Sample Final Exam – MAT 176

(11/4/2014)

This exam should be taken without text, notes or electronic devices. Additional material from the syllabus may be included.

1. (18pts.) Evaluate the indefinite integrals (find the general antiderivatives), and check by differentiating:
   (a) $\int (8x^3 + \frac{1}{x^2})\ dx =$
   (b) $\int\ te^t\ dt =$
   (c) $\int\ \frac{\cos\theta}{\sin^5\theta}\ d\theta =$
2. (5pts.) Find a formula for the derivative $F'(x)$ of the function $F(x)$ defined by:
$$F(x) = \int_0^x \frac{1}{\sqrt{1+t^3}}\ dt$$
   And evaluate $F'(x)$ at $x = 2$.
3. (12pts.) Evaluate the definite integrals:
   (a) $\int_0^{\pi/2}\ (\theta + 3\sin\theta)\ d\theta =$
   (b) $\int_0^2\ x\sqrt{4-x^2}\ dx =$
4. (5pts.) Set up an integral which equals the area of the region R in the xy-plane bounded by the curves $y = \sqrt{x}$ and $y = x^2$; do not evaluate the integral.
5. (5pts.) Set up an integral which equals the volume of the solid formed by rotating the region R in the previous problem around the x-axis; do not evaluate the integral.
6. (10pts.) Evaluate or show divergence:
   (a) $\int_1^\infty\ \frac{1}{\sqrt{x}}\ dx$
   (b) $\int_0^\infty\ e^{-y}\ dy$
7. (15pts.) Compute the limit of the sequence, or show divergence:
   (a) $\lim_{k\to\infty}\ \frac{e^k}{k^2}$
   (b) $\lim_{n\to\infty}\ \frac{\cos n}{n}$
   (c) $\lim_{n\to\infty}\ \sum_{k=0}^n \frac{3}{2^k}$
8. (5pts.) Determine if the series converges or diverges:
   (a) $\sum_{n=1}^\infty\ \frac{\sqrt{n}+100}{n^2+1}$
   (b) $\sum_{n=0}^\infty\ \frac{n^2-1}{n^2+1}$
9. (5pts.) Find the interval of convergence of the power series:
$$\sum_{n=2}^\infty \frac{5(x-2)^n}{n-1}$$
10. (5pts.) Find the interval of convergence of the power series:
$$\sum_{n=0}^\infty \frac{2^n}{n!}x^n$$
11. (5pts.) Write down the degree 4 Maclaurin polynomial $P_4(x) = \sum_{k=0}^4 \frac{f^{(k)}(0)}{k!}\ x^k$ for the function $f(x) = 3 + \cos x$.
12. (5pts.) Use the Maclaurin series $e^x = \sum_{n=0}^\infty \frac{x^n}{n!}$ to find the Maclaurin series for the function $g(x) = 2e^{3x}$.
13. (5pts.) Integrate the Maclaurin series $\frac{1}{1+x} = \sum_{n=0}^\infty (-1)^n x^n$ to find the Maclaurin series for $L(x) = \ln(1+x)$.