is not large enough *in relation to its volume* to get rid of the heat as it is produced. After another 10 million years the plasma has shrunk by another factor of 100 to the size of the sun ($D_\odot$ = 1 million miles), the density near the center has risen to 10 million °K (corresponding to an average speed of 280 miles per second for the protons). At this stage the plasma is contracting so quickly that is about one hour from a total collapse into a point. But in the nick of time, we get $\cdots$

§ Nuclear ignition. Nuclei stay together in spite of the strong repulsive electric forces between the protons. This is because there is a much stronger attractive nuclear force which acts like a glue between protons and protons (or p's and n's, or n's and p's): namely, when they get close enough to touch, the attraction due to the nuclear force overwhelms any electrical repulsion which may exist (like between p and p) [22]. It is also a property of elementary particles (like p's and n's) that they never sit still – they are always jiggling and moving about. This tends to greatly weaken the binding due to the glue. In the case of neutrons touching protons, the glue holds, the binding is stable, and we get a deuteron; see Fig. 15. If the Sun were to contain large numbers of neutrons, it would burn up immediately into deuterium.

For the case of protons touching other protons, the glue is slightly weakened by the presence of the electrical repulsion, and the net attractive force is just insufficient to overcome the jiggling and bind the pp to form ^{2}He. So in our star, the protons will collide, hang around each other a little, but no binding, no γ-ray, no "burning."

§ However, enters the weak force! There is another short range interaction between the nucleons which is extremely weak, and is generally completely masked by the strong force. This is called the weak interaction [23]. If two protons did bind, you would never know about the weak force. But they don't, so instead, once in every 10 billion collisions, the protons stick together to form a deuteron. But a deuteron has a charge +1, and the two

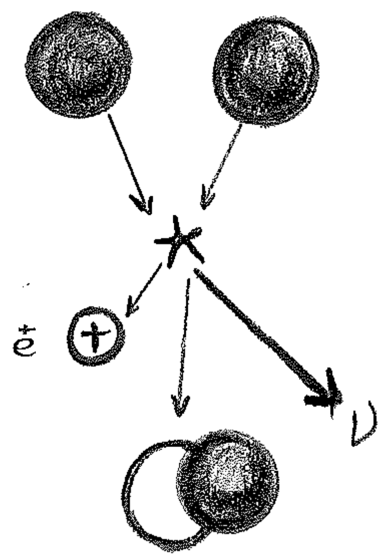

FIG. 16: $p + p \rightarrow D + e^+ + \nu$.

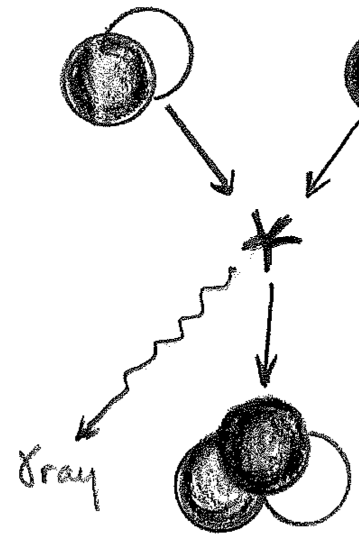

FIG. 17: $D + p \rightarrow {}^3\text{He} + \gamma$.

protons have charge +2, so we need something to carry off the extra positive charge: the carrier is a positron, a positive electron e^+. In addition to the positron, a neutrino, carries off a little of the energy and leaves the Sun.

The reaction shown in Fig. 16 is very unlikely for two reasons: *(i)* At 10 million °K, the protons get close enough to touch only very rarely: the electric repulsion is not fully overcome at this temperature. *(ii)* Even when they do touch the fusion reaction is so weak that it only occurs in a million times. Altogether, to see how unlikely the whole thing is, if you had a lb of hydrogen at the center of the sun, it would take 10 billion years for half of it to burn into deuterium.

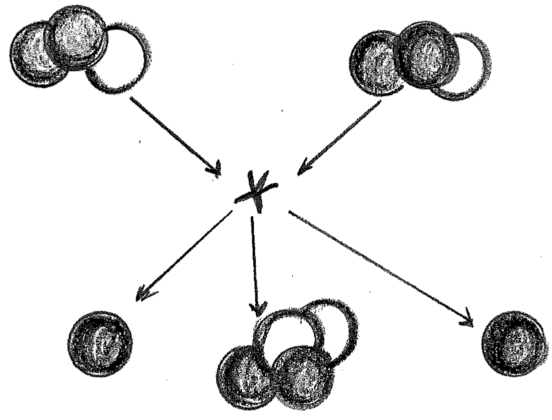

FIG. 18: ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$.

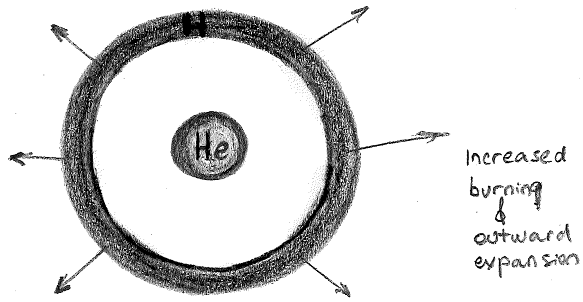

FIG. 19: Outward expansion into the red giant phase.

§ Cooking up the helium isotopes. We left off with the process $p^+ + p^+ \rightarrow D^+ + e^+ + \nu$. This is not the end of this sequence, however. The deuterium, in its many collisions with protons, undergoes rapid conversion to ^{3}He (see Fig. 17), and the ^{3}He's collide to form He^4 and two protons. The last reaction, shown in Fig. 18, proceeds slowly because the two ^{3}He, each having charge +2, have a hard time getting close to one another, in order to undergo their nuclear reaction.

The net effect of this sequence, which is called the *pp*-cycle, is for four protons to combine to form one ${}^4_2$He nucleus, plus two positrons, two neutrinos, and two gamma rays.[6] Now, nothing new happens for a long time except for the accumulation of helium at the center of the star.

§ Red giant phase. As the helium accumulates, it starts undergoing gravitational contraction, heating up

[6] The theory of the *pp* cycle as the source of energy for the Sun was first worked out by Bethe [24]. Interestingly, the same set of nuclear reactions that supply the energy of the Sun's radiation also produce neutrinos that can be searched for in the laboratory [25, 26].

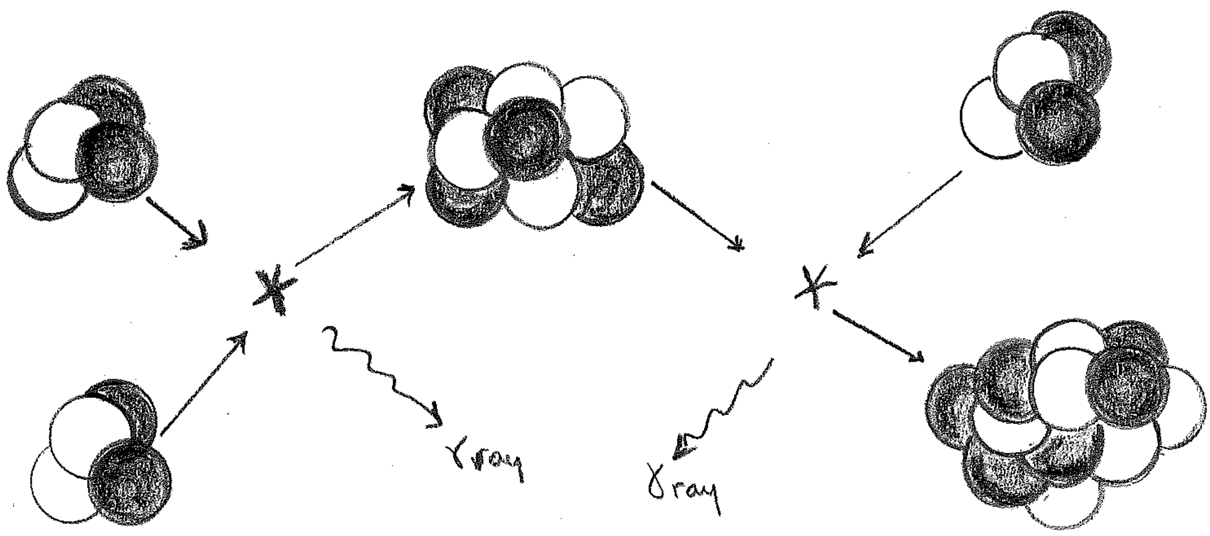

FIG. 20: ^{4}He + ^{4}He → ^{8}Be + γ and ^{4}He + ^{8}Be → ^{12}C + γ.

in the process (just like the hydrogen did before it). This heating and contracting proceeds at a rather rapid rate (over a period of tens of millions of years) leading to a great increase in temperature of the hydrogen surrounding the helium core, and a consequent acceleration of its burning. It also expands outward as shown in Fig. 19.

The end result is that the star becomes huge, the core becomes small and dense (20,000 miles across, 1 ton per in^3). The large size of the star allows the surface to remain "cool" (i.e., only red hot) so that the star at this stage is visible as a "red giant." An example is Betelgeuse (pronounced "beetle juice") which is visible as Orion's left shoulder in the night spring sky, looking south.

§ **Burning of the helium.** When the temperature of the core reaches 100 million °K, the helium begins to burn, undergoing the reaction

$$^4\text{He} + {^4\text{He}} \rightarrow {^8\text{Be}} \quad \text{(beryllium-8)}\,. \tag{27}$$

Now, there is another of those "just so" accidents. ^{8}Be is not stable; although it can exist for a short time, the nucleons out of which it is made do not attract each other strongly enough, and in less than 10^{-12} s it breaks up into separate helium nuclei again:

$$^8\text{Be} \rightarrow^4 \text{He} +^4 \text{He}\,. \tag{28}$$

However, at the temperature and density of the helium core, each helium nucleus undergoes about 10^{12} collisions per second. So it is not unlikely that ^{8}Be will be hit by another helium-4 before it breaks apart; see Fig. 20. This leads to the formation of carbon, and a γ-ray (which is energy).

§ **On to the white dwarf.** So now a carbon core begins to form. The heat generated by these interactions and the subsequent collapse of the carbon core lead to a blow-off of most of the outer gas layers. The star has become a hot carbon core, about 20,000 miles across, with a density of 10 tons per in^3. The electron gas exerts enough outward pressure to keep the carbon core from contracting much in diameter, if the star is less than one and a half times as massive as the Sun. But the inward pressures still generate a great deal of heat, and the carbon star glows with a white heat. This is a white dwarf. After a few million years, the dwarf cools some, becomes yellow, then red, then cools completely and the fire goes out. The star has met its death as a massive, dense lump of coal.

§ **Heavy stars.** An entirely different end awaits the 5% of stars whose masses exceed 1.5 times the mass of the sun [27]. For such masses, the inward gravitational pressure of the carbon core can generate a temperature sufficiently large (600 million °K) to ignite the carbon. The carbon burns to form magnesium and other elements.

The pressure and temperature mount, nuclear reactions continue, until finally the *iron* is reached. At this point, the process stops, because iron is a very special element. Any reaction that takes place involving an iron nucleus will *use up* energy. The iron, instead of providing more fuel to burn, puts the fire out. The center of the star commences to collapse again, but this time, because of the presence of the iron nuclei, the fire cannot be rekindled; it has gone out for the last time, and the entire star commences its final collapse.

The collapse is a catastrophic event. The materials of the collapsing star pile up at the center, creating exceedingly high temperatures and pressures. Finally, when all the nuclei are pressed against each other , the star can be compressed no further. The collapsed star, compressed like a giant spring, rebounds instantly in a great explosion. About half the material within the star disperses into space, the other half remains as a tiny core (about 10 miles in diameter), containing a mass equal to that of the Sun. This is now a *pulsar* or *neutron star* (for reason to be explained soon). The entire episode (collapse and explosion) lasts a few minutes.

The exploding star is called a supernova. The most famous supernova was recorded by Chinese astronomers in A.D. 1054, and its remnants are now visible as the Crab Nebula.

At the very high temperatures generated in the collapse and explosion, some of the nuclei in the star are broken up, and many neutrons and protons freed. These are captured by other nuclei, building up heavier elements, such as silver, gold, and uranium. In this way the remaining elements of the periodic table, extending beyond iron, are *perhaps* manufactured in the final moments of the star's life [28]. Because the time available for making these elements is so brief, they never become as abundant as the elements up to including iron.

The core of the collapsed star then contracts until all the nuclei are touching, and then stops. The forces are so great that all the nuclei (iron, etc.) disintegrate into their constituents (neutrons and protons); the protons combine with electrons to leave a dense core of neutrons – a nucleus about as large as Boston [29]. This object, a neutron star, rotates madly about its axis, emitting energy at a billion times the rate at which the sun does so. We see it in the sky as a pulsar.

If the mass of the neutron star is greater than about three solar masses, then the star further contracts under gravity and eventually collapses to the point of zero volume and infinite density [30]. As the density increases, the paths of light rays emitted from the star are bent and eventually wrapped irrevocably around the star. The "star" with infinite density is completely enclosed by a boundary known as the event horizon, inside which the gravitational force of the star is so strong that light cannot escape [31]. This is called a *black hole*, because no light escapes the event horizon.

VIII. THE BIG BANG THEORY

Sir Hubble also observed a persistent redshift in the spectra of known elements and that the shift was greater the greater the distance of the galaxy from the Earth. It was Hubble himself who explained the redshift as indicating that distant galaxies were radially moving away from the Earth [32]. In every direction, these vast accumulations of stars and interstellar matter were moving outward at enormous speeds. He called this motion, recession. He showed that the velocity of recession was greater at greater distances. Hubble's law of cosmic expansion states that an observer *at any point in the universe* will observe distant galaxies receding from him/her with *radial velocities V proportional to their distance d from the observer*,

$$V = H_0\, d \tag{29}$$

where H_0 is the Hubble's proportionality constant. Hubble's initial determination of H_0 was approximately 160 km/s per million-light-years. Most recent observations indicate that $H_0 \approx 22.4$ km/s per million-light-years [33–36].

Hubble's law is consistent with a general expansion of the space between galaxies (or galactic clusters), and is not a particular characteristic of the galaxies (clusters) themselves. This statement means that the galaxies themselves are not changing in any way; only the regions between them are expanding with time. If the expansion is run backward (as can be done with mathematics), then it would appear that, very long ago, all the matter of the universe was once compacted into a relatively small volume from which it was hurled outward by some titanic force. This idea is the basis for the hot *Big Bang* model [37–40].

Appendix A: Scientific Notation

Scientific notation refers to expressing a number as a product of any number between 1 and 10 to the 10th power. Scientific notation is mostly used when dealing with large quantities or numbers containing many digits since it shortens the notation, see Tables I and II.

To write a number in scientific notation:

- If the number is in decimal notation, move the decimal point to the right of its original position and place it after the first non-zero digit. The exponent of 10 will be the number of places the original decimal point was moved, and it will be negative since it was moved to the right. For example, the number 0.0000643 can be written as 6.43×10^{-5}.

- If the number to be changed to scientific notation is a whole number greater than 10, move the decimal point to the left of its original position and place it after the first digit. The exponent of 10 will be the number of places the original decimal point was moved, and it will be positive since it was moved to the left. For example, the number $125,000$ can be written as 1.25×10^{5}.

As an illustration, we estimate the volume of solar

TABLE I: Powers of 10 are shorthand for writing large numbers.

$10^0 = 1$	One
$10^1 = 10$	Ten (deca)
$10^2 = 100$	Hundred (centa)
$10^3 = 1,000$	Thousand (kilo)
$10^6 = 1,000,000$	Million (mega)
$10^9 = 1,000,000,000$	Billion (giga)
$10^{12} = 1,000,000,000,000$	Trillion (tera)
$10^{15} = \cdots$	Quadrillion (peta)
$10^{54} = \cdots$	Septendecillion

TABLE II: Powers of 10 also work for very small numbers.

$10^0 = 1$	One
$10^{-1} = 0.1$	One tenth (deci)
$10^{-2} = 0.01$	One hundredth (centi)
$10^{-3} = 0.001$	One thousandth (milli)
$10^{-6} = 0.000,001$	One millionth (micro)
$10^{-9} = 0.000,000,001$	One billionth (nano)
$10^{-12} = 0.000,000,000,001$	One trillionth (pico)
$10^{-15} = \cdots$	One quadrillionth (femto)
$10^{-54} = \cdots$	One septendecillionth

system taken up by stuff:

$$\begin{aligned} V_{\rm stuff} &= \frac{\text{volume of Sun}}{\text{volume of solar system}} \\ &= \frac{(\text{radius of Sun})^3}{(\text{radius of Pluto's orbit})^3} \\ &= \frac{\left(7\times 10^8\right)^3}{(6\times 10^{12})^3} \\ &= \frac{7^3\times 10^{3\times 8}}{6^3\times 10^{3\times 12}} \\ &= \frac{7^3}{6^3}\times 10^{3\times 8-3\times 12} \\ &= 1.6\times 10^{-12}\,, \end{aligned} \tag{A1}$$

which is about a millionth of a millionth. Recall that the volume of a sphere = $(4/3)\pi r^3$, where r is its radius.

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